# Parametric Equations and Vectors For Calculus

## Parametric Equations & Formulas for Calculus

If a smooth curve $C$ is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of $C$ at the point $(x, y)$ is given by $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ where $\frac{dx}{dt} \neq 0$, and the second derivative is given by

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] \cdot \frac{dt}{dx} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}.$$

## Parametric Arc Length

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ is the length of the arc from $t = a$ to $t = b$

## Displacement & Distance Traveled

Suppose a particle moves along a path in the plane so that its velocity at any time $t$ is $\vec{v}(t) = (x'(t), y'(t))$, then the **displacement** from $t = a$ to $t = b$ is given by the vector

$$\left\langle \int_a^b x'(t)\,dt, \int_a^b y'(t)\,dt \right\rangle.$$

The preceding vector is added to the position at time $t = a$ to get the **position** at time $t = b$.

The **distance traveled** from $t = a$ to $t = b$ is the arc length

$$\int_a^b \left|\vec{v}(t)\right| dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt.$$

**Horizontal and Vertical Velocity Component VECTORS**

- $x'(t) = \frac{dx}{dt}$ is the rate at which the $x$-coordinate is changing with respect to $t$ or the velocity of a particle in the horizontal direction.

- $y'(t) = \frac{dy}{dt}$ is the rate at which the $y$-coordinate is changing with respect to $t$ or the velocity of a particle in the vertical direction.

- $\vec{s} = \langle x(t), y(t) \rangle = (x(t), y(t))$ is the position at any time $t$.

- $\vec{v} = \langle x'(t), y'(t) \rangle = (x'(t), y'(t))$ is the velocity vector at any time $t$.

- $\vec{a} = \langle x''(t), y''(t) \rangle = (x''(t), y''(t))$ is the acceleration vector at any time $t$.

  *note: the vectors may or may not be contained within the chevrons $\langle \ \rangle$.

- $\frac{dy}{dx}$ is the rate of change of $y$ with respect to $x$ or the slope of the tangent line to the curve or the slope of the position vector.

- $\frac{d^2y}{dx^2}$ is the rate of change of the slope of the curve with respect to $x$.

- $|\vec{v}(t)| = \|\vec{v}(t)\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ is the **<u>speed of a particle</u>** or the **<u>magnitude</u>** or the **<u>length</u>** or the **<u>norm</u>** of the velocity vector.

- $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ is the **length of the arc** from $t = a$ to $t = b$ or the **distance traveled** by a particle from $t = a$ to $t = b$.

*Remember that $\int_a^b |v(t)|\, dt$ is the total distance traveled, whether it be along a straight line or curve.