

21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \leq t \leq 1$, is given by

- (A) $\int_0^1 \sqrt{t^2 + 1} dt$
(B) $\int_0^1 \sqrt{t^2 + t} dt$
(C) $\int_0^1 \sqrt{t^4 + t^2} dt$
(D) $\frac{1}{2} \int_0^1 \sqrt{4 + t^4} dt$
(E) $\frac{1}{6} \int_0^1 t^2 \sqrt{4t^2 + 9} dt$

77. If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

- (A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$
(D) $(e^{-t}, \cos t)$ (E) $(e^{-t}, -\cos t)$

2. In the xy -plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \leq t \leq 3$, is a line segment with slope

- (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

10. A particle moves on a plane curve so that at any time $t > 0$ its x -coordinate is $t^3 - t$ and its y -coordinate is $(2t - 1)^3$. The acceleration vector of the particle at $t = 1$ is

- (A) $(0, 1)$ (B) $(2, 3)$ (C) $(2, 6)$ (D) $(6, 12)$ (E) $(6, 24)$

21 C The length of this parametric curve is given by $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^1 \sqrt{(t^2)^2 + t^2} dt$.

77. E $f'(t) = (-e^{-t}, -\sin t)$; $f''(t) = (e^{-t}, -\cos t)$.

2. A $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = 3 \Rightarrow \frac{dy}{dx} = \frac{3}{5}$

10. E $v(t) = (3t^2 - 1, 6(2t - 1)^2)$ and $a(t) = (6t, 24(2t - 1)) \Rightarrow a(1) = (6, 24)$

15. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \leq t \leq \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t} dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t} dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t} dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t} dt$

18. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only

(B) 1 only

(C) 0 and $\frac{2}{3}$ only

(D) 0, $\frac{2}{3}$, and 1

(E) No value

15. D $x = \cos^3 t, y = \sin^3 t$ for $0 \leq t \leq \frac{\pi}{2}$. $L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

$$L = \int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2} dt = \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} dt$$

18. C $x = t^3 - t^2 - 1, y = t^4 + 2t^2 - 8t; \quad \frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t} = \frac{4t^3 + 4t - 8}{t(3t - 2)}$. Vertical tangents at $t = 0, \frac{2}{3}$

25. Consider the curve in the xy -plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is
- (A) 20.086 (B) 0.342 (C) -0.005 (D) -0.011 (E) -0.033
28. If a particle moves in the xy -plane so that at time $t > 0$ its position vector is $(\ln(t^2 + 2t), 2t^2)$, then at time $t = 2$, its velocity vector is
- (A) $\left(\frac{3}{4}, 8\right)$ (B) $\left(\frac{3}{4}, 4\right)$ (C) $\left(\frac{1}{8}, 8\right)$ (D) $\left(\frac{1}{8}, 4\right)$ (E) $\left(-\frac{5}{16}, 4\right)$
2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$
- (A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

25. D At $t = 3$, slope $= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-te^{-t} + e^{-t}}{e^t} = \frac{1-t}{e^{2t}}$ $\Big|_{t=3} = -\frac{2}{e^6} = -0.005$

28. A $v = \left(\frac{2t+2}{t^2+2t}, 4t \right)$, $v(2) = \left(\frac{6}{8}, 8 \right) = \left(\frac{3}{4}, 8 \right)$

2. E $x = e^{2t}$, $y = \sin(2t)$; $\frac{dy}{dx} = \frac{2\cos(2t)}{2e^{2t}} = \frac{\cos(2t)}{e^{2t}}$

6. If $x = t^2 + 1$ and $y = t^3$, then $\frac{d^2y}{dx^2} =$

- (A) $\frac{3}{4t}$ (B) $\frac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\frac{3}{2}$

23. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

- (A) $\int_0^4 \sqrt{4t+1} dt$
(B) $2\int_0^4 \sqrt{t^2+1} dt$
(C) $\int_0^4 \sqrt{2t^2+1} dt$
(D) $\int_0^4 \sqrt{4t^2+1} dt$
(E) $2\pi \int_0^4 \sqrt{4t^2+1} dt$

34. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is

- (A) $y = 2x$ (B) $y = 8x$ (C) $y = 2x - 1$
(D) $y = 4x - 5$ (E) $y = 8x + 13$

4. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

- (A) $-\frac{5}{2}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{4}{5}$ (E) $\frac{6}{5}$

6. A $\frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2$ thus $\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3}{2}t; \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{3}{2}}{2t} = \frac{3}{4t}$

23. D $L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^4 \sqrt{4t^2 + 1} dt$

34. C At $t = 1$, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3 + 4t}{3t^2 + 1} \Big|_{t=1} = \frac{8}{4} = 2$; the point at $t = 1$ is $(2, 3)$. $y = 3 + 2(x - 2) = 2x - 1$

4. B If $x = 2$ then $y = 5$. $x \frac{dy}{dt} + y \frac{dx}{dt} = 0; 2(3) + 5 \cdot \frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{6}{5}$

4. A particle moves in the xy -plane so that at any time t its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^3$. At $t = 1$, its acceleration vector is

(A) $(0, -1)$ (B) $(0, 12)$ (C) $(2, -2)$ (D) $(2, 0)$ (E) $(2, 8)$

15. For any time $t \geq 0$, if the position of a particle in the xy -plane is given by $x = t^2 + 1$ and $y = \ln(2t + 3)$, then the acceleration vector is

(A) $\left(2t, \frac{2}{(2t+3)}\right)$ (B) $\left(2t, \frac{-4}{(2t+3)^2}\right)$ (C) $\left(2, \frac{4}{(2t+3)^2}\right)$
(D) $\left(2, \frac{2}{(2t+3)^2}\right)$ (E) $\left(2, \frac{-4}{(2t+3)^2}\right)$

1. The asymptotes of the graph of the parametric equations $x = \frac{1}{t}$, $y = \frac{t}{t+1}$ are

(A) $x = 0, y = 0$ (B) $x = 0$ only (C) $x = -1, y = 0$
(D) $x = -1$ only (E) $x = 0, y = 1$

22. A particle moves on the curve $y = \ln x$ so that the x -component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln \frac{5}{2}\right)$
(D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln \frac{3}{2}\right)$

4. D $x(t) = t^2 - 1 \Rightarrow \frac{dx}{dt} = 2t$ and $\frac{d^2x}{dt^2} = 2$; $y(t) = t^4 - 2t^3 \Rightarrow \frac{dy}{dt} = 4t^3 - 6t^2$ and $\frac{d^2y}{dt^2} = 12t^2 - 12t$
 $a(t) = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right) = (2, 12t^2 - 12t) \Rightarrow a(1) = (2, 0)$

15. E $x = t^2 + 1$, $\frac{dx}{dt} = 2t$, $\frac{d^2x}{dt^2} = 2$; $y = \ln(2t+3)$, $\frac{dy}{dt} = \frac{2}{2t+3}$; $\frac{d^2y}{dt^2} = -\frac{4}{(2t+3)^2}$

1. C For horizontal asymptotes consider the limit as $x \rightarrow \pm\infty$: $t \rightarrow 0 \Rightarrow y = 0$ is an asymptote
For vertical asymptotes consider the limit as $y \rightarrow \pm\infty$: $t \rightarrow -1 \Rightarrow x = -1$ is an asymptote

22. C $x'(t) = t+1 \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + C$ and $x(0) = 1 \Rightarrow C = \frac{1}{2} \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + \frac{1}{2}$

$$x(1) = \frac{5}{2}, \quad y(1) = \ln \frac{5}{2}; \quad \left(\frac{5}{2}, \ln \frac{5}{2} \right)$$