Strategy for Testing Series

MAT V1102 – 004

Test	Description	Examples and Comments
geometric series	$\sum_{n=0}^{\infty} r^n$ converges to $1/(1-r)$ if $\lvert r\rvert < 1$ and diverges if $\lvert r\rvert \geq 1$.	$\sum_{n=0}^{\infty}(1/2)^n$ converges to 2; $\sum_{n=0}^{\infty} 2^n$ diverges.
limit test	If $\lvert a_n\rvert$ does not converge to 0, then $\sum a_n$ diverges.	If $a_n \to 0$, then $\sum a_n$ may converge ($\sum 1/n^2$) or it may not (the harmonic series $\sum 1/n$).
p-series	$\sum_{n=1}^{\infty} 1/n^p$ diverges if $0 \leq p \leq 1$ and converges if $p > 1$.	
integral test	Suppose $a_n = f(n) \geq 0$ and f is decreasing. If $\int_1^{\infty} f(x)dx$ converges then $\sum a_n$ converges. If $\int_1^{\infty} f(x)dx$ diverges then $\sum a_n$ diverges.	Use this test whenever $f(x)$ can easily be integrated.
comparison test	Suppose $0 \leq a_n \leq b_n$. If $\sum b_n$ converges then $\sum a_n$ also converges. If $\sum a_n$ diverges, then $\sum b_n$ also diverges.	It is not necessary that $a_n \leq b_n$ for all n, only for $n \geq N$ for some integer N; convergence or divergence of a series is not affected by the values of the first few terms.
ratio test	If $\lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert} = L$ then $\sum a_n$ converges if $L < 1$ and diverges when $L > 1$.	This is often the easiest test to apply; note that if $L = 1$, then the series may either converge ($\sum 1/n^2$) or diverge ($\sum 1/n$).
alternating series test	$\sum(-1)^n a_n$ converges if (i) $a_n \to 0$ as $n \to \infty$ and (ii) $0 \leq a_{n+1} < a_n$.	This test can only be applied when the terms are alternately positive and negative; if there are two or more positive (or negative) terms in a row, then try another test.
absolute convergence test	If $\sum \lvert a_n\rvert$ converges, then $\sum a_n$ converges.	To determine whether $\sum \lvert a_n\rvert$ converges, try any of the tests that apply to series with nonnegative terms.

Determine which of the following series converge.

1. $\displaystyle\sum_{n=1}^{\infty}(-5)^{-n}$

2. $\displaystyle\sum_{n=5}^{\infty}\frac{1}{n(n+6)}$

3. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{50n}$

4. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n\sqrt{n}}{\ln n}$

5. $\displaystyle\sum_{n=1}^{\infty}\frac{r^n}{n^r}\quad 0<r<1$

6. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n(n-1)}}$

7. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}\ln(n)}{n^3+2}$

8. $\displaystyle\sum_{n=2}^{\infty}\frac{e^{1/n}}{n^{3/2}}$

9. $\displaystyle\sum_{n=3}^{\infty}\frac{(-1)^n(n+2)(n+3)}{(n+1)^3}$

10. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n^n}{n!}$

11. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n!}{n^n}$

12. $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{n!}$

13. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2 n!}{(2n)!}$

14. $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{n!+2}$

15. $\displaystyle\sum_{n=2}^{\infty}\frac{n}{(\ln n)^n}$

16. $\displaystyle\sum_{n=1}^{\infty}\frac{n^6 5^n}{(n+1)!}$

17. $\displaystyle\sum_{n=2}^{\infty}\frac{e^n}{(\ln n)^n}$

18. $\displaystyle\sum_{n=3}^{\infty}\frac{\sin(n\pi/7)}{n^3}$

19. $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$

20. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(1+\frac{1}{n}\right)^n$

21. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}$

22. $\displaystyle\sum_{n=3}^{\infty}\frac{\ln n}{\ln(\ln n)}$

23. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)(n+2)}}$

24. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\sqrt{n^2-1}}$

25. $\displaystyle\sum_{n=1}^{\infty}\frac{3(-1)^{n+1}\sqrt{n+1}}{\sqrt{n}+1}$

26. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\ln\left(1+\frac{1}{n}\right)$

27. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}$

28. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^3-5}$

29. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2+2n+1}$

30. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{(2n)!}{2^n n! n}$

31. $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^{n+1}}{n+5^n}$

32. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sin n}{n^2}$

33. $\displaystyle\sum_{n=1}^{\infty}\frac{2}{1+e^n}$

34. $\displaystyle\sum_{n=1}^{\infty}n\sin\left(\tfrac{1}{n}\right)$

35. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}\quad p>1$

36. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$