AP Calculus AB Semester 1 Practice Final

1. Find the limit (if it exists).

$$\lim_{x \to 5} \frac{(\sqrt{x+4}-3)}{(x-5)} \cdot \frac{(\sqrt{x+4}+3)}{(\sqrt{x+4}+3)} \rightarrow \lim_{x \to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x+4}+3)} = \lim_{x \to 5} \frac{1}{\sqrt{x+4}+3} = \frac{1}{6}$$

a. 6 b. 1 c. 0 d. $\frac{1}{6}$ e. Limit does not exist.

or L' Hospital

2. Determine the limit (if it exists).

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\sin^4 x}{x^3} = \lim_{x \to 0} \underbrace{\frac{\sin x}{x}}_{1} \cdot \underbrace{\frac{\sin x}{x}}_{1} \cdot \underbrace{\frac{\sin x}{x}}_{1} \cdot \underbrace{\frac{\sin x}{1}}_{0}$$

a. 1 b. 0 c. 2 d. ∞ e. does not exist

3. Find $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ where $f(x)=4x-3$.

a. 1 b. 4 c. -3 d. 0 e. Limit does not exist.

this is the limit definition of the derivative

so $f'(x) = 4$

the actual work:

$$= \lim_{\Delta x \to 0} \frac{4(x+\Delta x)-3-(4x-3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{4x}+4\Delta x \cancel{-3} \cancel{-4x} \cancel{+3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4\cancel{\Delta x}}{\cancel{\Delta x}} = \boxed{4}$$

4. Find the x-values (if any) at which the function $f(x) = 13x^2 - 15x - 15$ is not continuous. Which of the discontinuities are removable?

a. $x = 4$, removable b. $x = 0$, removable

c. $x = \dfrac{15}{26}$, not removable.

d. continuous everywhere e. $x = \dfrac{15}{26}$, removable.

polynomials are continuous and differentiable everywhere

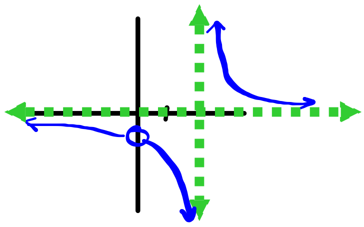
5. Find the x-values (if any) at which $f(x) = \dfrac{x}{x^2 - 2x}$ is not continuous.

a. $f(x)$ is not continuous at $x = 0$ and $f(x)$ has a removable discontinuity at $x = 0$.
b. $f(x)$ is not continuous at $x = 0, 2$ and both the discontinuities are nonremovable.
c. $f(x)$ is not continuous at $x = 2$ and $f(x)$ has a removable discontinuity at $x = 2$.
d. $f(x)$ is not continuous at $x = 0, 2$ and $f(x)$ has a removable discontinuity at $x = 0$.
e. $f(x)$ is continuous for all real x.

$$f(x) = \frac{x}{x(x-2)} \rightarrow$$

$x \neq 0$	$x \neq 2$
hole	vert. asy.
removable	non-removable

$$= \frac{1}{x-2}$$



6. Find the x-values (if any) at which the function $f(x) = \dfrac{x+2}{x^2 + 6x + 8}$ is not continuous. Which of the discontinuities are removable?

a. no points of discontinuity
b. $x = -2$ (not removable), $x = -4$ (removable)
c. $x = -2$ (removable), $x = -4$ (not removable)
d. no points of continuity
e. $x = -2$ (not removable), $x = -4$ (not removable)

$$\frac{\cancel{x+2}}{\cancel{(x+2)}(x+4)}$$

$x \neq -2$ removable

$x \neq -4$ non-removable

7. Find the constant a such that the function

$$f(x)=\begin{cases}-4\cdot\dfrac{\sin x}{x}, & x<0\\ a+7x, & x\ge 0\end{cases}$$

is continuous on the entire real line.

a. 1 b. -7 c. 7 d. 4 e. -4

$\lim_{x\to 0}\frac{\sin x}{x}=1$

$-4(1)=a+7(0)$

$a=-4$

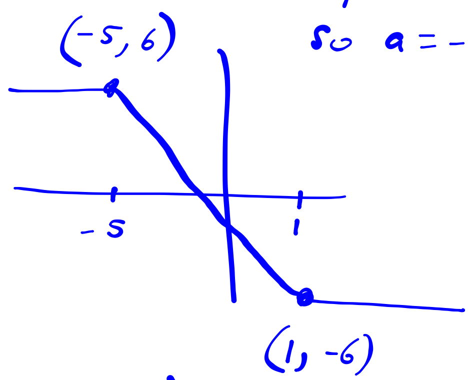
8. Find the constants a and b such that the function

$$f(x)=\begin{cases}6, & x\le -5\\ ax+b, & -5<x<1\\ -6, & x\ge 1\end{cases}$$

is continuous on the entire real line.

a. $a=2,b=0$ b. $a=2,b=-4$
c. $a=-2,b=-4$ d. $a=-2,b=4$
e. $a=2,b=4$

slope is -2
so $a=-2$



$y-6=-2(x--5)$

$y=-2x-4$

so $b=-4$

9. Find the value of c guaranteed by the Intermediate Value Theorem.

$f(x) = x^2 - 2x + 8, [2,6], f(c) = 11$

a. 0 b. 3 c. 5 d. 1 e. 4

$11 = x^2 - 2x + 8$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3$ ~~$x = -1$~~

interval $[2, 6]$

10. Find all values of c such that f is continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} 4 - x^2, & x \le c \\ x, & x > c \end{cases}$$

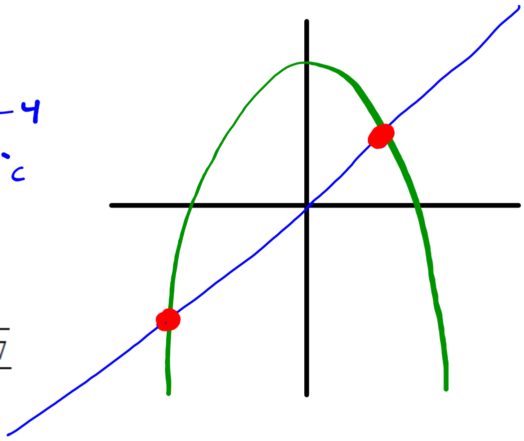
$4 - x^2 = x$

$0 = x^2 + x - 4$

use quadratic formula

a. $c = 3$ b. $c = 0$ c. $\dfrac{-1+\sqrt{17}}{2}$

d. $\dfrac{1+\sqrt{17}}{2}, \dfrac{1-\sqrt{17}}{2}$ e. $\dfrac{-1+\sqrt{17}}{2}, \dfrac{-1-\sqrt{17}}{2}$



11. Find the vertical asymptotes (if any) of the function

$f(x) = \dfrac{x^2 - 4}{x^2 + 3x + 2}$.

$$f(x) = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x+1)}$$

a. $x = 2$ b. $x = -1$ c. $x = 1$ d. $x = -2$
e. $x = -2$

$x \neq -2$ hole

$x \neq -1$ vertical asymptote

12. Find an equation of the tangent line to the graph of the function $f(x) = \sqrt{x - 2}$ at the point $(18, 4)$.

a. $y = \frac{x}{4} + \frac{7}{2}$ b. $y = \frac{x}{8} + \frac{7}{4}$ c. $y = \frac{x}{8} + \frac{9}{2}$
d. $y = \frac{x}{4} + \frac{9}{2}$ e. $y = \frac{x}{8} + \frac{9}{4}$

$$f'(x) = \frac{1}{2\sqrt{x-2}}$$

$$f'(18) = \frac{1}{2\sqrt{18-2}} = \frac{1}{8}$$

slope

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{8}(x - 18)$$

$$y = \frac{1}{8}x - \frac{9}{4} + 4$$

$$y = \frac{1}{8}x - \frac{9}{4} + \frac{16}{4}$$

$$y = \frac{1}{8}x + \frac{7}{4}$$

13. Find an equation of the line that is tangent to the graph of f and parallel to the given line.

$f(x) = 3x^3$, $9x - y + 9 = 0$

a. $y = -9x + 6$ b. $y = -3x + 6$ c. $y = 9x - 3$ and $y = 9x + 3$ d. $y = -9x - 6$ e. $y = 9x - 6$ and $y = 9x + 6$

$y = 9x + 9$
$m = 9$

$f'(x) = 9x^2$
$9 = 9x^2$
$x = \pm 1$

$f(1) = 3$ $(1, 3)$
$f(-1) = -3$ $(-1, -3)$

$y - 3 = 9(x - 1)$ and $y + 3 = 9(x + 1)$
$y = 9x - 6$ $y = 9x + 6$

14. Find the derivative of the function $f(x) = -4x^2 - 4\cos(x)$.

a. $f'(x) = -4x + 4\sin(x)$
b. $f'(x) = -8x + 4\sin(x)$
c. $f'(x) = -8x + 4\cos(x)$
d. $f'(x) = -8x - 4\sin(x)$
e. $f'(x) = -8x - 4\cos(x)$

15. Find the derivative of the function
$f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x$.

a. $f'(x) = \frac{2}{3x^{4/3}} - 3\sin x$

b. $f'(x) = -\frac{2}{3x^{4/3}} - 3\sin x$

c. $f'(x) = -\frac{2}{3x^{4/3}} + 3\sin x$

d. $f'(x) = -\frac{2}{3x^{3/4}} - 3\sin x$

e. $f'(x) = -\frac{2}{3x^{3/4}} + 3\sin x$

$f(x) = 2x^{-\frac{1}{3}} + 3\cos x$

$f'(x) = -\frac{2}{3}x^{-\frac{4}{3}} - 3\sin x$

$f'(x) = -\frac{2}{4x^{\frac{4}{3}}} - \sin x$

16. Suppose the position function for a free-falling object on a certain planet is given by $s(t) = -16t^2 + v_0 t + s_0$. A silver coin is dropped from the top of a building that is 1372 feet tall. Determine the average velocity of the coin over the time interval $[3, 4]$.

a. –113 ft/sec b. 80 ft/sec c. 112 ft/sec
d. –112 ft/sec e. –80 ft/sec

$= \frac{s(4) - s(3)}{4 - 3}$

$= \frac{-16(4^2) - -16(3^2)}{1}$

$= -16(16 - 9)$

$= -16(7)$

$= -112$

17. Suppose the position function for a free-falling object on a certain planet is given by $s(t) = -12t^2 + v_0 t + s_0$. A silver coin is dropped from the top of a building that is 1372 feet tall. Find the instantaneous velocity of the coin when $t = 4$.

a. –96 ft/sec b. –32 ft/sec c. –20 ft/sec
d. –144 ft/sec e. –48 ft/sec

$v(t) = -24t + v_0$

$v_0 = 0$

$v(t) = -24t$

$v(4) = -24(4) = -96$

18. The volume of a cube with sides of length s is given by $V = s^3$. Find the rate of change of volume with respect to s when $s = 6$ centimeters.

a. 648 cm² b. 216 cm² c. 36 cm² d. 108 cm²
e. 72 cm²

$\frac{dV}{ds} = ?$ when $s = 6$

$\frac{dV}{ds} = 3s^2$

$\left.\frac{dV}{ds}\right|_{s=6} = 3(6)^2 = 108$

19. Find $\frac{dy}{dx}$ by implicit differentiation.

$x^2 + 5x + 9xy - y^2 = 4$

a. $\frac{dy}{dx} = \frac{2x-5+9y}{2y-9x}$ b. $\frac{dy}{dx} = \frac{2x+5+9y}{2x-9y}$

c. $\frac{dy}{dx} = \frac{x+5+9y}{y-9x}$ d. $\frac{dy}{dx} = \frac{2x+5+9y}{2y-9x}$

e. $\frac{dy}{dx} = \frac{2x+5-9y}{2y-9x}$

$2x + 5 + 9(y + xy') - 2yy' = 0$

$2x + 5 + 9y + 9xy' - 2yy' = 0$

$y'(9x - 2y) = -2x - 5 - 9y$

$y' = \frac{-1(-2x - 5 - 9y)}{-1(9x - 2y)}$

$y' = \frac{2x + 5 + 9y}{2y - 9x}$

20. Use implicit differentiation to find an equation of the tangent line to the ellipse $\frac{x^2}{2} + \frac{y^2}{200} = 1$ at $(1, 10)$.

a. $y = -9x + 20$ b. $y = -5x + 14$

c. $y = -10x + 20$ d. $y = -3x + 14$

e. $y = -6x + 20$

$100x^2 + y^2 = 200$

$200x + 2yy' = 0$

$2yy' = -200x$

$y' = -\frac{200x}{2y}$

$\frac{dy}{dx} = -\frac{100x}{y}$

$\left.\frac{dy}{dx}\right|_{(1,10)} = -\frac{100}{10}$

$y - y_1 = m(x - x_1)$

$y - 10 = -10(x - 1)$

$y = -10x + 20$

21. Find $\frac{d^2y}{d^2x}$ in terms of x and y given that

$3x^2 + 3y^2 = 8$. Use the original equation to simplify your answer.

$x^2+y^2=\frac{8}{3}$

a. $y'' = -8y^3$ b. $y'' = -9y^3$ c. $y'' = -3y^3$

d. $y'' = -\frac{8}{3y^3}$ e. $y'' = -\frac{8}{9y^3}$

$6x+6yy'=0$

$x+yy'=0$

$y'=\frac{-x}{y}$

$y''=\frac{(-1)(y)-(y')(-x)}{y^2}$

$y''=\frac{-y+\left(\frac{-x}{y}\right)x}{y^2}\cdot\frac{y}{y}$

$y''=-\frac{y^2+x^2}{y^3}$

$y''=-\frac{8}{3}\left(\frac{1}{y^3}\right)$

$y''=-\frac{8}{3y^3}$

22. Find $\frac{d^2y}{dx^2}$ in terms of x and y given that $8-9xy=8x-9y$.

a. $\frac{d^2y}{dx^2}=0$ b. $\frac{d^2y}{dx^2}=-9y^2$ c. $\frac{d^2y}{dx^2}=-9y^3$

d. $\frac{d^2y}{dx^2}=8y^2$ e. $\frac{d^2y}{dx^2}=-8y^3$

$9y-9xy=8x-8$

$y(9-9x)=8x-8$

$y=\frac{8x-8}{9-9x}$

$y=\frac{8x-8}{9-9x}$

$y=\frac{8\cancel{(x-1)}}{-9\cancel{(x-1)}}$

$y=-\frac{8}{9}$

$y'=0$

$\boxed{y''=0}$

23. A point is moving along the graph of the function $y = \sin 6x$ such that $\frac{dx}{dt} = 2$ centimeters per second. Find $\frac{dy}{dt}$ when $x = \frac{\pi}{7}$.

a. $\frac{dy}{dt} = 6\cos\frac{2\pi}{7}$ b. $\frac{dy}{dt} = 12\cos\frac{6\pi}{7}$

c. $\frac{dy}{dt} = 6\cos\frac{6\pi}{7}$ d. $\frac{dy}{dt} = 12\cos\frac{2\pi}{7}$

e. $\frac{dy}{dt} = 12\cos\frac{12\pi}{7}$

$$\frac{d}{dt}[y] = \frac{d}{dt}[\sin 6x]$$

$$\frac{dy}{dt} = \cos 6x\left(6\frac{dx}{dt}\right)$$

$$\frac{dy}{dt} = \cos\left(6\left(\frac{\pi}{7}\right)\right)(6\cdot 2)$$

$$\frac{dy}{dt} = 12\cos\frac{6\pi}{7}$$

24. All edges of a cube are expanding at a rate of 5 centimeters per second. How fast is the volume changing when each edge is 2 centimeters?

a. 20 cm^3 / sec b. 150 cm^3 / sec

c. 40 cm^3 / sec d. 60 cm^3 / sec

e. 50 cm^3 / sec

$$V = S^3 \qquad \frac{dS}{dt} = 5$$

$$\frac{dV}{dt} = 3S^2\frac{dS}{dt}$$

$$\left.\frac{dV}{dt}\right|_{S=2} = 3(2)^2(5) = 60$$

25. The radius r of a sphere is increasing at a rate of 6 inches per minute. Find the rate of change of the volume when $r = 11$ inches.

a. $\frac{dV}{dt} = 3630\pi \text{ in}^3 / \text{min}$

b. $\frac{dV}{dt} = 1452\pi \text{ in}^3 / \text{min}$

c. $\frac{dV}{dt} = 2904\pi \text{ in}^3 / \text{min}$

d. $\frac{dV}{dt} = \frac{1}{1452\pi} \text{ in}^3 / \text{min}$

e. $\frac{dV}{dt} = \frac{1}{2904\pi} \text{ in}^3 / \text{min}$

$\frac{dr}{dt} = 6 \qquad V = \frac{4}{3}\pi r^3$

$\left.\frac{dV}{dt}\right|_{r=11} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{r=11} = 4\pi(11)^2 6$

26. A spherical balloon is inflated with gas at the rate of 500 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 centimeters?

a. $\frac{dr}{dt} = 54\pi \text{cm/min}$ b. $\frac{dr}{dt} = \frac{5}{18\pi} \text{cm/min}$

c. $\frac{dr}{dt} = \frac{5}{54\pi} \text{cm/min}$ d. $\frac{dr}{dt} = \frac{5}{36\pi} \text{cm/min}$

e. $\frac{dr}{dt} = 36\pi \text{cm/min}$

$\frac{dV}{dt} = 500 \qquad \left.\frac{dr}{dt}\right|_{r=30} = ?$

$V = \frac{4}{3}\pi r^3$

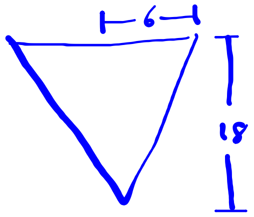
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$500 = 4\pi(30)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{500}{3600\pi} = \frac{5}{36\pi}$

27. A conical tank (with vertex down) is 12 feet across the top and 18 feet deep. If water is flowing into the tank at a rate of 18 cubic feet per minute, find the rate of change of the depth of the water when the water is 10 feet deep.

a. $\frac{9}{40\pi}$ ft/min b. $\frac{9}{100\pi}$ ft/min c. $\frac{81}{20\pi}$ ft/min
d. $\frac{81}{50\pi}$ ft/min e. $\frac{81}{200\pi}$ ft/min



$\frac{dv}{dt} = 18$

$\left.\frac{dh}{dt}\right|_{h=10} = ?$

$V = \frac{1}{3}\pi r^2 h$ $\qquad \frac{1}{3}h = r$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{\pi}{27}h^3$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

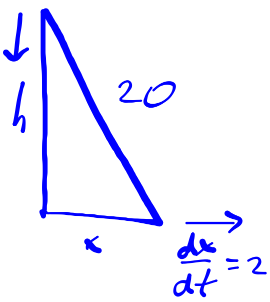
$18 = \frac{\pi}{9}(10)^2\frac{dh}{dt}$

$18 = \frac{100\pi}{9}\frac{dh}{dt}$

$\frac{81}{50\pi} = \frac{dh}{dt}$

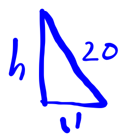
28. A ladder 20 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when its base is 11 feet from the wall? Round your answer to two decimal places.

a. –5.33 ft/sec b. 3.00 ft/sec c. –9.00 ft/sec
d. 5.33 ft/sec e. –1.32 ft/sec



$\frac{dx}{dt} = 2$

$\left.\frac{dh}{dt}\right|_{x=11}$



$h = \sqrt{400-121}$

$20^2 = x^2 + h^2$

$0 = \cancel{2}x\frac{dx}{dt} + \cancel{2}h\frac{dh}{dt}$

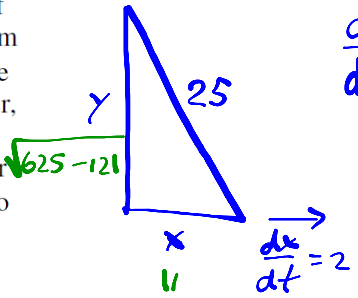
$0 = (11)(2) + \left(\sqrt{400-121}\right)\frac{dh}{dt}$

29. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changed when the base of the ladder is 11 feet from the wall. Round your answer to two decimal places.

a. 16.67 ft^2/sec b. 119.10 ft^2/sec

c. 66.34 ft^2/sec d. 20.06 ft^2/sec

e. 40.13 ft^2/sec



$\left.\frac{dA}{dt}\right|_{x=11} = ?$

oof! we need $\frac{dy}{dt}$

$25^2 = x^2 + y^2$

$0 = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$

$0 = 11(2) + \sqrt{504}\frac{dy}{dt}$

$\frac{dy}{dt} = -0.98$

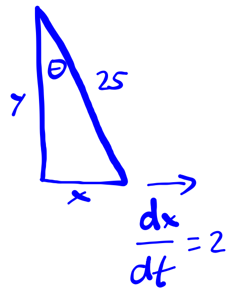
$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}y + \frac{dy}{dt}x\right)$

$\frac{dA}{dt} = \frac{1}{2}\left(2(\sqrt{504}) + (-.98)(11)\right) = 17.06 \text{ ft}^2/s$

30. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 9 feet from the wall. Round your answer to three decimal places.

a. 0.320 rad/sec b. 0.090 rad/sec c. 0.109 rad/sec d. 2.254 rad/sec e. 2.234 rad/sec



$y = \sqrt{25^2 - 9^2}$

$\left.\frac{d\theta}{dt}\right|_{x=9} = ?$

$\sin\theta = \frac{x}{25}$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{25}(2)\frac{1}{\cos\theta}$

$\frac{d\theta}{dt} = \frac{2}{25}\left(\frac{1}{\frac{y}{25}}\right)$

$\frac{d\theta}{dt} = \frac{2}{\cancel{25}}\left(\frac{\cancel{25}}{\sqrt{25^2-9^2}}\right)$

$\frac{d\theta}{dt} \approx .0857$

31. Find any critical numbers of the function $g(t) = t\sqrt{2-t}$, $t < 2$.

a. 0 b. $-\frac{2}{3}$ c. $-\frac{4}{3}$ d. $\frac{2}{3}$ e. $\frac{4}{3}$

$$g'(t) = \sqrt{2-t} + t\left(\frac{-1}{2\sqrt{2-t}}\right)$$

$$0 = \sqrt{2-t} - \frac{t}{2\sqrt{2-t}}$$

$$\frac{t}{2\sqrt{2-t}} = \sqrt{2-t}$$

cross multiply

$$t = 2(2-t)$$

$$t = 4 - 2t$$

$$3t = 4$$

$$t = \frac{4}{3}$$

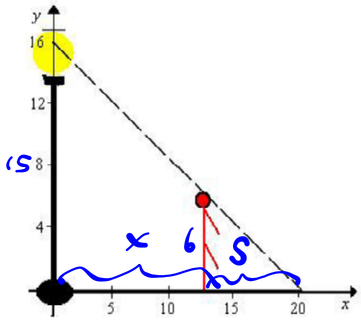
32. An airplane is flying in still air with an airspeed of 255 miles per hour. If it is climbing at an angle of 21°, find the rate at which it is gaining altitude. Round your answer to four decimal places.

a. 103.7178 mi/hr b. 78.7993 mi/hr
c. 111.7846 mi/hr d. 92.4589 mi/hr
e. 91.3838 mi/hr

$$\sin 21° = \frac{\frac{dh}{dt}}{255}$$

$$\frac{dh}{dt} = 255 \sin 21° \approx 91.383$$

33. A man 6 feet tall walks at a rate of 10 feet per second away from a light that is 15 feet above the ground (see figure). When he is 13 feet from the base of the light, at what rate is the tip of his shadow moving?



a. $\frac{1}{2}$ ft/sec b. 50 ft/sec c. $\frac{3}{50}$ ft/sec d. $\frac{9}{2}$ ft/sec e. $\frac{50}{3}$ ft/sec

$\frac{dx}{dt} = 10$

$\frac{S}{6} = \frac{x+S}{15}$

$15S = 6x + 6S$

$9S = 6x$

$3S = 2x$

$3\frac{dS}{dt} = 2\frac{dx}{dt}$

$\frac{dS}{dt} = \frac{20}{3}$

$= \left.\frac{dx}{dt}\right|_{x=13} + \left.\frac{dS}{dt}\right|_{x=13}$

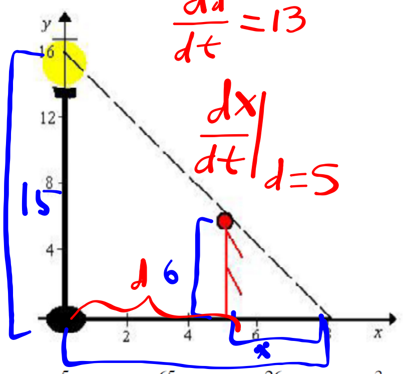
$= 10 + \frac{20}{3}$

$= \frac{50}{3}$ ft/s

34. A man 6 feet tall walks at a rate of 13 feet per second away from a light that is 15 feet above the ground (see figure). When he is 5 feet from the base of the light, at what rate is the length of his shadow changing?

$\frac{dd}{dt} = 13$

$\left.\frac{dx}{dt}\right|_{d=5}$



a. $\frac{5}{2}$ ft/sec b. $\frac{65}{3}$ ft/sec c. $\frac{26}{3}$ ft/sec d. $\frac{3}{65}$ ft/sec e. $\frac{1}{2}$ ft/sec

$x + d$

$\frac{x}{x+d} = \frac{6}{15}$

$15x = 6x + 6d$

$9x = 6d$

$3x = 2d$

$3\frac{dx}{dt} = 2\frac{dd}{dt}$

$\frac{dx}{dt} = \frac{2}{3}(13)$

35. Locate the absolute extrema of the function $f(x) = x^3 - 12x$ on the closed interval $[0, 4]$.

a. absolute max: $(2, -16)$; absolute min: $(4, 16)$
b. no absolute max; absolute min: $(4, 16)$
c. absolute max: $(4, 16)$; absolute min: $(2, -16)$
d. absolute max: $(4, 16)$; no absolute min e. no absolute max or min

check crit numbers and endpoints

$f'(x) = 3x^2 - 12$

$0 = 3x^2 - 12$

$x = \pm 2$

$f(0) = 0$

$f(2) = -16$ min

$f(4) = 16$ max

36. Determine whether Rolle's Theorem can be applied to $f(x) = -x^2 + 10x$ on the closed interval $[0, 10]$. If Rolle's Theorem can be applied, find all values of c in the open interval $(0, 10)$ such that $f'(c) = 0$.

a. Rolle's Theorem applies; $c = 6$, $c = 5$
b. Rolle's Theorem applies; $c = 4$, $c = 6$
c. Rolle's Theorem applies; $c = 5$ d. Rolle's Theorem applies; $c = 4$ e. Rolle's Theorem does not apply

continuous ✓
differentiable ✓
$f(a) = f(b)$

$f(0) = 0 = f(10)$ ✓

$f'(x) = -2x + 10$

$0 = -2x + 10$

$x = 5$

37. Determine whether Rolle's Theorem can be applied to $f(x) = \frac{x^2 - 13}{x}$ on the closed interval $[-13, 13]$. If Rolle's Theorem can be applied, find all values of c in the open interval $(-13, 13)$ such that $f'(c) = 0$.

a. $c = 8$ b. $c = 12, c = 11$ c. $c = 11, c = 8$ d. $c = 12$ e. Rolle's Theorem does not apply

not continuous on $[-13, 13]$

38. Determine whether the Mean Value Theorem can be applied to the function $f(x) = x^3$ on the closed interval [0,16]. If the Mean Value Theorem can be applied, find all numbers c in the open interval (0,16) such that $f'(c) = \frac{f(16) - f(0)}{16 - 0}$.

a. MVT applies; $-\frac{16\sqrt{3}}{3}$ b. MVT applies; 4

c. MVT applies; $\frac{16\sqrt{3}}{3}$ d. MVT applies; 8

e. MVT does not apply

continuous and differentiable

$f'(c) = \frac{16^3 - 0}{16 - 0} = 256$

$3c^2 = 256$

$c^2 = \frac{256}{3}$

$c = \pm \frac{16\sqrt{3}}{3}$

only positive $[0, 16]$

39. The height of an object t seconds after it is dropped from a height of 550 meters is $s(t) = -4.9t^2 + 550$. Find the average velocity of the object during the first 7 seconds.

a. 34.30 m/sec b. –34.30 m/sec c. –49.00 m/sec d. 49 m/sec e. –16.00 m/sec

$$\frac{s(7) - s(0)}{7 - 0} = -34.3$$

40. A company introduces a new product for which the number of units sold S is $S(t) = 300\left(5 - \frac{10}{3+t}\right)$ where t is the time in months since the product was introduced. During what month does $S'(t)$ equal the average value of $S(t)$ during the first year?

a. October b. July c. December d. April e. March

$$s(t) = 1500 - 3000(3+t)^{-1}$$

$$s'(t) = +3000(3+t)^{-2}$$

$$s'(t) = \frac{3000}{(3+t)^2}$$

$$\frac{3000}{(3+t)^2} = \frac{s(12) - s(0)}{12 - 0}$$

$$\frac{3000}{(3+t)^2} = \frac{1300 - 500}{12}$$

solve for t → algebra

41. For the function $f(x) = (x-1)^{\frac{2}{3}}$:

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

Use a graphing utility to confirm your results.

a. (a) $x = 0$
(b) increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$
(c) relative max: $f(0) = 1$

b. (a) $x = 1$
(b) increasing: $(-\infty, 1)$; decreasing: $(1, \infty)$
(c) relative max: $f(1) = 0$

c. (a) $x = 1$
(b) decreasing: $(-\infty, 1)$; increasing: $(1, \infty)$
(c) relative min: $f(1) = 0$

d. (a) $x = 0, 1$
(b) decreasing: $(-\infty, 0) \cup (1, \infty)$; increasing: $(0, 1)$
(c) relative min: $f(0) = 1$; relative max: $f(1) = 0$

e. (a) $x = 0$
(b) decreasing: $(-\infty, 0)$; increasing: $(0, \infty)$
(c) relative min: $f(0) = 1$

$f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x-1}}$

a) crit #: $x = 1$

	$-\infty$	1	∞
$f'(x)$	$-$		$+$
	dec	min	inc

42. A ball bearing is placed on an inclined plane and begins to roll. The angle of elevation of the plane is θ radians. The distance (in meters) the ball bearing rolls in t seconds is $s(t) = 4.1(\sin\theta)t^2$. Determine the speed of the ball bearing after t seconds.

a. speed: $5.1(\sin\theta)t^2$ meters per second
b. speed: $(\sin\theta)t^2$ meters per second
c. speed: $8.2(\sin\theta)t$ meters per second
d. speed: $8.2(\cos\theta)t$ meters per second
e. speed: $4.1(\sin\theta)t^2$ meters per second

$s'(t) = 8.2(\sin\theta)t$

43. Determine the open intervals on which the graph of $f(x) = 3x^2 + 7x - 3$ is concave downward or concave upward.

a. concave downward on $(-\infty, \infty)$ b. concave upward on $(-\infty, 0)$; concave downward on $(0, \infty)$ c. concave upward on $(-\infty, 1)$; concave downward on $(1, \infty)$ d. concave upward on $(-\infty, \infty)$ e. concave downward on $(-\infty, 0)$; concave upward on $(0, \infty)$

$f'(x) = 6x + 7$

$f''(x) = 6$

concave up, always

44. Find the point of inflection of the graph of the function $f(x) = 2\sin\frac{x}{7}$ on the interval $[0, 14\pi]$.

a. $(0,0)$ b. $(2\pi, 0)$ c. $(8\pi, 0)$ d. $(7\pi, 0)$ e. $(7\pi, 2)$

$f'(x) = \frac{2}{7}\cos\frac{x}{7}$

$f''(x) = -\frac{2}{49}\sin\frac{x}{7}$

0 7π | 7π 14π

− cd | + cu

45. Find the points of inflection and discuss the concavity of the function.

$f(x) = 4x^3 - 5x^2 + 5x - 7$

a. inflection point at $x = -\frac{5}{12}$; concave upward on $\left(-\infty, -\frac{5}{12}\right)$; concave downward on $\left(-\frac{5}{12}, \infty\right)$

b. inflection point at $x = \frac{5}{24}$; concare downward on $\left(-\infty, \frac{5}{24}\right)$; concave upward on $\left(\frac{5}{24}, \infty\right)$

c. inflection point at $x = \frac{5}{12}$; concave downward on $\left(-\infty, \frac{5}{12}\right)$; concave upward on $\left(\frac{5}{12}, \infty\right)$

d. inflection point at $x = -\frac{5}{12}$; concave downward on $\left(-\infty, -\frac{5}{12}\right)$; concave upward on $\left(-\frac{5}{12}, \infty\right)$

e. inflection point at $x = \frac{5}{12}$; concave upward on $\left(-\infty, \frac{5}{12}\right)$; concave downward on $\left(\frac{5}{12}, \infty\right)$

$f'(x) = 12x^2 - 10x + 5$

$f''(x) = 24x - 10$

	$-\infty \quad \frac{5}{12}$	$\frac{5}{12} \quad \infty$
$f''(x)$	$-$ cd	$+$ cu

46. Find the points of inflection and discuss the concavity of the function $f(x) = x\sqrt{x+16}$.

a. no inflection points; concave up on $(-16, \infty)$

b. no inflection points; concave down on $(-16, \infty)$

c. inflection point at $x = 16$; concave up on $(-16, \infty)$

d. inflection point at $x = 0$; concave up on $(-16, 0)$; concave down on $(0, \infty)$

e. inflection point at $x = 16$; concave down on $(-16, \infty)$

47. Find the points of inflection and discuss the concavity of the function $f(x) = -\sin x + \cos x$ on the interval $(0, 2\pi)$.

a. no inflection points. concave up on $(0, 2\pi)$

b. concave upward on $\left(0, \frac{1}{2}\pi\right)$; concave downward on $\left(\frac{5}{2}\pi, 2\pi\right)$; inflection point at $\left(0, \frac{1}{2}\pi\right)$

c. no inflection points. concave down on $(0, 2\pi)$

d. concave downward on $\left(0, \frac{1}{2}\pi\right)$; concave upward on $\left(\frac{5}{2}\pi, 2\pi\right)$; inflection point at $\left(0, \frac{1}{2}\pi\right)$

e. none of the above

$f'(x) = -\cos x - \sin x$

$f''(x) = \sin x - \cos x$

$0 = \sin x - \cos x$

$\sin x = \cos x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

	0 $\frac{\pi}{4}$	$\frac{\pi}{4}$ $\frac{5\pi}{4}$	$\frac{5\pi}{4}$ 2π
$f''(x)$	$-$ cd	$+$ cu	$-$ cd

48. Find the limit.

$$\lim_{x \to \infty} \left(5 + \frac{3}{x^2}\right) = 5 + 0$$

a. ∞ b. 3 c. $-\infty$ d. -3 e. 5

49. Find the limit.

$$\lim_{x \to \infty} \frac{3x+2}{-6x-6} = \frac{3}{-6} = \frac{-1}{2}$$

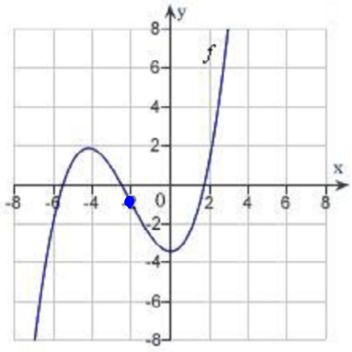
a. 1 b. 0 c. $-\frac{1}{3}$ d. $-\frac{1}{2}$ e. does not exist

50. Find the limit.

$$\lim_{x \to \infty} \frac{-6x \cdot \frac{1}{x}}{\sqrt{64x^2 - 5} \cdot \frac{1}{x}} = \lim_{x \to \infty} \frac{-6}{\sqrt{64 - \frac{5}{x^2}}}$$

a. $-\frac{3}{32}$ b. $-\frac{3}{4}$ c. 1 d. -6 e. $-\infty$

51. The graph of f is shown below. For which value of x is $f''(x)$ zero? poi

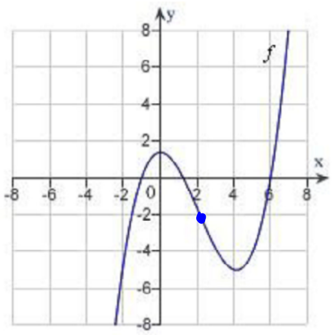


a. $x=2$ b. $x=0$ c. $x=-2$ d. $x=6$
e. $x=4$

52. The graph of f is shown below. On what interval is f' an increasing function?

f' increasing means f'' is positive

f'' positive means concave up

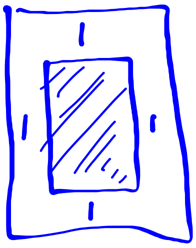


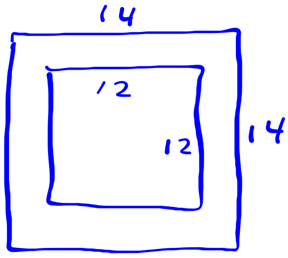
a. $(0,\infty)$ b. $(-1,\infty)$ c. $(-2,\infty)$ d. $(1,\infty)$
e. $(2,\infty)$

53. A rectangular page is to contain 144 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.

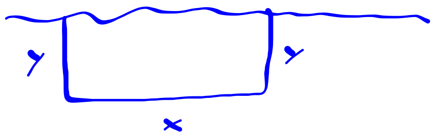
has to be a perfect square

a. 16, 16 b. 13, 13 c. 15, 15 d. 25, 25
e. 14, 14





54. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 720,000 square meters in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing?



a. $x = 600$ and $y = 1200$ b. $x = 1000$ and $y = 720$
c. $x = 1200$ and $y = 600$ d. $x = 720$ and $y = 1000$
e. none of the above

$xy = 720{,}000$

$y = \dfrac{720000}{x}$

$P = x + 2y$

$P = x + \dfrac{1{,}440{,}000}{x}$

$P' = 1 - \dfrac{1440{,}000}{x^2}$

$x = 1200$

$y = 600$

55. Find the indefinite integral $\int 11\tan^2 x + 16\ dx$.

a. $11\tan x + 5x + C$ b. $\frac{11}{3}\tan^3 x + 16x + C$

c. $-\frac{11}{3}\tan^3 x + 16x + C$ d. $11\tan x - 5x + C$

e. $11\tan x + 7x + C$

$\int (11(\sec^2 x - 1) + 16)\,dx$

$\int (11\sec^2 x - 11 + 16)\,dx$

$\int (11\sec^2 x + 5)\,dx$

$11\tan x + 5x + C$

56. An evergreen nursery usually sells a certain shrub after 4 years of growth and shaping. The growth rate during those 4 years is approximated by $\frac{dh}{dt} = 2.5t + 6$, where t is the time in years and h is the height in centimeters. The seedlings are 15 centimeters tall when planted ($t = 0$). Find the height after t years.

a. $h(t) = 1.25t^2 + 21t$ b. $h(t) = 1.25t^2 + 6t + 15$

c. $h(t) = 1.25t + 15$ d. $h(t) = 2.5t^2 + 6t + 15$

e. $h(t) = 2.5t + 21$

$\int dh = \int (2.5t + 6)\,dt$

$h(t) = 1.25t^2 + 6t + C$

$C = 15$

57. The rate of growth $\frac{dP}{dt}$ of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days $(0 \le t \le 10)$. That is, $\frac{dP}{dt} = k\sqrt{t}$. The initial size of the population is 500. After one day the population has grown to 600. Estimate the population after 5 days. Round your answer to the nearest integer.

a. $P(5) \approx 824$ bacteria b. $P(5) \approx 792$ bacteria
c. $P(5) \approx 1618$ bacteria d. $P(5) \approx 724$ bacteria
e. $P(5) \approx 1718$ bacteria

$\int dP = \int k\sqrt{t}\,dt$

$P(t) = k\frac{2}{3}t^{\frac{3}{2}} + C$

$C = 500$

$P(t) = \frac{2}{3}kt^{\frac{3}{2}} + 500$

$600 = \frac{2}{3}k + 500$

$k = 150$

$P(t) = 100t^{\frac{3}{2}} + 500$

$P(5) = 100(5)^{\frac{3}{2}} + 500$

58. A ball is thrown vertically upwards from a height of 9 ft with an initial velocity of 70 ft per second. How high will the ball go? Note that the acceleration of the ball is given by $a(t) = -32$ feet per second per second.

a. 238.6875 ft b. 220.6875 ft c. 66.4219 ft
d. 85.5625 ft e. 240.0875 ft

$v_0 = 70$ ft/s height $= 9 = x_0$

$0^2 = 70^2 + 2(-32)(x-9)$

$-4900 = -64x + 576$

$-5476 = -64x$

$= 85.56$

or

$v_0 = 70$ ft/s

$t = \frac{70}{32} \times 35 + 9$

59. The maker of an automobile advertises that it takes 13 seconds to accelerate from 25 kilometers per hour to 75 kilometers per hour. Assuming constant acceleration, compute the distance, in meters, the car travels during the 13 seconds. Round your answer to two decimal places.

a. 27.69 m b. 361.11 m c. 180.56 m
d. 234.72 m e. 90.28 m

$25\frac{km}{h}\left(\frac{1\ h}{60\ min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{10^3 m}{1\ km}\right)$

do the same for 75

$\frac{125\ m}{18\ s}$ $\frac{125\ m}{6\ s}$

v_0 v_{13}

$v = v_0 + at$

$\frac{125}{6} = \frac{125}{18} + a(13)$ $a = \frac{125\ m}{117\ s^2}$

$\Delta x = v_0 t + \frac{1}{2}at^2$

$\frac{125}{18}(13) + \frac{1}{2}\left(\frac{125}{117}\right)(13)^2$

$= \frac{1625}{9} m \approx 180.56 m$

60. Evaluate the definite integral of the function.

$$\int_0^{5\pi} (2t + 5\cos t)\,dt \rightarrow \int_0^{5\pi} 2t\,dt + \int_0^{5\pi} 5\cos t\,dt$$

Use a graphing utility to verify your results.

a. $50\pi^2$ b. $25\pi^2 + 10$ c. $25\pi^2$ d. $25\pi^2 - 10$
e. $5\pi^2$

$= \left[t^2\right]_0^{5\pi} + \left[5\sin t\right]_0^{5\pi}$

$\left[t^2 + 5\sin t\right]_0^{5\pi}$ $(25\pi^2 + 0) - (0 + 0)$

0

61. Find the average value of the function $f(x) = 48 - 12x^2$ over the interval $-5 \le x \le 5$.

a. -152 b. -52 c. 148 d. 248 e. -252

$$\frac{1}{5--5}(2)\int_0^5 (48-12x^2)\,dx$$

$$\frac{12}{5}\int_0^5 (4-x^2)\,dx$$

62. Find the average value of the function over the given interval and all values t in the interval for which the function equals its average value.

$f(t) = \dfrac{t^2+4}{t^2}$, $1 \le t \le 5$

Use a graphing utility to verify your results.

a. The average is $\frac{9}{5}$ and the point at which the function is equal to its mean value is $\sqrt{5}$.

b. The average is $\frac{9}{5}$ and the point at which the function is equal to its mean value is $\sqrt{5}$ and $-\sqrt{5}$.

c. The average is $\frac{9}{20}$ and the point at which the function is equal to its mean value is $-\sqrt{5}$.

d. The average is $\frac{9}{20}$ and the point at which the function is equal to its mean value is $\sqrt{5}$.

e. The average is $\frac{9}{20}$ and the point at which the function is equal to its mean value is $\sqrt{5}$ and $-\sqrt{5}$.

$$\frac{1}{5-1}\int_1^5 (1+4t^{-2})\,dt$$

$$\frac{1}{4}\left[t-\frac{4}{t}\right]_1^5$$

$$\frac{1}{4}\left[\frac{21}{5}+\frac{15}{5}\right] = \frac{9}{5}$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$

63. Find the indefinite integral of the following function and check the result by differentiation.

$$\int \frac{4u^3}{\sqrt{u^4+5}}\,du$$

a. $2\sqrt{u^3+5}+C$ b. $\sqrt{u^4+5}+C$

c. $\frac{1}{2}\sqrt{u^4+5}+C$ d. $\frac{1}{2}\sqrt{u^3+5}+C$

e. $2\sqrt{u^4+5}+C$

$z = u^4+5$

$dz = 4u^3\,du$

$\int \frac{dz}{\sqrt{z}}$

$\int z^{-\frac{1}{2}}\,dz$

$2z^{\frac{1}{2}}+C$

$2\sqrt{u^4+5}+C$

64. Find the indefinite integral of the following function and check the result by differentiation.

$$\int (7-s)\sqrt{s}\,ds$$

a. $\frac{7}{3}s^{\frac{3}{2}}-\frac{2}{5}s^{\frac{5}{2}}+C$ b. $\frac{7}{3}s^{\frac{3}{2}}-\frac{5}{2}s^{\frac{5}{2}}+C$

c. $\frac{14}{3}s^{\frac{3}{2}}-\frac{2}{5}s^{\frac{5}{2}}+C$ d. $\frac{14}{3}s^{\frac{3}{2}}+\frac{5}{2}s^{\frac{5}{2}}+C$

e. $\frac{14}{3}s^{\frac{3}{2}}-\frac{5}{2}s^{\frac{5}{2}}+C$

$\int (7s^{\frac{1}{2}}-s^{\frac{3}{2}})\,ds$

$\frac{14}{3}s^{\frac{3}{2}}-\frac{2}{5}s^{\frac{5}{2}}+C$

65. Find the indefinite integral $\int 5z^4 \cos z^5 \, dz$.

a. $\cos z^5 + C$ b. $\dfrac{\sin z^6}{6} + C$ c. $\sin z^4 + C$

d. $\sin z^5 + C$ e. $\dfrac{\sin z^5}{5} + C$

$u = z^5$

$du = 5z^4 dz$

$\int \cos u \, du$

$\sin u + C$

$\sin z^5 + C$

66. Find an equation for the function $f(x)$ whose derivative is $f'(x) = 7\sin(49x)$ and whose graph passes through the point $\left(\frac{\pi}{49}, -\frac{6}{7}\right)$.

a. $f(x) = -\frac{1}{7}\cos(49x) - 1$

b. $f(x) = \frac{1}{14}\cos(49x) + 1$

c. $f(x) = \frac{1}{49}\cos(98x) + 14$

d. $f(x) = -\frac{1}{7}\cos(49x) - 49$

e. $f(x) = -\frac{1}{98}\cos(7x) + 2$

$u = 49x$

$du = 49\,dx$

$\frac{1}{7} du = 7\,dx$

$f(x) = \frac{1}{7}\int \sin u \, du$

$f(x) = \frac{-1}{7}\cos u + C$

$f(x) = -\frac{1}{7}\cos u + C$

$f(x) = -\frac{1}{7}\cos 49x + C$

$-\frac{6}{7} = -\frac{1}{7}\cos \pi + C$

$-\frac{6}{7} = \frac{1}{7} + C$

$-1 = C$