Parametric and Vector Calculus Worksheet

1. (No Calculator) The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$, $y(t) = \frac{2}{3}t^3$.

   (a) Find the magnitude of the velocity vector at $t = 2$.

   (b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

   (c) Find $\frac{dy}{dx}$ as a function of x.

   (d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

2. (No Calculator) An object moving along a curve in the *xy*-plane has position $\langle x(t), y(t) \rangle$ at time t with the velocity vector $\vec{v}(t) = \left(\frac{1}{t+1}, 2t \right)$. At time $t = 1$, the object is at $(\ln 2, 4)$.

(a) Find the position vector.

(b) Write an equation for the line tangent to the curve when $t = 1$.

(c) Find the magnitude of the velocity vector when $t = 1$.

(d) At what time $t > 0$ does the line tangent to the particle at $\langle x(t), y(t) \rangle$ have a slope of 12?

3. A particle moving along a curve in the *xy*-plane has position $\langle x(t), y(t) \rangle$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \le t \le 10$. Find the velocity vector at the time when the particle's vertical position is $y = 7$.

4. A particle moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dx}{dt} = 1 + \sin\left(t^3\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. For any $t \geq 0$, the line tangent to the curve at $\langle x(t), y(t) \rangle$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

5. An object moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dx}{dt} = \cos\left(e^t\right)$ and $\frac{dy}{dt} = \sin\left(e^t\right)$ for $0 \leq t \leq 2$. At time $t = 1$, the object is at the point $(3, 2)$.

   (a) Find the equation of the tangent line to the curve at the point where $t = 1$.

   (b) Find the speed of the object at $t = 1$.

   (c) Find the total distance traveled by the object over the time interval $0 \leq t \leq 2$.

   (d) Find the position of the object at time $t = 2$.

6. A particle moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dx}{dt} = \sin\left(t^3 - t\right)$ and $\frac{dy}{dt} = \cos\left(t^3 - t\right)$. At time $t = 3$, the particle is at the point $(1, 4)$.

(a) Find the acceleration vector for the particle at $t = 3$.

(b) Find the equation of the tangent line to the curve at the point where $t = 3$.

(c) Find the magnitude of the velocity vector at $t = 3$.

(d) Find the position of the particle at time $t = 2$.

7. An object moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dy}{dt} = 2 + \sin\left(e^t\right)$. The derivative of $\frac{dx}{dt}$ is not explicitly given. At $t = 3$, the object is at the point $(4, 5)$.

(a) Find the y-coordinate of the position at time $t = 1$.

(b) At time $t = 3$, the value of $\frac{dy}{dx}$ is -1.8. Find the value of $\frac{dx}{dt}$ when $t = 3$.

(c) Find the speed of the object at time $t = 3$.