

## Chapter 9 Solutions

1. Identify the most appropriate test to be used to

determine whether the series  $\sum_{n=1}^{\infty} \frac{11(-1)^{n+1}}{n}$

converges or diverges.

- a. Ratio Test
- b. Alternating Series Test
- c. Root Test
- d.  $\rho$ -Series Test
- e. Telescoping Series Test

2. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3}{5n}$$

- a. diverges; Integral Test
- b. converges; Ratio Test
- c. converges; Integral Test
- d. converges; Alternating Series Test
- e. diverges; Ratio Test

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3. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{2}{n^2}$$

- a. converges;  $p$ -series
- b. diverges; Ratio Test
- c. converges; both  $p$ -series and Integral Test
- d. diverges;  $p$ -series
- e. converges; Integral Test

$$2 \int_1^{\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \left[ -\frac{2}{x} \right]_1^b$$

$$\lim_{b \rightarrow \infty} \frac{-2}{b} - \frac{-2}{1} = 2$$

1. Determine the convergence or divergence of the sequence with the given  $n$ th term. If the sequence converges, find its limit.

$$a_n = \frac{\ln(n^2)}{8n}$$

converges to 0,  
use L'Hospital

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2. Determine the convergence or divergence of the sequence with the given  $n$ th term. If the sequence converges, find its limit.

$$a_n = \frac{3^n}{5^n}$$

converges to 0

3. Find the sum of the convergent series.

$$\sum_{n=0}^{\infty} 5\left(-\frac{10}{11}\right)^n$$

$$\frac{5}{1 - -\frac{10}{11}} = \frac{5}{\frac{21}{11}} = 5\left(\frac{11}{21}\right) = \frac{55}{21}$$

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4. Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} \frac{2}{(n+5)(n+7)} = \frac{A}{x+5} + \frac{B}{x+7}$$

$$2 = A(x+7) + B(x+5)$$

$$\text{let } x=-5 \quad A=1$$

$$\text{let } x=-7 \quad B=-1$$

$$= \frac{1}{6} - \cancel{\frac{1}{8}} + \frac{1}{7} - \cancel{\frac{1}{9}} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{10}} + \cancel{\frac{1}{9}} - \cancel{\frac{1}{11}} \dots$$

$$= \frac{1}{6} + \frac{1}{7} = \frac{13}{42}$$

5. Find the sum of the convergent series

$$6 - 1 + \frac{1}{6} - \frac{1}{36} + \dots$$

$$\frac{6}{1 - \left(-\frac{1}{6}\right)} = \frac{6}{\frac{7}{6}} = \frac{36}{7}$$

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6. Find the sum of the convergent series

$$\sum_{n=0}^{\infty} \left( \frac{1}{11^n} - \frac{1}{12^n} \right) = \sum_{n=0}^{\infty} \left( \frac{1}{11} \right)^n - \sum_{n=0}^{\infty} \left( \frac{1}{12} \right)^n$$

$$\frac{1}{1 - \frac{1}{11}} - \frac{1}{1 - \frac{1}{12}}$$

$$\frac{11}{10} - \frac{12}{11} = \boxed{\frac{1}{110}}$$

7. Write the repeating decimal 0.73 as a geometric series.

$\bullet .7373737373 \text{ blah blah } \dots$

$$\frac{73}{100} + \frac{73}{100^2} + \frac{73}{100^3} + \dots$$

$\sum_{n=0}^{\infty} \frac{73}{100} \left( \frac{1}{100} \right)^n \text{ or } \sum_{n=1}^{\infty} 73 \left( \frac{1}{100} \right)^n$

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8. Use the Integral Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} ne^{-\frac{n}{10}}$$

$$\int_1^b xe^{-\frac{x}{10}} dx$$

$$u = x \quad v = -10e^{-\frac{x}{10}}$$

$$du = dx \quad dv = e^{-\frac{x}{10}} dx$$

$$-10xe^{-\frac{x}{10}} - \int -10e^{-\frac{x}{10}} dx$$

the integral converges so the series also converges

$$\lim_{b \rightarrow \infty} \left[ -10xe^{-\frac{x}{10}} - 100e^{-\frac{x}{10}} \right]_1^b$$

$$\lim_{b \rightarrow \infty} \left[ \frac{-10x - 100}{e^{\frac{x}{10}}} \right]_1^b \Rightarrow \infty \text{ use L'Hospital}$$

$$\lim_{b \rightarrow \infty} \left[ \frac{-10}{\frac{1}{10}e^{\frac{x}{10}}} \right]_1^b = 0 - \frac{-10}{\frac{1}{10}e^{\frac{1}{10}}} > 0$$

9. Use the Integral Test to determine the convergence or divergence of the series.

$$\sum_{n=2}^{\infty} \frac{8}{n\sqrt{\ln n}}$$

$$\int_2^{\infty} \frac{8}{x\sqrt{\ln x}} dx$$

$$u = \ln x \quad du = \frac{1}{x} dx$$

since the integral div. the series also diverges

$$\int_{\ln 2}^{\infty} \frac{8}{\sqrt{u}} du$$

$$\int_{\ln 2}^{\infty} 8u^{-\frac{1}{2}} du$$

$$16 \lim_{b \rightarrow \infty} \left[ u^{\frac{1}{2}} \right]_{\ln 2}^b = \infty$$

diverges

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10. Determine the convergence or divergence of the

series  $\sum_{n=1}^{\infty} \frac{20(-1)^{n+1}}{n}$  using any appropriate test.

converges by alternating series test

11. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$\sum_{n=1}^{\infty} \frac{7n}{n+7}$  diverges by  $n$ th term  
test for divergence

$$\lim_{n \rightarrow \infty} \frac{7n}{n+7} = 7 \neq 0$$

## Chapter 9 Solutions

12. Identify the most appropriate test to be used to

determine whether the series  $\sum_{n=1}^{\infty} \frac{n}{7n^2 + 1}$

converges or diverges.

lim comparison test with  $\frac{1}{n}$ , diverges

$$\lim_{n \rightarrow \infty} \left| \frac{n}{7n^2 + 1} \cdot \frac{n}{1} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{n^2}{7n^2 + 1} \right| = \frac{1}{7} > 0 \text{ (finite and positive)} \\ \text{so both behave the same}$$

13. Identify the most appropriate test to be used to

determine whether the series  $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$  converges

or diverges.

direct comparison test with

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \geq \sum_{n=1}^{\infty} \left| \frac{\cos n}{2^n} \right|$$

*Larger series converges*      *smaller series converges*

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14. The terms of a series  $\sum_{n=1}^{\infty} a_n$  are defined recursively. Determine the convergence or divergence of the series. Explain your reasoning.

$$a_1 = 3, a_{n+1} = \frac{-6n+3}{-3n+5} a_n$$

use ratio test

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{-6n+3}{-3n+5} a_n}{a_n} \right| = 2 > 1 \text{ so diverges}$$

15. Find the values of  $x$  for which the series

$$\sum_{n=0}^{\infty} 2\left(\frac{x}{6}\right)^n$$
 converges.

geometric series

$$|r| < 1 \text{ to converge}$$

$$\left|\frac{x}{6}\right| < 1$$

$$\text{so } -6 < x < 6$$

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16. Find the values of  $x$  for which the series

$$\sum_{n=0}^{\infty} 4(x-1)^n \text{ converges.}$$

geometric series

$|r| < 1$  to converge

$$|x-1| < 1$$

$$\text{so } 0 < x < 2$$

17. Find a first-degree polynomial function  $P_1$  whose value and slope agree with the value and slope of

$$f(x) = \frac{16}{\sqrt{x}} \text{ at } x = 16.$$

$$f(16) = \frac{16}{\sqrt{16}} = 4$$

$$f(x) = 16x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}(16)x^{-\frac{3}{2}}$$

$$f'(x) = \frac{-8}{\sqrt{x^3}}$$

$$f'(16) = \frac{-8}{\sqrt{16^3}} = -\frac{1}{8}$$

$$\frac{f(16)(x-16)^0}{0!} + \frac{f'(16)(x-16)^1}{1!}$$

$$4 - \frac{1}{8}(x-16)$$

$$\boxed{6 - \frac{1}{8}x}$$

18. Find the Maclaurin polynomial of degree 3 for the function.

$$f(x) = e^{-3x} \quad | + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$| - 3x + \frac{9x^2}{2} - \frac{27x^3}{6}$$

19. Find the Maclaurin polynomial of degree 5 for the function.

$$f(x) = \sin(4x) \quad | - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$| - 4x - \frac{(4x)^3}{3!} + \frac{(4x)^5}{5!}$$

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20. Determine the degree of the Maclaurin polynomial required for the error in the approximation of the function  $\sin(0.1)$  to be less than 0.001.

$$0.1 - \frac{(0.1)^3}{3!}$$

$$0.00016$$

21. Determine the values of  $x$  for which the function  $f(x) = \sin x$  can be replaced by the Taylor

polynomial  $f(x) = \sin x \approx x - \frac{x^3}{3!}$  if the error

cannot exceed 0.009. Round your answer to four decimal places.

*error is bounded  
by the next term*

$$\frac{x^5}{5!} \leq .009$$

$$x^5 \leq .009(5!)$$

$$x \leq \sqrt[5]{.009(5!)} \approx 1.0155$$

$$-1.0155 \leq x \leq 1.0155$$

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22. Find the radius of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{8^n}$$

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{8^{n+1}} \cdot \frac{8^n}{x^n} \right| < 1$$

$$|x| \lim_{n \rightarrow \infty} \left| \frac{1}{8} \right| < 1$$

$$R = 8$$

Or the easy way

$$\sum_{n=0}^{\infty} \left( \frac{-x}{8} \right)^n$$

$$\left| -\frac{x}{8} \right| < 1$$

$$-8 \leq x \leq 8$$

$$R = 8$$

23. Find the radius of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(9x)^{2n}}{(2n)!}$$

$$\lim_{n \rightarrow \infty} \left| \frac{(9x)^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{(9x)^{2n}} \right| < 1$$

$$\lim_{n \rightarrow \infty} \left| \frac{(9x)^2}{(2n+1)(2n+2)} \right| < 1$$

$$|9x|^2 \lim_{n \rightarrow \infty} \left| \frac{1}{(2n+1)(2n+2)} \right| < 1$$

$$|9x|^2 \cdot 0 < 1$$

$$R = \infty$$

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24. Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\sum_{n=0}^{\infty} \left(\frac{x}{6}\right)^n$$

$$(-6, 6)$$

$\downarrow$  Geometric series b/c  
 $r = \frac{x}{6} \Rightarrow |r| < 1 \Rightarrow -6 < x < 6$

25. Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\begin{aligned} & \text{ratio test:} \\ & \sum_{n=0}^{\infty} \frac{(8x)^n}{(3n)!} \lim_{n \rightarrow \infty} \left| \frac{(8x)^{n+1}}{(3(n+1))!} \cdot \frac{(3n)!}{(8x)^n} \right| \\ & \quad (3n+3)(3n+2)(3n+1)3^n \\ & = \lim_{n \rightarrow \infty} \left| \frac{8x}{(3n+3)(3n+2)(3n+1)} \right| < 1 \\ & 8x \cdot 0 < 1 \quad 0 < 1 \\ & R = \infty \\ & (-\infty, \infty) \end{aligned}$$

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26. Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\sum_{n=0}^{\infty} \frac{(-1)^n n! (x-5)^n}{(2)^n}$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)! (x-5)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n! (x-5)^n} \right| < 1$$

$$|x-5| \lim_{n \rightarrow \infty} \left| \frac{n+1}{2} \right| < 1$$

$x-5 = 0$

$x = 5$

$(0) \infty < 1$

27. Write an equivalent series of the series  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

with the index of summation beginning at  $n = 3$ .

$$\sum_{n=3}^{\infty} \frac{x^{n-3}}{(n-3)!}$$

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28. Write an equivalent series of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{3n+1} \text{ with the index of summation}$$

beginning at  $n=4$ .

$$\sum_{n=4}^{\infty} \frac{(-1)^{n-4} x^{3(n-4)+1}}{3(n-4)+1}$$

$$\sum_{n=4}^{\infty} \frac{(-1)^n \cdot x^{3n-11}}{3n-11}$$

29. Consider the function given by

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-9)^n}{n} . \text{ Find the interval of}$$

convergence for  $\int f(x) dx$ .

$$\lim_{n \rightarrow \infty} \left| \frac{(x-9)^{n+1}}{n+1} \cdot \frac{n}{(x-9)^n} \right| < 1$$

$$|x-9| \lim_{n \rightarrow \infty} \left| \frac{n}{n+1} \right| < 1$$

$$|x-9| < 1 \quad c = 9 \quad R = 1$$

$$\int f(x) dx = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-9)^{n+1}}{n(n+1)}$$

when  $x=8 \rightarrow \text{conv.}$   
 so  $[8, 10]$

when  $x=8$   $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (-1)^n}{n}$  diverges

$(8, 10]$

when  $x=10$   $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 1^n}{n}$  converges

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30. Find a geometric power series for the function

$\frac{1}{10+x}$  centered at 0.

$$\frac{1}{10 - (-x)} \Rightarrow \frac{\frac{1}{10}}{1 - \left(-\frac{x}{10}\right)} = \frac{a}{1-r}$$

$$\sum_{n=0}^{\infty} \frac{1}{10} \left(\frac{-x}{10}\right)^n .$$

31. Find a power series for the function  $\frac{1}{10-x} = \frac{1}{9-(x-1)}$  centered at 1.

$$\frac{1}{9 - (x-1)} \Rightarrow \frac{1}{9} \sum_{n=0}^{\infty} \left(\frac{x-1}{9}\right)^n$$

or  $\sum_{n=0}^{\infty} \frac{(x-1)^n}{9^{n+1}}$

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32. Find a power series for the function  $\frac{3}{3+x^2}$

centered at 0.

$$\frac{3}{3+x} \xrightarrow{\quad} \frac{1}{1-\left(\frac{-x}{3}\right)}$$

$$\sum_{n=0}^{\infty} \left(-\frac{x}{3}\right)^n$$

$$\text{so } \frac{3}{3+x^2} = \sum_{n=0}^{\infty} \left(-\frac{x^2}{3}\right)^n$$

$$\text{or} \\ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{3^n}$$

33. Use the power series  $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$  to

determine a power series centered at 0 for the

$$\text{function } f(x) = -\frac{3}{(3x+1)^2} = \frac{d}{dx} \left[ \frac{1}{3x+1} \right].$$

$$\frac{1}{1-(-3x)} = \sum_{n=0}^{\infty} (-1)^n (-3x)^n \quad \text{now take the derivative}$$

$$\text{so } f(x) = \sum_{n=0}^{\infty} (-1)^n 3n (-3x)^{n-1} \quad \text{or} \quad \sum_{n=0}^{\infty} (-3)^n n x^{n-1}$$

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34. Explain how to use the geometric series

$g(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1$  to find the series for  
the function  $\frac{5}{1+x}$ .

replace  $x$  with  $(-x)$  and multiply the series by 5

35. Use the definition to find the Taylor series  
(centered at  $c$ ) for the function.

$$f(x) = e^{4x}, c = 0$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

$$e^{4x} = \sum_{n=0}^{\infty} \frac{4^n}{n!} x^n \text{ or } \sum_{n=0}^{\infty} \frac{(4x)^n}{n!}$$