

1. Find the indefinite integral.

$$\int \frac{3m}{m-9} dm$$

$$\begin{aligned} m-9 &\sqrt{\frac{3m}{m-9}} \\ &= \frac{3m-27}{27} \\ &\downarrow \\ &\boxed{3m + 27 \ln|m-9| + C} \end{aligned}$$

- a. $27 \ln|m-9| - 3m + C$
- b. $-27 \ln|m-9| + 3m + C$
- c. $27 \ln|m-9| + 3m^2 + C$
- d. $\cancel{27 \ln|m-9|} + 3m + C$
- e. $-27 \ln|m-9| + 3m^2 + C$

2. Find the indefinite integral $\int 4 \csc^2 x e^{\cot x} dx$.

- a. $\cancel{-4e^{\cot x}} + C$
- b. $-4 \cot x \csc x + C$
- c. $-4 \cot x + C$
- d. $e^{5 \cot x} + C$
- e. $-4 \csc x + C$

$$\begin{aligned} u &= \cot x \\ du &= -\csc^2 x dx \\ dx &= -\frac{1}{\csc^2 x} du \\ -4 \int \frac{e^{\cot x} \cdot e^u}{\csc^2 x} du &\cancel{=} \\ \hookrightarrow -4 \int e^u du &\rightarrow -4e^u + C \\ \boxed{-4e^{\cot x} + C} \end{aligned}$$

3. Solve the differential equation

$$(36 + \tan^2 x)y' = \sec^2 x.$$

$$\int \frac{\sec^3 x}{36 + \tan^2 x} dx \quad u = \tan x \\ du = \sec^2 x dx \\ a = 6$$

a. $y = \frac{1}{6} \arctan\left(\frac{\sec x}{6}\right) + C$

b. $y = \arctan\left(\frac{\sec x}{6}\right) + C$

c. $y = \frac{1}{6} \arctan\left(\frac{\tan x \sec x}{6}\right) + C$

d. $\textcircled{O} \quad y = \frac{1}{6} \arctan\left(\frac{\tan x}{6}\right) + C$

e. $y = \arctan\left(\frac{\tan x}{6}\right) + C$

$$\int \frac{du}{a^2 + u^2} \rightarrow \frac{1}{a} \arctan \frac{u}{a} + C$$

$\frac{1}{6} \arctan\left(\frac{\tan x}{6}\right) + C$

4. Find the indefinite integral.

$$u = \ln x \quad v = \frac{x^5}{5} \\ du = \frac{1}{x} dx \quad dv = x^4 dx$$

$$\int x^4 \ln x dx$$

$$uv - \int v du \\ \frac{x^5 \ln x}{5} - \int \frac{x^5}{5} \cdot \frac{1}{x} dx$$

a. $\frac{x^5}{25} [4 \ln(x) - 1] + C$

b. $\frac{x^4}{25} [\ln(x^4) - 1] + C$

c. $\textcircled{O} \quad \frac{x^5}{25} [5 \ln(x) - 1] + C$

d. $\frac{x^5}{16} [\ln(x^5) - 1] + C$

e. $\frac{x^3}{25} [\ln(x^3) - 1] + C$

$$\frac{5}{5} \frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$$

$\frac{x^5}{25} [5 \ln(x) - 1] + C$

7. Find $\int 8x^{15} \cos(x^8) dx$. $= \int 8x^7 \cdot x^8 \cos x^8 dx$

$$\frac{z = x^8}{dz = 8x^7 dx}$$

a. $\int 8x^{15} \cos(x^8) dx = x^8 \sin(x^8) - \cos(x^8) + C$

b. $\int 8x^{15} \cos(x^8) dx = x^9 \sin(x^9) + \cos(x^9) + C$

c. $\int 8x^{15} \cos(x^8) dx = x^8 \sin(x^8) + \cos(x^8) + C$

d. $\int 8x^{15} \cos(x^8) dx = x^{15} \sin(x^{15}) - \cos(x^{15}) + C$

e. $\int 8x^{15} \cos(x^8) dx = x^{15} \sin(x^{15}) + \cos(x^{15}) + C$

$$\int z \cos z dz$$

$$u = z \quad v = \sin z$$

$$du = dz \quad dv = \cos z dz$$

$$z \sin z - \int \sin z dz$$

$$z \sin z + \cos z + C$$

$$x^8 \sin x^8 + \cos x^8 + C$$

8. Write the form of the partial fraction decomposition for the following rational expression.

$$\frac{6x-5}{x(x^2+4)^2}$$

a. $\frac{A}{x} + \frac{Bx+C}{(x^2+4)^2}$

b. $\frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

c. $\frac{A}{x} + \frac{B}{x+4} + \frac{C}{x-4}$

d. $\frac{A}{x} + \frac{B}{x^2+4} + \frac{C}{(x^2+4)^2}$

e. $\frac{A}{x} + \frac{B}{x+4} + \frac{C}{(x+4)^2} + \frac{D}{x-4} + \frac{E}{(x-4)^2}$

9. Use partial fractions to find the integral

$$\int \frac{16x - 136}{x^2 - 16x + 60} dx.$$

$$\frac{16x - 136}{(x-6)(x-10)} = \frac{A}{x-6} + \frac{B}{x-10}$$
$$16x - 136 = A(x-10) + B(x-6)$$
$$A = 10 \quad B = 6$$

$$\int \left(\frac{10}{x-6} + \frac{6}{x-10} \right) dx$$
$$10 \ln|x-6| + 6 \ln|x-10| + C$$

- a. $10x + 60 \ln|6-x| + 6 \ln|10-x| + C$
- b. $10x + 60 \ln|6-x| + 6x + 60 \ln|10-x| + C$
- c. $10 \ln|x-6| - 6 \ln|x-10| + C$
- d. $\textcircled{10} \ln|x-6| + 6 \ln|x-10| + C$
- e. $10x + 60 \ln|6-x| - 6x + 60 \ln|10-x| + C$

10. Evaluate the limit $\lim_{x \rightarrow 0} \frac{\arcsin(13x)}{2x}$ using L'Hopital's Rule if necessary.

- a. $\frac{2}{13}$
- b. 0
- c. $\frac{1}{2}$
- d. $\frac{13}{2}$ $\textcircled{13}$
- e. does not exist

$$\frac{0}{0} \text{ apply LH}$$
$$\lim_{x \rightarrow 0} \frac{\frac{13}{\sqrt{1-(13x)^2}}}{2}$$
$$= \lim_{x \rightarrow 0} \frac{13}{2\sqrt{1-169x^2}} = \frac{13}{2\sqrt{1}} = \frac{13}{2}$$

11. Evaluate the limit $\lim_{x \rightarrow -\infty} \frac{2e^{-\frac{1}{2}x}}{x^3}$ using L'Hopital's Rule if necessary.

- a. $\frac{1}{3}$
- b. $\frac{1}{24}$
- c. $-\infty$
- d. ∞
- e. 2

$$\text{L'H} \quad \lim_{x \rightarrow -\infty} \frac{-e^{-\frac{1}{2}x}}{3x^2}$$

$$\lim_{x \rightarrow -\infty} \frac{\frac{1}{2}e^{-\frac{1}{2}x}}{6x}$$

$$\lim_{x \rightarrow -\infty} \frac{-\frac{1}{4}e^{-\frac{1}{2}x}}{6} = -\infty$$

12. Evaluate the limit $\lim_{x \rightarrow \infty} 7x \sin\left(\frac{8}{x}\right)$ using

L'Hopital's Rule if necessary.

- a. $\frac{8}{7}$
- b. 56
- c. ∞
- d. 0
- e. does not exist

$$\begin{aligned} & \lim_{x \rightarrow \infty} \left(\frac{\sin \frac{8}{x}}{\frac{1}{x}} \right) \\ \xrightarrow{\text{L'H}} \quad & \lim_{x \rightarrow \infty} \frac{-\frac{8}{x^2} \cos \frac{8}{x}}{-\frac{1}{x^2}} \\ \xrightarrow{\text{simplify}} \quad & \lim_{x \rightarrow \infty} 8 \cos \frac{8}{x} \\ & = 7(8) = 56 \end{aligned}$$

13. Determine whether the improper integral

$$\int_9^{11} \frac{2}{(x-10)^2} dx$$

diverges or converges. Evaluate the

integral if it converges.

a. 100

b. 2

c. $\frac{1}{50}$

d. $\frac{22}{9}$

e. diverges

$$\lim_{b \rightarrow 10^-} \int_9^b \frac{2}{(x-10)^2} dx + \lim_{b \rightarrow 10^+} \int_b^{11} \frac{2}{(x-10)^2} dx$$

$$\lim_{b \rightarrow 10^-} \left[\frac{-2}{x-10} \right]_9^b$$

$$\lim_{b \rightarrow 10^-} \frac{-2}{b-10} - \frac{-2}{9-10}$$

div.

14. Determine whether the improper integral $\int_4^{\infty} \frac{2}{x^3} dx$

diverges or converges. Evaluate the integral if it converges.

a. $\frac{1}{2}$

b. $\frac{1}{16}$

c. 2

d. $\frac{1}{32}$

e. diverges

$$\lim_{b \rightarrow \infty} 2 \int_4^b \frac{1}{x^3} dx$$

$$\lim_{b \rightarrow \infty} \left[-\frac{1}{x^2} \right]_4^b$$

$$\lim_{b \rightarrow \infty} -\frac{1}{b^2} + \frac{1}{4^2}$$

$$0 + \frac{1}{16}$$

15. Determine whether the improper integral

∞

$\int_0^{\infty} xe^{-x/3} dx$ diverges or converges. Evaluate the integral if it converges.

- a. 9
- b. $e^{-1/3}$
- c. $-\frac{1}{3}$
- d. 3
- e. diverges

$$u = x \quad v = -3e^{-\frac{1}{3}x}$$

$$du = dx \quad dv = e^{-\frac{1}{3}x} dx$$

$$\lim_{b \rightarrow \infty} \int_0^b \left[-3xe^{-\frac{1}{3}x} - \int -3e^{-\frac{1}{3}x} dx \right]$$

$$\hookrightarrow \lim_{b \rightarrow \infty} \left[-3xe^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x} \right]_0^b$$

$$\hookrightarrow \lim_{b \rightarrow \infty} \left[\frac{-3x}{e^{\frac{1}{3}x}} - \frac{9}{e^{\frac{1}{3}x}} \right]_0^b$$

$$\lim_{b \rightarrow \infty} \left[\frac{-3b}{e^{\frac{1}{3}b}} - \frac{9}{e^{\frac{1}{3}b}} \right] - \left[\frac{-3(0)}{e^{\frac{1}{3}(0)}} - \frac{9}{e^{\frac{1}{3}(0)}} \right]$$

$$\lim_{b \rightarrow \infty} \left[\frac{9}{e^{\frac{1}{3}b}} - 0 \right] - \left[0 - 9 \right]$$

$$0 - 0 - 0 + 9$$

$\boxed{9}$

16. Determine whether the improper integral

∞

$\int_0^{\infty} \frac{e^{2x}}{1+e^{4x}} dx$ diverges or converges. Evaluate the integral if it converges.

- a. diverges
- b. converges

$$u = e^{2x}$$

$$du = 2e^{2x} dx$$

$$dx = \frac{1}{2e^{2x}} du$$

$$\lim_{b \rightarrow \infty} \int_0^b \frac{e^{2x}}{1+e^{4x}} \cdot \frac{1}{2e^{2x}} du$$

$$\hookrightarrow \lim_{b \rightarrow \infty} \frac{1}{2} \left[\arctan(e^{2x}) \right]_0^b$$

17. Determine whether the improper integral

$$\int_0^1 \frac{1}{\sqrt[3]{1-x}} dx$$

diverges or converges. Evaluate the integral if it converges.

- a. 3
- b. $\frac{2}{3}$
- c. $\frac{3}{2}$
- d. 2
- e. diverges

$$\lim_{b \rightarrow 1^-} \int_0^b (1-x)^{-\frac{1}{3}} dx$$

$u = -x$
 $du = -dx$
 $dx = -du$

$$\lim_{b \rightarrow 1^-} \left[\frac{-(1-x)^{\frac{2}{3}}}{2/3} \right]_0^b$$

$$0 - \left(-\frac{3}{2} \right) \rightarrow \boxed{\frac{3}{2}}$$

18. Determine whether the improper integral

$$\int_7^8 \frac{1}{\sqrt{64-x^2}} dx$$

diverges or converges. Evaluate the integral if it converges.

- a. $\frac{\pi}{2} - \arcsin\left(\frac{7}{8}\right)$
- b. $\frac{\pi}{16} - \arcsin\left(\frac{7}{8}\right)$
- c. $\frac{\pi}{2} - \arcsin\left(\frac{7}{64}\right)$
- d. $\frac{\pi}{16} - \arcsin\left(\frac{7}{64}\right)$
- e. diverges

$$\lim_{b \rightarrow 8^-} \int_7^b \frac{1}{\sqrt{64-x^2}} dx$$

\uparrow
 \uparrow
 x^2 u^2

$$\lim_{b \rightarrow 8^-} \left[\arcsin\left(\frac{x}{8}\right) \right]_7^b$$

$$\boxed{\frac{\pi}{2} - \arcsin\left(\frac{7}{8}\right)}$$

19. Determine whether the improper integral

$$\int_0^{\infty} \frac{10}{\sqrt{x}(x+81)} dx$$

diverges or converges. Evaluate the integral if it converges.

a. $\frac{10}{81}\pi$

b. $\frac{10}{9}\pi$

c. diverges

d. $\frac{10}{81}$

e. $\frac{10}{9}$

$$\int_0^1 \frac{10}{\sqrt{x}(x+81)} dx + \int_1^{\infty} \frac{10}{\sqrt{x}(x+81)} dx$$

$$\lim_{a \rightarrow 0^+} \int_a^1 \frac{10}{\sqrt{x}(x+81)} dx + \lim_{b \rightarrow \infty} \int_1^b \frac{10}{\sqrt{x}(x+81)} dx$$

$$u = \sqrt{x} \rightarrow x^{\frac{1}{2}} \\ du = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \frac{1}{2\sqrt{x}} dx \\ dx = 2\sqrt{x} du$$

$$\int_0^1 \frac{10}{\sqrt{x}(x+81)} dx \rightarrow \int_0^1 \frac{10}{u(u+81)} du$$

$$\rightarrow 20 \int_0^1 \frac{1}{u+81} du$$

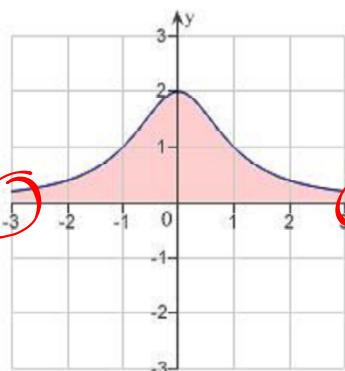
$$\lim_{a \rightarrow 0^+} \left[\frac{20}{9} \arctan \frac{\sqrt{x}}{9} \right]_a^1 + \lim_{b \rightarrow \infty} \left[\frac{20}{9} \arctan \frac{\sqrt{x}}{9} \right]_1^b$$

$$\left[\frac{20}{9} \arctan \frac{1}{9} - 0 \right] + \left[\frac{20\pi}{9} - \frac{20}{9} \arctan \frac{1}{9} \right]$$

$$\frac{20\pi}{18} \rightarrow \boxed{\frac{10\pi}{9}}$$

20. Find the area between the x -axis and the graph of

the function $y = \frac{2}{x^2+1}$.



- a. 2π
b. 2
c. 3
d. 3π
e. 0

$$\int_{-\infty}^{\infty} \frac{2}{x^2+1} dx$$

$$\lim_{a \rightarrow -\infty} \int_a^0 \frac{2}{x^2+1} dx + \lim_{b \rightarrow \infty} \int_0^b \frac{2}{x^2+1} dx$$

$$2 \lim_{a \rightarrow -\infty} [\arctan x]_a^0 + 2 \lim_{b \rightarrow \infty} [\arctan x]_0^b$$

$$2 \left[0 + \frac{\pi}{2} \right] + 2 \left[\frac{\pi}{2} - 0 \right]$$

$$\boxed{2\pi}$$