Chapter 4 Practice Test

1. Find the indefinite integral $\int \left(-9t^2 + 14t - 2\right) dt$.

$$-9\int t^2\,dt + 14\int t\,dt - 2\int dt$$

$$-9\left(\frac{t^3}{3}\right) + 14\left(\frac{t^2}{2}\right) - 2(t) + C$$

$$-3t^3 + 7t^2 - 2t + C$$

a. $-9t^3 + 14t^2 - 2t + C$

b. $-3t^3 + 14t^2 - 2t + C$

c. $-3t^3 + 7t^2 - 2t + C$

d. $-18t^2 + 14t + C$

e. $-9t^3 + 14t^2 - 2 + C$

2. Find the indefinite integral and check the result by differentiation.

$$\int \frac{3z^2+12z-9}{z^4}\,dz$$

$$\int \left(3z^{-2}+12z^{-3}-9z^{-4}\right)dz$$

$$3\left(\frac{z^{-1}}{-1}\right)+12\left(\frac{z^{-2}}{-2}\right)-9\left(\frac{z^{-3}}{-3}\right)+C$$

$$-\frac{3}{z}-\frac{6}{z^2}+\frac{3}{z^3}+C$$

a. $-\frac{3}{z}+\frac{12}{z^2}+\frac{9}{z^3}+C$

b. $-\frac{3}{z}-\frac{6}{z^2}+\frac{3}{z^3}$

c. $\frac{3}{z}-\frac{6}{z^2}+\frac{3}{z^3}+C$

d. $-\frac{3}{z}+\frac{6}{z^2}+\frac{3}{z^3}$

e. $-\frac{3}{z}-\frac{6}{z^2}+\frac{3}{z^3}+C$

3. Find the indefinite integral

$$\int 2\sec y\left(\tan y - \sec y\right)dy.$$

$2\int(\sec y \tan y - \sec^2 y)\,dy$

$2(\sec y - \tan y) + C$

a. $2\sec y \tan y + C$

b. $2\sec y - 2\tan y + C$

c. $2\sec y + \tan y + C$

d. $2 + 2\sec y + C$

e. $2\sec y \tan y - 2\sec y + C$

4. An evergreen nursery usually sells a certain shrub after 4 years of growth and shaping. The growth rate during those 4 years is approximated by $\frac{dh}{dt} = 2.5t + 6$, where t is the time in years and h is the height in centimeters. The seedlings are 15 centimeters tall when planted ($t = 0$). Find the height after t years.

initial condition $(0, 15)$ $\rightarrow$ $C = 15$

$$\int dh = \int (2.5t + 6)\,dt$$

$$h(t) = \frac{2.5t^2}{2} + 6t + C$$

$$h(t) = 1.25t^2 + 6t + 15$$

a. $h(t) = 1.25t^2 + 21t$

b. $h(t) = 1.25t^2 + 6t + 15$

c. $h(t) = 1.25t + 15$

d. $h(t) = 2.5t^2 + 6t + 15$

e. $h(t) = 2.5t + 21$

5. Find the limit of $s(n)$ as $n \to \infty$.

$$s(n) = \frac{5}{n^3}\left[\frac{n^3(n+1)^2}{7}\right]$$

a. 5/7

b. 5

c. 1/7

d. 10/7

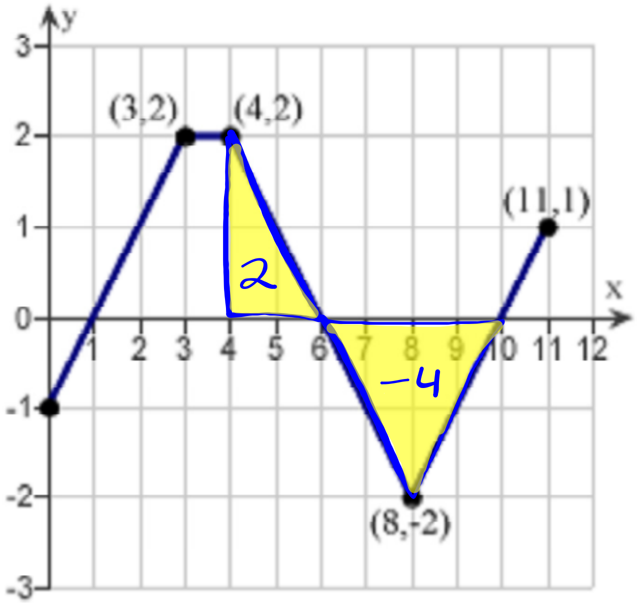
e. unbounded

$$\frac{5n^5 + \cdots}{7n^3}$$

$5 > 3$

6. The graph of f consists of line segments, as shown in the figure. Evaluate the definite integral

$$\int_4^{10} f(x)\,dx$$ using geometric formulas.



a. –3
b. –1
c. –2
d. 1
e. 0

7. Find the area of the region bounded by the graphs of the equations $y = x^3 + x$, $x = 4$, $y = 0$. Round your answer to the nearest whole number.

a. 768
b. 96
c. 49
d. 72
e. 16

$$\int_0^4 (x^3 + x)\,dx$$

$$\left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^4 = (64 + 8) - (0 + 0) = 72$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$

8. Find the average value of $f(x) = \dfrac{5\left(x^2+5\right)}{x^2}$ on the interval $[1,3]$.

a. $\dfrac{40}{3}$

b. 35

c. $\dfrac{25}{3}$

d. $\dfrac{80}{3}$

e. 40

$$\frac{1}{3-1}\int_1^3 \left(5+25x^{-2}\right)dx$$

$$\frac{1}{2}\left[5x-\frac{25}{x}\right]_1^3$$

$$\frac{1}{2}\left[15-\frac{25}{3}-(5-25)\right]$$

$$\frac{1}{2}\left[\frac{20}{3}+20\right] = \boxed{\frac{40}{3}}$$

9. Determine all values of x in the interval $[1,3]$ for which the function $f(x) = \dfrac{2(x^2+4)}{x^2}$ equals its average value $\dfrac{14}{3}$.

a. $x = 4$

b. $x = \pm\sqrt{2}$

c. $x = 2$

d. $x = \sqrt{3}$

e. $x = \pm\sqrt{3}$

$$\frac{2(x^2+4)}{x^2} = \frac{14}{3}$$

$$14x^2 = 6x^2 + 24$$

$$8x^2 - 24 = 0$$

$$8(x^2 - 3) = 0$$

$$x = \pm\sqrt{3}$$

only $\sqrt{3}$

10. Find the indefinite integral $\int 5z^4 \cos z^5 \, dz$.

$u = z^5$

$du = 5z^4 dz$

a. $\cos z^5 + C$

b. $\dfrac{\sin z^6}{6} + C$

c. $\sin z^4 + C$

d. $\sin z^5 + C$

e. $\dfrac{\sin z^5}{5} + C$

$\int \cos u \, du$

$\sin u + c$

$\sin(z^5) + c$

11. Find the indefinite integral $\int u^3\sqrt{4+u^4}\,du$.

a. $\dfrac{2\left(4+u^4\right)^{\frac{2}{3}}}{12}+C$

b. $\dfrac{\left(4+u^4\right)^{\frac{3}{2}}}{8}+C$

c. $\dfrac{\left(4+u^4\right)^{\frac{3}{2}}}{12}+C$

d. $\dfrac{2\left(4+u^4\right)^{\frac{5}{2}}}{20}+C$

e. $\dfrac{2\left(4+u^4\right)^{\frac{3}{2}}}{12}+C$

$x = 4+u^4$

$dx = 4u^3du$

$\frac{1}{4}dx = u^3du$

$\frac{1}{4}\int \sqrt{x}\,dx$

$\frac{1}{4}\left(\frac{2}{3}x^{\frac{3}{2}}\right)+c$

$\frac{2}{12}(4+u^4)^{\frac{3}{2}}+C$

12. Find the indefinite integral of the following function and check the result by differentiation.

$$\int \frac{4u^3}{\sqrt{u^4+5}}\,du$$

a. $2\sqrt{u^3+5}+C$

b. $\sqrt{u^4+5+C}$

c. $\frac{1}{2}\sqrt{u^4+5}+C$

d. $\frac{1}{2}\sqrt{u^3+5}+C$

e. $2\sqrt{u^4+5}+C$

$z = u^4+5$

$dz = 4u^3 du$

$= \int \frac{dz}{\sqrt{z}}$

$= \int z^{-\frac{1}{2}} dz$

$= 2z^{\frac{1}{2}} + C$

$= 2\sqrt{u^4+5} + C$

13. Solve the differential equation.

$$\frac{df}{dz} = 8z + \frac{9z}{\sqrt{9-z^2}}$$

a. $f(z) = 4z^2 - \frac{9}{2}\sqrt{9-z^2} + C$

b. $f(z) = 4z^2 - \frac{9}{\sqrt{9-z^2}} + C$

c. $f(z) = 4z^2 + 9\sqrt{9-z^2} + C$

d. $f(z) = 4z^2 - 9\sqrt{9-z^2} + C$

e. $f(z) = 4z - 9\sqrt{9-z^2} + C$

$$\int df = \int \left(8z + 9\frac{z}{\sqrt{9-z^2}}\right) dz$$

$$f(z) = 4z^2 + 9\int \frac{z\,dz}{\sqrt{9-z^2}}$$

$u = 9 - z^2$

$du = -2z\,dz$

$-\frac{1}{2}du = z\,dz$

$$f(z) = 4z^2 + 9\left(-\frac{1}{2}\int \frac{1}{\sqrt{u}}du\right)$$

$$f(z) = 4z^2 - \frac{9}{2}\left(2u^{\frac{1}{2}}\right) + C$$

$$\boxed{f(z) = 4z^2 - 9\sqrt{9-z^2} + C}$$

14. Find the indefinite integral of the following function.

$$\int \sin 2x \, dx$$

$u = 2x$

$du = 2dx$

$\frac{1}{2} du = dx$

$\frac{1}{2} \int \sin u \, du$

$\frac{1}{2}(-\cos u) + c$

$\boxed{-\frac{\cos 2x}{2} + C}$

a. $-2\cos 2x + C$

b. $-\cos 2x + C$

c. $\dfrac{-\cos 2x}{3} + C$

d. $\dfrac{-\cos 2x}{2} + C$

e. $\sin 2x + C$

15. Find the indefinite integral of the following function.

$$\int \pi \cos 3\pi s \, ds$$

a. $3\sin 3\pi s + C$

b. $\dfrac{\cos 3\pi s}{3} + C$

c. $\dfrac{\sin 3\pi s}{3} + C$

d. $\dfrac{\sin 3s}{3} + C$

e. $\dfrac{\sin 3\pi s}{4} + C$

$u = 3\pi s$

$du = 3\pi \, ds$

$\frac{1}{3} du = \pi \, ds$

$\frac{1}{3} \int \cos u \, du$

$\frac{1}{3} \sin u + c$

$\frac{1}{3} \sin 3\pi s + C$

16. Find the indefinite integral of the following function.

$$\int \frac{\sin t}{\cos^7 t}\,dt$$

a. $\dfrac{(\cos t)^{-6}}{7} + C$

b. $\dfrac{(\cos t)^{-6}}{6} + C$

c. $\dfrac{(\cos t)^{-7}}{6} + C$

d. $\dfrac{(\sin t)^{-6}}{7} + C$

e. $\dfrac{(\sin t)^{-6}}{6} + C$

method 1

$u = \cos t$

$du = -\sin t\,dt$

$-\int \frac{du}{u^7}$

$-\int u^{-7}\,du$

$\frac{u^{-6}}{6} + C$

$\frac{(\cos t)^{-6}}{6} + C$

method 2

$\int (\tan t \sec t)(\sec^5 t)\,dt$

$u = \sec t$

$du = \sec t \tan t\,dt$

$\int u^5\,du$

$\frac{u^6}{6} + C$

$\frac{\sec^6 t}{6} + C$

17. Find an equation for the function $f(x)$ whose derivative is $f'(x) = 7\sin(49x)$ and whose graph passes through the point $\left(\frac{\pi}{49}, -\frac{6}{7}\right)$.

a. $f(x) = -\frac{1}{7}\cos(49x) - 1$

b. $f(x) = \frac{1}{14}\cos(49x) + 1$

c. $f(x) = \frac{1}{49}\cos(98x) + 14$

d. $f(x) = -\frac{1}{7}\cos(49x) - 49$

e. $f(x) = -\frac{1}{98}\cos(7x) + 2$

$$f(x) = \frac{-\cos 49x}{7} + c$$

$$-\frac{6}{7} = \frac{-\cos\left(\cancel{49}\left(\frac{\pi}{\cancel{49}}\right)\right)}{7} + c$$

$$-\frac{6}{7} = \frac{1}{7} + c$$

$$-1 = c$$

18. Find $F'(x)$ given

$$F(x) = \int_{-3x}^{3x} t^4\,dt. = 2\int_0^{3x} t^4\,dt$$

$$F'(x) = 2\left(\underset{\uparrow}{3}\left(3x\right)^4\right) = 486x^4$$

from chain rule

19. Find $F'(x)$ given

$$F(x) = \int_{-3x}^{3x} t^5 \, dt. = 0$$

odd function

20. Find $F'(x)$ given

$$F(x) = \int_{x}^{x+3} (10t+1)\,dt.$$

$F(x) = \left[5t^2 + t\right]_x^{x+3}$

$F(x) = \left[5(x+3)^2 + (x+3)\right] - \left[5x^2 + x\right]$

$F(x) = \left[5(\cancel{x^2} + 6x + 9) + \cancel{x} + 3\right] - \cancel{5x^2} - \cancel{x}$

$F(x) = 30x + 48$

$F'(x) = 30$

or FTC pt. 2

$\int_x^2 + \int_2^{x+3}$

$\int_2^{x+3} - \int_2^x$

$10(x+3)+1 - (10x+1)$

$10x + 31 - 10x - 1$

$= 30$

MULTIPLE CHOICE

1. C
2. E
3. B
4. B
5. E
6. C
7. D
8. A
9. D
10. D
11. E
12. E
13. D
14. D
15. C
16. B
17. A

SHORT ANSWER

18. $F'(x) = \cancel{162x}^4$ 486x^4
19. $F'(x) = 0$
20. $F'(x) = 30$