Name: ________________________ Class: ___________________ Date: __________ ID: A

AP Calculus AB Chapter 3 Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

$g'(t) = \sqrt{7-t} + t\left(\frac{-1}{2\sqrt{7-t}}\right)$

____ 1. Find any critical numbers of the function $g(t) = t\sqrt{7-t}$, $t < 7$.

a. $\frac{7}{3}$
b. $\frac{14}{3}$
c. $-\frac{7}{3}$
d. $-\frac{14}{3}$
e. 0

(1) derive
(2) set to 0
(3) solve for t

$\frac{t}{2\sqrt{7-t}} = \sqrt{7-t}$
$t = 2(7-t)$
$t = 14 - 2t$
$3t = 14$
$t = \frac{14}{3}$

____ 2. Find all critical numbers of the function $f(x) = \sin^2 6x + \cos 6x$, $0 < x < \frac{\pi}{3}$.

a. $\frac{\pi}{36}, \frac{\pi}{6}, \frac{2\pi}{9}$
b. $\frac{\pi}{6}, \frac{5\pi}{24}, \frac{7\pi}{24}$
c. $\frac{\pi}{18}, \frac{\pi}{12}, \frac{\pi}{9}, \frac{\pi}{4}$
d. $\frac{\pi}{24}, \frac{\pi}{6}, \frac{\pi}{2}$
e. $\frac{\pi}{18}, \frac{\pi}{6}, \frac{5\pi}{18}$

$f'(x) = 2(\sin 6x)(\cos 6x)(6) - (\sin 6x)(6)$
$f'(x) = 12\sin 6x\cos 6x - 6\sin 6x$
$0 = 6\sin 6x(2\cos 6x - 1)$
$\sin 6x = 0$ $\cos 6x = \frac{1}{2}$
$6x = 0, \pi, 2\pi, 3\pi$ $6x = \frac{\pi}{3}, \frac{5\pi}{3}$
$x = \cancel{0}, \frac{\pi}{6}, \cancel{\frac{\pi}{3}}, \cancel{\frac{\pi}{2}}$ $x = \frac{\pi}{18}, \frac{5\pi}{18}$

____ 3. Locate the absolute extrema of the function $f(x) = x^3 - 3x$ on the closed interval $[0, 4]$.

a. absolute max: $(4, 52)$; no absolute min
b. absolute max: $(1, -2)$; absolute min: $(4, 52)$
c. no absolute max; absolute min: $(4, 52)$
d. absolute max: $(4, 52)$; absolute min: $(1, -2)$
e. no absolute max or min

check endpoints

$f'(x) = 3x^2 - 3$
$0 = 3(x^2 - 1)$
$x = \pm 1$
$f(0) = 0$
$f(1) = -2$ min
$f(4) = 52$ max

Name: ______________________ $g'(t) = \frac{2t(t^2+3) - 2t(t^2)}{(t^2+3)^2} = \frac{6t}{(t^2+3)^2}$ ID: A

____ 4. Locate the absolute extrema of the function $g(t) = \frac{t^2}{t^2+3}$ on the closed interval $[-2, 2]$.

crit # $x = 0$

$f(0) = 0$ min

$\left.\begin{aligned} f(-2) &= \tfrac{4}{7} \\ f(2) &= \tfrac{4}{7} \end{aligned}\right\}$ max

a. The absolute maximum is $\frac{4}{7}$, and it occurs only at the right endpoint $x = 2$
The absolute minimum is 0 and it occurs at the critical number $x = 0$.

b. The absolute maximum is $\frac{4}{7}$, and it occurs only at the left endpoint $x = -2$.
The absolute minimum is 0 and it occurs at the critical number $x = 0$.

c. The absolute maximum is $\frac{4}{7}$, and it occurs at either endpoint $x = \pm 2$.
The absolute minimum is 0, and it occurs at the critical number $x = 0$.

d. The absolute maximum is $\frac{4}{7}$, and it occurs at the critical number $x = 0$.
The absolute minimum is $\frac{4}{13}$, and it occurs at the left endpoint $x = -2$.

e. The absolute maximum is $\frac{4}{7}$, and it occurs at the critical number $x = 0$.
The absolute minimum is $\frac{4}{13}$, and it occurs at the right endpoint $x = 2$.

____ 5. Determine whether Rolle's Theorem can be applied to the function $f(x) = x^2 - 6x + 8$ on the closed interval [2,4]. If Rolle's Theorem can be applied, find all values of c in the open interval (2,4) such that $f'(c) = 0$.

polynomials are both continuous and differentiable

$f'(x) = 2x - 6$

crit # $x = 3$

$f(2) = 0 = f(4)$

$f'(c) = 0$ in the interval $(2, 4)$

a. Rolle's Theorem applies; $c = 3$
b. Rolle's Theorem applies; c = 3.5
c. Rolle's Theorem applies; $c = 2.5$
d. Rolle's Theorem applies; $c = -3$
e. Rolle's Theorem does not apply

____ 6. Determine whether Rolle's Theorem can be applied to $f(x) = \frac{x^2 - 15}{x}$ on the closed interval $[-15, 15]$. If Rolle's Theorem can be applied, find all values of c in the open interval $(-15, 15)$ such that $f'(c) = 0$.

a. $c = 7, c = 2$
b. $c = 2, c = 15$
c. $c = 15$
d. $c = 7$
e. Rolle's Theorem does not apply

$x \neq 0$
discontinuity in the interval

____ 7. Determine whether the Mean Value Theorem can be applied to the function $f(x)=x^2$ on the closed interval [–2,8]. If the Mean Value Theorem can be applied, find all numbers c in the open interval (–2,8) such that $f'(c)=\dfrac{f(8)-f(-2)}{8-(-2)}$. $= \dfrac{64-4}{8+2} = 6$ $\quad f'(x)=2x$

a. MVT applies; $c = 3$
b. MVT applies; c = 2
c. MVT applies; $c = 4$
d. MVT applies; $c = 1$
e. MVT applies; $c = 5$

$2x = 6$

when $x=3$

____ 8. The height of an object t seconds after it is dropped from a height of 650 meters is $s(t)=-4.9t^2+650$. Find the time during the first 6 seconds of fall at which the instantaneous velocity equals the average velocity.

a. 3 seconds
b. 2.45 seconds
c. 22.11 seconds
d. 14.7 seconds
e. 18 seconds

$\dfrac{s(6)-s(0)}{6-0} = s'(t) \longrightarrow \dfrac{473.6-650}{6} = -9.8t$

$-29.4 = -9.8t$

$3 = t$

____ 9. Identify the open intervals where the function $f(x)=x\sqrt{6-x^2}$ is increasing or decreasing. $-\sqrt{6} \le x \le \sqrt{6}$

a. increasing: $\left(-\sqrt{6},\sqrt{3}\right)\cup\left(\sqrt{3},\sqrt{6}\right)$; decreasing: $\left(-\sqrt{3},\sqrt{3}\right)$
b. increasing: $\left(-\infty,\sqrt{6}\right)$; decreasing: $\left(\sqrt{6},\infty\right)$
c. decreasing on $\left(-\infty,\infty\right)$
d. increasing: $\left(-\sqrt{3},\sqrt{3}\right)$; decreasing: $\left(-\sqrt{6},-\sqrt{3}\right)\cup\left(\sqrt{3},\sqrt{6}\right)$
e. decreasing: $\left(-\infty,\sqrt{3}\right)$; increasing: $\left(\sqrt{3},\infty\right)$

$f'(x)=\sqrt{6-x^2}+x\left(\dfrac{-2x}{2\sqrt{6-x^2}}\right)$

$f'(x)=\dfrac{2(3-x^2)}{\sqrt{6-x^2}}$

crit #s $x=\pm\sqrt{3}$

	$-\sqrt{6}$ – $-\sqrt{3}$	$-\sqrt{3}$ – $\sqrt{3}$	$\sqrt{3}$ – $\sqrt{6}$
$f'(x)$	−	+	−

____ 10. Identify the open intervals on which the function $y=3x-6\cos x,\ 0<x<2\pi$ is increasing or decreasing.

a. increasing on $\left(\frac{7\pi}{6},\frac{5\pi}{6}\right)$ and $\left(\frac{5\pi}{6},2\pi\right)$; decreasing on $\left(\frac{5\pi}{6},\frac{11\pi}{6}\right)$
b. increasing on $\left(0,\frac{7\pi}{6}\right)$ and $\left(\frac{11\pi}{6},2\pi\right)$; decreasing on $\left(\frac{7\pi}{6},\frac{11\pi}{6}\right)$
c. increasing on $\left(0,\frac{11\pi}{6}\right)$ and $\left(\frac{7\pi}{6},2\pi\right)$; decreasing on $\left(0,\frac{7\pi}{6}\right)$
d. increasing on $\left(0,\frac{7\pi}{6}\right)$ and $\left(\frac{5\pi}{6},2\pi\right)$; decreasing on $\left(\frac{7\pi}{6},\frac{5\pi}{6}\right)$
e. increasing on $\left(\frac{7\pi}{6},\frac{11\pi}{6}\right)$; decreasing on $\left(0,\frac{7\pi}{6}\right)$ and $\left(\frac{11\pi}{6},2\pi\right)$

$y' = 3+6\sin x$

$0 = 3+6\sin x$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

	0 $\frac{2\pi}{6}$	$\frac{7\pi}{6}$ $\frac{11\pi}{6}$	$\frac{11\pi}{6}$ 2π
$f'(x)$	+ inc	− dec	+ inc

3

____ 11. For the function $f(x) = (x-1)^{\frac{4}{7}}$:

$f'(x) = \frac{4}{7}(x-1)^{-\frac{3}{7}} = \frac{4}{7(x-1)^{3/7}}$

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

crit #s $x = 1$

Use a graphing utility to confirm your results.

	$-\infty$ |1| ∞	
$f'(x)$	$-$ inc	$+$ dec

min at $(1, 0)$

a. (a) $x = 1$
(b) increasing: $(-\infty, 1)$; decreasing: $(1, \infty)$
(c) relative max: $f(1) = 0$

b. (a) $x = 0, 1$
(b) decreasing: $(-\infty, 0) \cup (1, \infty)$; increasing: $(0, 1)$
(c) relative min: $f(0) = 1$; relative max: $f(1) = 0$

c. (a) $x = 0$
(b) increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$
(c) relative max: $f(0) = 1$

d. (a) $x = 1$
(b) decreasing: $(-\infty, 1)$; increasing: $(1, \infty)$
(c) relative min: $f(1) = 0$

e. (a) $x = 0$
(b) decreasing: $(-\infty, 0)$; increasing: $(0, \infty)$
(c) relative min: $f(0) = 1$

____ 12. Find all points of inflection on the graph of the function $f(x) = \frac{1}{2}x^4 + 2x^3$.

a. $(0,0)$
b. $(0,0)(-2,-8)$
c. $(-2,-8)$
d. $(-2,-8)$
e. $(0,0),(-4,0)$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x$

$0 = 6x(x+2)$

$x = 0 \quad x = -2$

	$-\infty$ -2	-2 0	0 ∞
$f''(x)$	$+$ cu	$-$ cd	$+$ cu

both are poi

Name: ________________________ **ID: A**

____ 13. Find the point of inflection of the graph of the function $f(x) = 4\sin\frac{x}{8}$ on the interval $[0, 16\pi]$.

a. $(9\pi, 0)$
b. $(8\pi, 4)$
c. $(0, 0)$
d. $(4\pi, 0)$
e. $(8\pi, 0)$

$f'(x) = \frac{1}{2}\cos\frac{x}{8}$

$f''(x) = \frac{-1}{16}\sin\frac{x}{8}$

$0 = -\frac{1}{16}\sin\frac{x}{8}$

$x = 0, 8\pi, 16\pi$

	0 8π	8π 16π
$f''(x)$	$-$ cd	$+$ cu

$f(8\pi) = 4\sin\pi = 0$

____ 14. Find the points of inflection and discuss the concavity of the function $f(x) = x + 7\cos x$ on the interval $[0, 2\pi]$.

a. concave down on $(0, 2\pi)$; no points of inflection

b. concave upward on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; concave downward on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$; inflection points at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

c. concave downward on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; concave upward on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$; inflection points at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

d. concave up on $(0, 2\pi)$; no points of inflection

e. none of the above

$f'(x) = 1 - 7\sin x$

$f''(x) = -7\cos x$

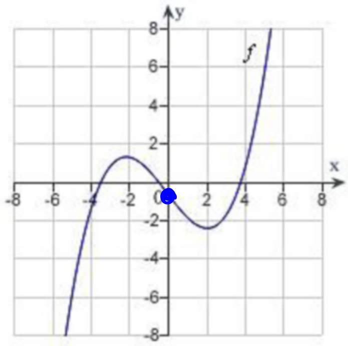
$0 = -7\cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

	0 $\frac{\pi}{2}$	$\frac{\pi}{2}$ $\frac{3\pi}{2}$	$\frac{3\pi}{2}$ 2π
$f''(x)$	$-$ cd	$+$ cu	$-$ cd

Name: ________________________ ID: A

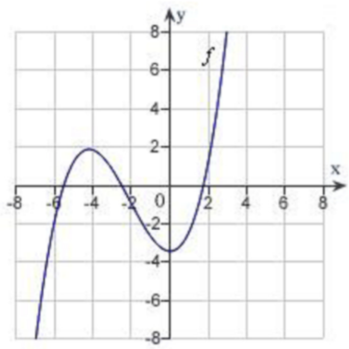
____ 15. The graph of f is shown below. For which value of x is $f''(x)$ zero?



this means "where is poi"

a. $x = 2$
b. $x = 0$
c. $x = 4$
d. $x = -4$
e. $x = -6$

____ 16. The graph of f is shown below. On what interval is f' an increasing function?

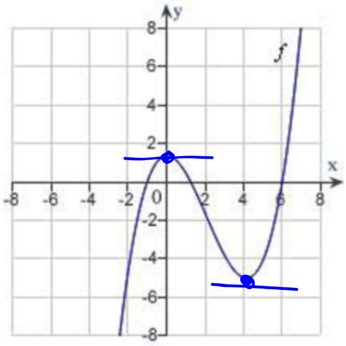


this means "where is it concave up"

a. $(0, \infty)$
b. $(1, \infty)$
c. $(-2, \infty)$
d. $(2, \infty)$
e. $(-4, \infty)$

Name: ________________________ ID: A

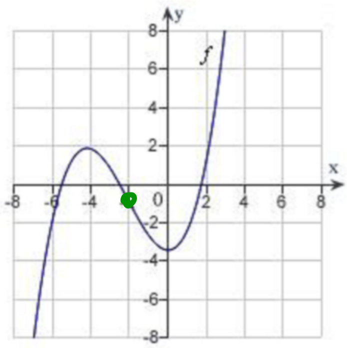
____ 17. The graph of f is shown below. For which values of x is $f'(x)$ zero?



where is the tangent line or slope = 0

a. $x = 0; x = 4$
b. $x = 0; x = -1$
c. $x = -2; x = 1$
d. $x = 2; x = 0$
e. $x = 0; x = 6$

____ 18. The graph of f is shown below. For which value of x is $f'(x)$ minimum?



this might be the trickiest

this means "where does $f''(x)$ go from negative (cd) to positive (cu)

poi!

a. $x = 6$
b. $x = 2$
c. $x = 4$
d. $x = -2$
e. $x = 0$

Name: ________________________ **ID: A**

____ 19. A farmer plans to fence a rectangular pasture adjacent to a river (see figure). The pasture must contain 245,000 square meters in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing?

$245000 = xy$

$y = \frac{245000}{x}$

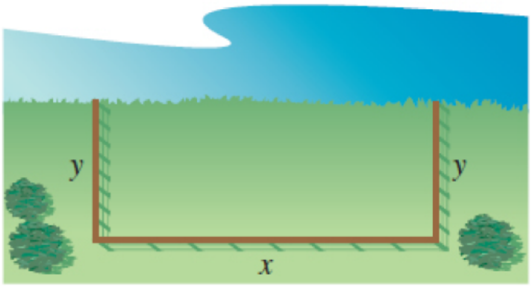
$P = 2y + x$

$P = \frac{490000}{x} + x$

$P' = -\frac{490000}{x^2} + 1$

$x = 700$

$y = 350$



a. $x = 1000$ and $y = 245$
b. $x = 350$ and $y = 700$
c. $x = 700$ and $y = 350$
d. $x = 245$ and $y = 1000$
e. none of the above

____ 20. Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $(6,0)$.

a. $\left(\sqrt{\frac{13}{2}}, \frac{11}{2}\right)$

b. $\left(\frac{13}{2}, \sqrt{\frac{13}{2}}\right)$

c. $\left(\frac{11}{2}, \sqrt{\frac{11}{2}}\right)$

d. $\left(\sqrt{\frac{11}{2}}, \frac{11}{2}\right)$

e. $\left(\frac{11}{2}, \sqrt{\frac{13}{2}}\right)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-6)^2 + (y-0)^2}$

$d = \sqrt{(x-6)^2 + (\sqrt{x} - 0)^2}$

$d = \sqrt{x^2 - 12x + 36 + x}$

$d = \sqrt{x^2 - 11x + 36}$

$d = x^2 - 11x + 36$

$d' = 2x - 11$

$0 = 2x - 11$

$x = \frac{11}{2}$

AP Calculus AB Chapter 3 Practice Test
Answer Section

MULTIPLE CHOICE

1. B
2. E
3. D
4. C
5. A
6. E
7. A
8. A
9. D
10. B
11. D
12. B
13. E
14. B
15. B
16. C
17. A
18. D
19. ~~A~~ C
20. C