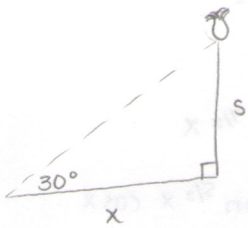
Chapter 2 Review

pg 158-160 # 3, 5, 8, 9-10, 12, 15-30 (odd), 33-36, 39, 41-53 (odd), 58-59, 64, 67-77 (odd), 101-105 (odd), 109-110, 113

3.



$s(t) = 60 - 4.9t^2$

$\frac{dx}{dt} = ?$ when $s = 35$ m

$\frac{ds}{dt} = -9.8t$

$\tan 30 = \frac{s}{x}$

$x = \frac{s}{\tan 30}$

$35 = 60 - 4.9t^2$

$t = 2.259$

$\frac{dx}{dt} = \frac{1}{\tan 30}\frac{ds}{dt}$

$= \frac{1}{\tan 30}(-9.8t)$

$= \frac{1}{\tan 30}(-9.8(2.259))$

$= -38.34$ m/s

10. $SA = 6s^2$ $\quad \frac{ds}{dt} = 5$ cm/sec

$\frac{dSA}{dt} = ?$ when $s = 4.5$ cm

$\frac{dSA}{dt} = 12s\frac{ds}{dt}$

$= 12(4.5)(5)$

$= 270$ cm^2/sec

09. $y = \sqrt{x}$ $\quad \frac{dy}{dt} = 2$ u/sec

$\frac{dx}{dt} = ?$

$\frac{dy}{dt} = \frac{1}{2}x^{-1/2}\frac{dx}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt}$

(a) $x = \frac{1}{2}$

$2 = \frac{1}{2\sqrt{\frac{1}{2}}}\frac{dx}{dt}$

$4\sqrt{\frac{1}{2}} = \frac{dx}{dt}$

$2\sqrt{2}$ u/sec $= \frac{dx}{dt}$

(b) $x = 1$

$2 = \frac{1}{2\sqrt{1}}\frac{dx}{dt}$

4 u/sec $= \frac{dx}{dt}$

(c) $x = 4$

$2 = \frac{1}{2\sqrt{4}}\frac{dx}{dt}$

8 u/sec $= \frac{dx}{dt}$

105. $x\sin y = y\cos x$

$\sin y + x\cos y\frac{dy}{dx} = \frac{dy}{dx}\cos x + y(-\sin x)$

$x\cos y\frac{dy}{dx} - \frac{dy}{dx}\cos x = -\sin y - y\sin x$

$\frac{dy}{dx}(x\cos y - \cos x) = -\sin y - y\sin x$

$\frac{dy}{dx} = \frac{-\sin y - y\sin x}{x\cos y - \cos x}$

103. $y\sqrt{x} - x\sqrt{y} = 16$

$yx^{1/2} - xy^{1/2} = 16$

$\frac{dy}{dx}x^{1/2} + y(\frac{1}{2}x^{-1/2}) - [y^{1/2} + x\frac{1}{2}y^{-1/2}\frac{dy}{dx}] = 0$

$\frac{dy}{dx}x^{1/2} + \frac{1}{2}yx^{-1/2} - y^{1/2} - \frac{1}{2}xy^{-1/2}\frac{dy}{dx} = 0$

$\frac{dy}{dx}x^{1/2} - \frac{1}{2}xy^{-1/2}\frac{dy}{dx} = -\frac{1}{2}yx^{-1/2} + y^{1/2}$

$$\frac{dy}{dx}\left(x^{1/2} - \frac{1}{2}xy^{-1/2}\right) = -\frac{1}{2}yx^{-1/2} + y^{1/2}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}yx^{-1/2} + y^{1/2}}{x^{1/2} - \frac{1}{2}xy^{-1/2}}$$

$$= \frac{\frac{-y}{2\sqrt{x}} + \sqrt{y}}{\sqrt{x} - \frac{x}{2\sqrt{y}}}$$

$$= \frac{\frac{-y + 2\sqrt{xy}}{2\sqrt{x}}}{\frac{2\sqrt{xy} - x}{2\sqrt{y}}}$$

$$= \frac{-y + 2\sqrt{xy}}{2\sqrt{x}} \cdot \frac{2\sqrt{y}}{2\sqrt{xy} - x}$$

$$= \frac{-2y\sqrt{y} + 4y\sqrt{x}}{4x\sqrt{y} - 2x\sqrt{x}}$$

$$= \frac{-y\sqrt{y} + 2y\sqrt{x}}{2x\sqrt{y} - x\sqrt{x}}$$

101. $x^2 + 3xy + y^3 = 10$

$$2x + 3y + 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx}(3x + 3y^2) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{3x + 3y^2}$$

77. $y = \dfrac{\sin \pi x}{x+2}$

$$y' = \frac{\pi\cos(\pi x)(x+2) - [\sin \pi x]}{(x+2)^2}$$

$$= \frac{\pi(x+2)\cos(\pi x) - \sin(\pi x)}{(x+2)^2}$$

75. $y = \frac{2}{3}\sin^{3/2} x - \frac{2}{7}\sin^{7/2} x$

$$y' = \sin^{1/2} x \cos x - \sin^{5/2} x \cos x$$

$$= \sin^{1/2} x \cos x (1 - \sin^2 x)$$

$$= \sin^{1/2} x \cos x \cos^2 x$$

$$= \sin^{1/2} x \cos^3 x$$

73. $y = \dfrac{x}{2} - \dfrac{\sin(2x)}{4}$

$$y' = \frac{1}{2} - \frac{1}{4}\cos(2x)(2)$$

$$= \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$= \frac{1}{2}(1 - \cos(2x))$$

$$= \sin^2 x$$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

71. $y = 3\cos(3x+1)$

$$y' = 3(-\sin(3x+1))(3)$$

$$= -9\sin(3x+1)$$

69. $f(s) = (s^2-1)^{5/2}(s^3+5)$

$$f'(s) = \frac{5}{2}(s^2-1)^{3/2}(2s)(s^3+5) + (s^2-1)^{5/2}(3s^2)$$

$$= 5s(s^2-1)^{3/2}(s^3+5) + 3s^2(s^2-1)^{5/2}$$

$$= s(s^2-1)^{3/2}[5(s^3+5) + 3s(s^2-1)]$$

$$= s(s^2-1)^{3/2}(5s^3 + 25 + 3s^3 - 3s)$$

$$= s(s^2-1)^{3/2}(8s^3 - 3s + 25)$$

57. $h(x) = \left(\frac{x-3}{x^2+1}\right)^2$

$$h'(x) = 2\left(\frac{x-3}{x^2+1}\right)\left(\frac{(x^2+1) - [(x-3)(2x)]}{(x^2+1)^2}\right)$$

$$= 2\left(\frac{(x-3)(x^2+1) - (2x^2-6x)}{(x^2+1)^3}\right)$$

$$= 2\left(\frac{x^3+x-3x^2-3-2x^2+6x}{(x^2+1)^3}\right)$$

$$= 2\left(\frac{x^3-5x^2+7x-3}{(x^2+1)^3}\right)$$

54. $h(t) = 4\sin t - 5\cos t$

$h'(t) = 4\cos t + 5\sin t$

$h''(t) = -4\sin t + 5\cos t$

9. $v(t) = 36 - t^2$ $\quad v(4) = 36 - 16 = 20$ m/s

$a(t) = -2t$ $\quad a(4) = -2(4) = -8$ m/s^2

8. $f(x) = \frac{1+\sin x}{1-\sin x}$ tangent line at $(\pi, 1)$

$$f'(x) = \frac{\cos x(1-\sin x) - [(1+\sin x)(-\cos x)]}{(1-\sin x)^2}$$

$$= \frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{(1-\sin x)^2}$$

$$f'(\pi) = \frac{\cos\pi(1-\sin\pi) + \cos\pi(1+\sin\pi)}{(1-\sin\pi)^2}$$

$$= \frac{(-1)(1) + (-1)(1)}{1}$$

$$= -2$$

$y - 1 = -2(x - \pi)$

$y - 1 = -2x + 2\pi$

$y = -2x + 2\pi + 1$

53. $y = x\cos x - \sin x$

$y' = \cos x + x(-\sin x) - \cos x$

$= \cos x - x\sin x - \cos x$

$= -x\sin x$

51. $y = 3x^2\sec x$

$y' = 6x\sec x + 3x^2\sec x\tan x$

49. $y = \frac{x^2}{\cos x}$

$$y' = \frac{2x\cos x - [x^2(-\sin x)]}{\cos^2 x}$$

$$= \frac{2x\cos x + x^2\sin x}{\cos^2 x}$$

47. $f(x) = \frac{1}{4-3x^2} = (4-3x^2)^{-1}$

$f'(x) = -(4-3x^2)^{-2}(-6x)$

$= \frac{6x}{(4-3x^2)^2}$

45. $f(x) = \frac{x^2+x-1}{x^2-1}$

$$f'(x) = \frac{(2x+1)(x^2-1) - [(x^2+x-1)(2x)]}{(x^2-1)^2}$$

$$= \frac{2x^3-2x+x^2-1-2x^3-2x^2+2x}{(x^2-1)^2}$$

$$= \frac{-x^2-1}{(x^2-1)^2}$$

$$= \frac{-(x^2+1)}{(x^2-1)^2}$$

43. $h(x) = \sqrt{x}\sin x$

$h'(x) = \frac{1}{2}x^{-1/2}\sin x + x^{1/2}\cos x$

$= \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$

11. $f(x) = (3x^2+7)(x^2-2x+3)$

$f'(x) = 6x(x^2-2x+3) + (3x^2+7)(2x-2)$

$= 6x^3 - 12x^2 + 18x + 6x^3 - 6x^2 + 14x - 14$

$= 12x^3 - 18x^2 + 32x - 14$

$= 2(6x^3 - 9x^2 + 16x - 7)$

9. $x(t) = t^2 - 3t + 2$

(a) $v(t) = 2t - 3$

(b) $2t - 3 < 0$

$t < \frac{3}{2}$

(c) $0 = 2t - 3$

$t = \frac{3}{2}$

$s\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2$

$= \frac{9}{4} - \frac{9}{2} + 2$

$= \frac{9 - 18 + 8}{4}$

$= -\frac{1}{4}$

(d) $t^2 - 3t + 2 = 0$

$(t-1)(t-2) = 0$

$t = 1, 2$

$|v(1)| = |2(1) - 3| = 1$

$|v(2)| = |2(2) - 3| = 1$

6. $s(t) = -16t^2 + v_0 t + s_0$

$s(t) = -16t^2 + 14{,}400$

$-16t^2 + 14{,}400 = 0$

$t = 30$ sec

$600 \text{ mi/hr} = \frac{1}{6} \text{ mi/sec}$

$\frac{1}{6}(30) = 5$ miles

35. $s(t) = -16t^2 + s_0$

$-16(9.2)^2 + s_0 = 0$

$s_0 = 1{,}354.24$ ft

34. $s(t) = -16t^2 + 100$ (ball a)

$-16t^2 + 100 = 0$

$t = 2.5$ sec

$s(t) = -16t^2 + 75$ (ball b)

$-16t^2 + 75 = 0$

$t = 2.17$ sec

First ball

8. $f(x) = \begin{cases} x^2 + 4x + 2 & x < -2 \\ 1 - 4x - x^2 & x \geq -2 \end{cases}$

(a) $-2, -11$

Non-removable discontinuity

(b) Discontinuous $\Rightarrow$ not differentiable

3. $f(x) = \frac{x+1}{x-1}$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{x+\Delta x+1}{x+\Delta x-1} - \frac{x+1}{x-1}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{(x+\Delta x+1)(x-1) - (x+1)(x+\Delta x-1)}{(x-1)(x+\Delta x-1)}}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{\dfrac{x^2 + x\Delta x + x - x - \Delta x - 1 - x^2 - x\Delta x + x - x - \Delta x + 1}{(x-1)(x+\Delta x-1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x\,(x-1)(x+\Delta x-1)}$$

$$= \lim_{\Delta x \to 0} \frac{-2}{(x-1)(x+\Delta x-1)}$$

$$= \frac{-2}{(x-1)^2}$$