AP Calculus

Chapter 1 Test

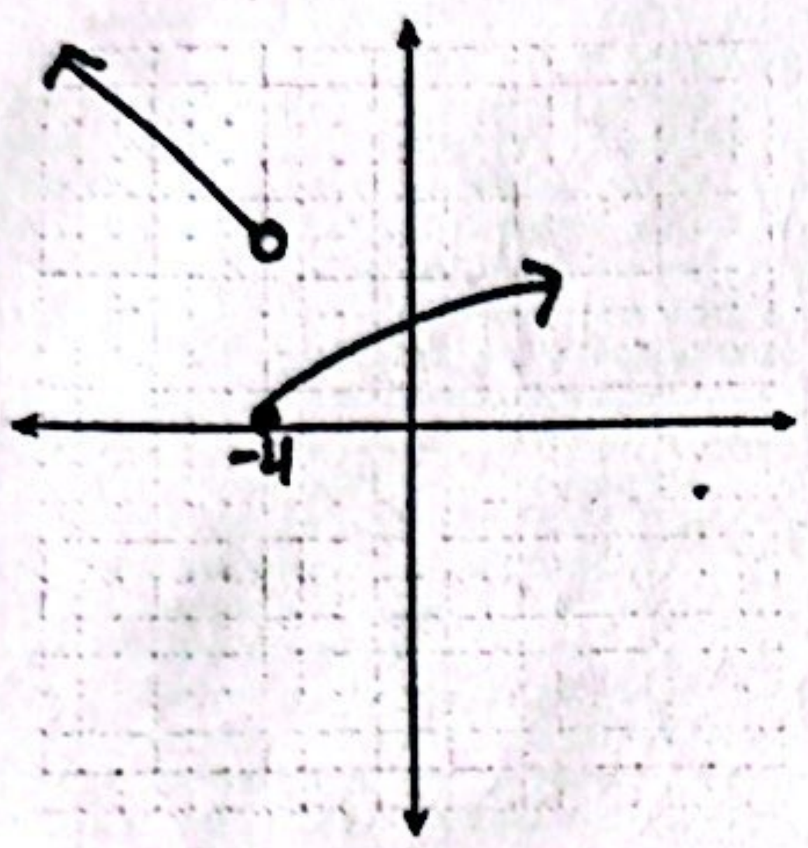
Name: Key

$\frac{\quad}{37}$

Date: ______________ Per: ______

You may not use a calculator... Good Luck!

1. Evaluate the following limits by using the graph of the function



(a) $\lim_{x\to -4^+} f(x) =$ 0

(b) $\lim_{x\to -4^-} f(x) =$ 5

(c) $\lim_{x\to -4} f(x) =$ DNE

3pts

2. Suppose you are given $\lim_{x\to c} f(x) = -7$ and $\lim_{x\to c} g(x) = 14$, calculate the following limits:

2pts (a) $\lim_{x\to c}[f(x)g(x)] =$ $(-7)(14) = -98$

2pts (b) $\lim_{x\to c} -9f(x) =$ $-9(-7) = 63$

2pts (c) $\lim_{x\to c}\frac{f(x)}{g(x)} =$ $\frac{-7}{14} = -\frac{1}{2}$

2pts (d) $\lim_{x\to c} g(x)^2 =$ 14^2 $= 196$

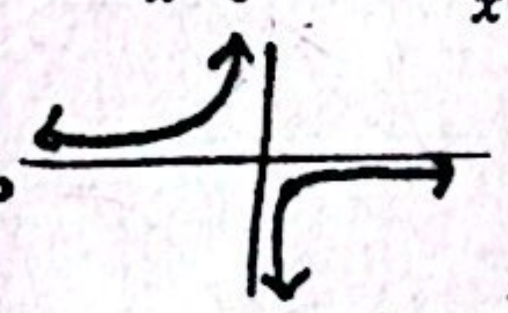
Evaluate the following limits

3. $\lim_{x\to 0}\frac{-2(1-\cos x)}{x}$ $= \left(\lim_{x\to 0} -2\right)\left(\lim_{x\to 0} \frac{1-\cos x}{x}\right)$

2pts

$= -2(0) = 0$

4. $\lim_{x\to 0^-} x^4 - \frac{1}{x}$ $= 0 + \infty = \infty$



2pts

5. $\lim_{x\to\pi} \tan\left(\frac{5x}{6}\right)$ $= \tan\left(\frac{5\pi}{6}\right)$

1pt

c s

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $= -\frac{\sqrt{3}}{3}$

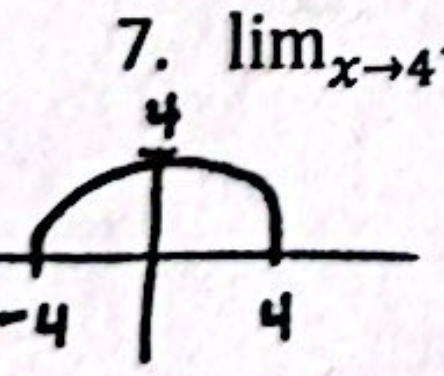
$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

6. $\lim_{x\to -3}\frac{5x+15}{x^2-2x-15}$ $= \lim_{x\to -3} \frac{5(x+3)}{(x+3)(x-5)}$

2pts

$= \lim_{x\to -3} \frac{5}{x-5} = -\frac{5}{8}$

7. $\lim_{x\to 4^+}\sqrt{16-x^2}$ $=$ DNE



1pt

8. $\lim_{x\to 6^+}\frac{x-8}{-x+6}$ $= \infty$

2pts

$f(6.001) = \frac{-}{-} = +$

9. $\lim_{x\to 0}\frac{\sin(3x)\cos(x)}{3x}$ $= \left(\lim_{x\to 0} \frac{\sin(3x)}{3x}\right)\left(\lim_{x\to 0} \cos x\right)$

2pts

$= (1)(1) = 1$

10. $\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-(x+\Delta x)-6-(x^2-x-6)}{\Delta x}$ $= \lim_{\Delta x\to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 \cancel{-x} - \Delta x \cancel{-6} \cancel{-x^2} \cancel{+x} \cancel{+6}}{\Delta x}$

3pts

$= \lim_{\Delta x\to 0} \frac{\Delta x(2x+\Delta x-1)}{\Delta x} = 2x-1$

25

iscuss the continuity of each function. Be sure to clearly justify your answers. If there is discontinuity, specify whether it is removable or not, and where it occurs.

11. $f(x) = \frac{x^2-25}{x^2-15x+50}$ 2 pts	Removable discontinuity at $x=5$ Non-removable discontinuity in the form of an asymptote at $X=10$.
12. $f(x) = \begin{cases} 3-x, & x \neq 1 \\ 0, & x = 1 \end{cases}$ 1 pt	removable discontinuity at $x=1$. (hole)
13. $f(x) = \sin(x) - 4x^2$ 2pts	Everywhere continuous because $f(x)$ is the sum of two continuous functions $\sin(x)$ and $-4x^2$

14. Find the value of a such that $f(x) = \begin{cases} 5, & x \leq 3 \\ ax-7, & x > 3 \end{cases}$ is everywhere continuous.

$a(3)-7=5$

$3a=12$

$a=4$

2pts

15. Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - 5x^2 - 10x + 5$ has a zero in the interval $[-1,2]$.

$f(-1) = 2(-1)^3 - 5(-1)^2 - 10(-1) + 5$

$= -2 - 5 + 10 + 5$

$= 8$

5 pts

$f(2) = 2(2)^3 - 5(2)^2 - 10(2) + 5$

$= 16 - 20 - 20 + 5$

$= -19$

$f(x)$ is everywhere continuous because it is a polynomial.

Since $f(2) = -19 \leq 0 \leq 8 = f(-1)$, $f(x)$ must have a zero in the interval $[-1,2]$.