Name: ________________________ Class: ___________________ Date: __________ ID: A

Chapter 15 Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

   $f(x,y) = 10x^2 + 8xy + 3y^2$

   a. $\nabla f(x,y) = (6y + 8x)\hat{\mathbf{i}} - (20x + 8y)\hat{\mathbf{j}}$

   b. $\nabla f(x,y) = 6y\hat{\mathbf{i}} - 20x\hat{\mathbf{j}}$

   c. $\nabla f(x,y) = 6y\hat{\mathbf{i}} + 20x\hat{\mathbf{j}}$

   d. $\nabla f(x,y) = (20x + 8y)\hat{\mathbf{i}} + (6y + 8x)\hat{\mathbf{j}}$

   e. $\nabla f(x,y) = 20x\hat{\mathbf{i}} + 6y\hat{\mathbf{j}}$

2. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

   $\vec{\mathbf{F}}(x, y) = \frac{3y}{x}\hat{\mathbf{i}} - \frac{x^3}{y^3}\hat{\mathbf{j}}$

   a. conservative with potential function $f(x, y) = \frac{3x^3}{y}$

   b. conservative with potential function $f(x, y) = \frac{x^3}{3y}$

   c. conservative with potential function $f(x, y) = \frac{x^3}{4y}$

   d. conservative with potential function $f(x, y) = \frac{4x^3}{y}$

   e. not conservative

3. Find the curl for the vector field at the given point.

   $\vec{\mathbf{F}}(x, y, z) = 7xyz\hat{\mathbf{i}} + 7y\hat{\mathbf{j}} + 7z\hat{\mathbf{k}}, \quad (7,8,7)$

   a. $42\hat{\mathbf{i}} + 343\hat{\mathbf{j}}$

   b. $392\hat{\mathbf{i}} + 343\hat{\mathbf{k}}$

   c. $392\hat{\mathbf{j}} - 343\hat{\mathbf{k}}$

   d. $56\hat{\mathbf{i}} + 343\hat{\mathbf{k}}$

   e. $294\hat{\mathbf{i}} + 343\hat{\mathbf{j}}$

4. Find the divergence of the vector field.

   $\vec{\mathbf{F}}(x,y,z) = 10x^6\,\hat{\mathbf{i}} - xy^5\,\hat{\mathbf{j}}$

   a. $\text{div}\vec{\mathbf{F}}(x,y,z) = 60x^7 - 6xy^6$

   b. $\text{div}\vec{\mathbf{F}}(x,y,z) = 60x^5 - 5xy^4$

   c. $\text{div}\vec{\mathbf{F}}(x,y,z) = 10x^6\,\hat{\mathbf{i}} - 6xy^5\,\hat{\mathbf{j}}$

   d. $\text{div}\vec{\mathbf{F}}(x,y,z) = \frac{10}{7}x^7 - \frac{1}{6}xy^6$

   e. $\text{div}\vec{\mathbf{F}}(x,y,z) = \frac{10}{7}x^7\,\hat{\mathbf{i}} - \frac{1}{5}xy^4\,\hat{\mathbf{j}}$

5. Find the divergence of the vector field **F** given by $\mathbf{F}(x,y,z) = \sin(7x)\mathbf{i} + \cos(6y)\mathbf{j} + z^3\hat{\mathbf{k}}$.

   a. $7\cos(x) - 6\sin(y) + 3z^2$

   b. $\cos(x) + \sin(y) + z^4$

   c. $7\cos(7x) + 6\sin(6y) + 3z^4$

   d. $7\cos(7x) - 6\sin(6y) + 3z^2$

   e. $\cos(7x) - \sin(6y) + z^2$

6. Find the divergence at $(4,0,0)$ for the vector field $\mathbf{F}(x,y,z) = e^x \sin y\mathbf{i} - e^x \cos y\mathbf{j} + z^2\mathbf{k}$

   a. 8
   b. 16
   c. 5
   d. 33
   e. 0

7. Find $\mathbf{curl}(\vec{\mathbf{F}} \times \vec{\mathbf{G}})$.

   $\vec{\mathbf{F}}(x,y,z) = 3\hat{\mathbf{i}} + 4x\hat{\mathbf{j}} + 5y\hat{\mathbf{k}}$

   $\vec{\mathbf{G}}(x,y,z) = 3x\hat{\mathbf{i}} - 3y\hat{\mathbf{j}} + 3z\hat{\mathbf{k}}$

   a. $\mathbf{curl}(\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = 36x\hat{\mathbf{i}} - 15\hat{\mathbf{k}}$

   b. $\mathbf{curl}(\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = 36x\hat{\mathbf{i}} + 15\hat{\mathbf{j}}$

   c. $\mathbf{curl}(\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = 15x\hat{\mathbf{i}} - 36\hat{\mathbf{j}}$

   d. $\mathbf{curl}(\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = 15x\hat{\mathbf{i}} + 36\hat{\mathbf{k}}$

   e. $\mathbf{curl}(\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = 36x\hat{\mathbf{j}} - 15y\hat{\mathbf{k}}$

Name: ________________________ **ID: A**

8. Find $\mathbf{curl}\left(\mathbf{curl}\left(\vec{\mathbf{F}}\right)\right) = \nabla \times \left(\nabla \times \vec{\mathbf{F}}\right)$.

$\vec{\mathbf{F}}\left(x,y,z\right) = 5xyz\hat{\mathbf{i}} + 3y\hat{\mathbf{j}} + 10z\hat{\mathbf{k}}$

a. $\mathbf{curl}\left(\mathbf{curl}\left(\vec{\mathbf{F}}\right)\right) = 3z\hat{\mathbf{i}} + 3y\hat{\mathbf{k}}$

b. $\mathbf{curl}\left(\mathbf{curl}\left(\vec{\mathbf{F}}\right)\right) = 5z\hat{\mathbf{i}} + 5y\hat{\mathbf{k}}$

c. $\mathbf{curl}\left(\mathbf{curl}\left(\vec{\mathbf{F}}\right)\right) = 5x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}} + 10z\hat{\mathbf{k}}$

d. $\mathbf{curl}\left(\mathbf{curl}\left(\vec{\mathbf{F}}\right)\right) = 5z\hat{\mathbf{j}} + 5y\hat{\mathbf{k}}$

e. $\mathbf{curl}\left(\mathbf{curl}\left(\vec{\mathbf{F}}\right)\right) = 3z\hat{\mathbf{j}} + 3y\hat{\mathbf{k}}$

9. Find $\mathbf{div}\left(\vec{\mathbf{F}} \times \vec{\mathbf{G}}\right)$.

$\vec{\mathbf{F}}\left(x,y,z\right) = 8\hat{\mathbf{i}} + 9x\hat{\mathbf{j}} + 10y\hat{\mathbf{k}}$

$\vec{\mathbf{G}}\left(x,y,z\right) = 8x\hat{\mathbf{i}} - 8y\hat{\mathbf{j}} + 8z\hat{\mathbf{k}}$

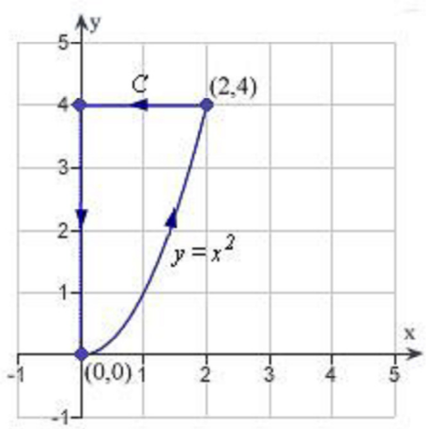
a. $80x + 72y$
b. $72y + 80z$
c. $80x + 72z$
d. $72x + 80z$
e. $80y + 72z$

10. Find div(curl $\mathbf{F}$) = $\nabla \cdot (\nabla \times \mathbf{F})$ for the vector field given by $\mathbf{F}\left(x,y,z\right) = x^8 z\mathbf{i} - 2xz\mathbf{j} + yz\mathbf{k}$.

a. 65
b. 129
c. 8
d. 0
e. 3

Name: ________________________ **ID: A**

11. Find a piecewise smooth parametrization of the path C given in the following graph.



a. $\mathbf{r}(t) = \begin{cases} t\mathbf{i} + t\mathbf{j} & 0 \le t \le 2 \\ \left(4 + t^2\right)\mathbf{i} - 4\mathbf{j} & 2 \le t \le 4 \\ \left(8 - t^2\right)\mathbf{j} & 4 \le t \le 10 \end{cases}$

b. $\mathbf{r}(t) = \begin{cases} t\mathbf{i} - t^2\mathbf{j} & 0 \le t \le 2 \\ (4 + t)\mathbf{i} + 4\mathbf{j} & 2 \le t \le 4 \\ (8 + t)\mathbf{j} & 4 \le t \le 8 \end{cases}$

c. $\mathbf{r}(t) = \begin{cases} t\mathbf{i} + t^2\mathbf{j} & 0 \le t \le 2 \\ (4 + t)\mathbf{i} - 4\mathbf{j} & 2 \le t \le 4 \\ \left(8 - t^2\right)\mathbf{j} & 4 \le t \le 8 \end{cases}$

d. $\mathbf{r}(t) = \begin{cases} t\mathbf{i} + t^2\mathbf{j} & 0 \le t \le 2 \\ (4 - t)\mathbf{i} + 4\mathbf{j} & 2 \le t \le 4 \\ (8 - t)\mathbf{j} & 4 \le t \le 8 \end{cases}$

e. $\mathbf{r}(t) = \begin{cases} t\mathbf{i} + t\mathbf{j} & 0 \le t \le 2 \\ \left(4 - t^2\right)\mathbf{i} + 4\mathbf{j} & 2 \le t \le 4 \\ \left(8 - t^2\right)\mathbf{j} & 4 \le t \le 10 \end{cases}$

Name: ________________________ **ID: A**

12. Evaluate $\int_C \left(x^2 + y^2\right) ds$ along the path C, defined as counterclockwise along the circle $x^2 + y^2 = 4$ from $(2,0)$ to $(0,2)$.

a. 2π
b. 4π
c. 2
d. 0
e. 4

13. Find the total mass of the wire with density ρ.

$\vec{\mathbf{r}}(t) = t^2\,\hat{\mathbf{i}} + 2t\hat{\mathbf{j}},\ \rho(x,y) = \frac{11}{12}y,\ 0 \le t \le 1$

a. $\frac{11}{18}\left(3\sqrt{2}+1\right)$

b. $\frac{11}{9}\left(2\sqrt{2}-1\right)$

c. $\frac{11}{9}\left(3\sqrt{2}-1\right)$

d. $\frac{11}{18}\left(3\sqrt{2}-1\right)$

e. $\frac{11}{9}\left(2\sqrt{2}+1\right)$

14. Evaluate $\int_C \vec{\mathbf{F}} \cdot d\,\vec{r}$ where C is represented by $\hat{\mathbf{r}}(t)$.

$\vec{\mathbf{F}}(x,y) = 7x\hat{\mathbf{i}} + 7y\hat{\mathbf{j}}$

$C\!:\vec{\mathbf{r}}(t) = t\hat{\mathbf{i}} + \sqrt{4-t^2}\,\hat{\mathbf{j}},\quad -2 \le t \le 2$

a. 196
b. 0
c. 14
d. 343
e. 49

15. Find the work done by the force field $\vec{\mathbf{F}}$ on a particle moving along the given path.

$$\vec{\mathbf{F}}(x,y) = -y\hat{\mathbf{i}} - x\hat{\mathbf{j}},\ C:\ y = \sqrt{25-x^2}\ \text{from}\ (5,0)\ \text{to}\ (-5,0).$$

a. 0
b. 25
c. $5\sqrt{26}$
d. 5
e. $\sqrt{26}$

16. Evaluate the integral $\int_c (4x-y)dx + (x+6y)dy$ along the path C, defined as y-axis from $y=0$ to $y=4$.

a. 24
b. 48
c. 128
d. 12
e. 96

17. Evaluate the integral $\int_c (4x-y)dx + (x+5y)dy$ along the path C, defined as $y = 9 - x^2$ from $(0,9)$ to $(3,0)$.

a. $-\frac{1755}{7}$

b. $-\frac{303}{2}$

c. $-\frac{909}{7}$

d. $-\frac{1323}{8}$

e. $-\frac{441}{2}$

Name: ________________________ **ID: A**

18. Find the work done by a person weighing 180 pounds walking exactly one revolution up a circular helical staircase of radius 4 feet if the person rises 14 feet.

 a. $1,260 \text{ ft} \cdot \text{lb}$
 b. $10,080 \text{ ft} \cdot \text{lb}$
 c. $2,520 \text{ ft} \cdot \text{lb}$
 d. $5,040 \text{ ft} \cdot \text{lb}$
 e. $180 \text{ ft} \cdot \text{lb}$

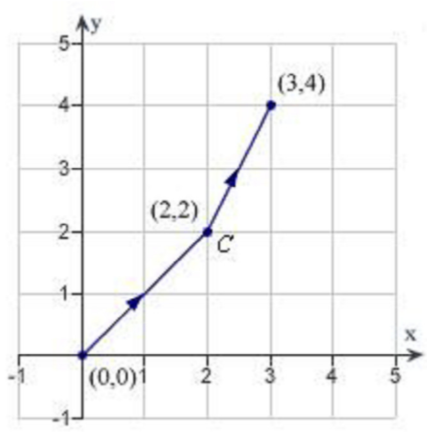
19. Find the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ on the closed path consisting of line segments from $(0,2)$ to $(0,0)$, from $(0,0)$ to $(2,0)$, and then from $(2,0)$ to $(0,2)$, where $\mathbf{F}(x,y) = 3ye^{3xy}\mathbf{i} + 3xe^{3xy}\mathbf{j}$.
(*Hint:* If **F** is conservative, the integration may be easier on an alternate path.)

 a. 99
 b. 12
 c. 36
 d. 30
 e. 0

Name: ________________________ **ID: A**

20. Find the value of the line integral $\int_C y^3\,dx + 3xy^2\,dy$.

(*Hint:* If $\mathbf{F}(x,y) = y^3\mathbf{i} + 3xy^2\mathbf{j}$ is conservative, the integration may be easier on an alternate path.)



a. 243
b. 192
c. 108
d. 0
e. 12

21. Find the value of the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = 4\mathbf{i} + 2z\mathbf{j} + 2y\mathbf{k}$ and $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t^2\mathbf{k}$, $0 \le t \le \pi$.

(*Hint:* If $\mathbf{F}$ is conservative, the integration may be easier on an alternate path.)

a. 4
b. 8
c. 4
d. –4
e. –8

22. Find the value of the line integral $\int_C \mathbf{F}.\,d\mathbf{r}$, where $\mathbf{F}(x,y) = e^{y}\left(z\mathbf{i} + xz\mathbf{j} + x\mathbf{k}\right)$ and $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 8\mathbf{k}$, $0 \le t \le \pi$.

(*Hint:* If **F** is conservative, the integration may be easier on an alternate path.)

a. 48
b. 24
c. –24
d. 0
e. –48

23. Evaluate the line integral $\int_C \sin x \sin y\, dx - \cos x \cos y\, dy$ using the Fundamental Theorem of Line Integrals, where C is the line segment from $(0,-\pi)$ to $\left(\frac{3\pi}{2},\frac{\pi}{2}\right)$.

a. –2
b. –1
c. 2
d. 1
e. 0

24. A stone weighing 2 pounds is attached to the end of a four-foot string and is whirled horizontally with one end held fixed. It makes 1 revolution per second. Find the work done by the force **F** that keeps the stone moving in a circular path. [*Hint:* Use Force = (mass)(centripetal acceleration).] Round your answer to two decimal places, if required.

a. 201.06 ft · lbs
b. 0.00 ft · lbs
c. 804.25 ft · lbs
d. 256.00 ft · lbs
e. 12.57 ft · lbs

Name: ________________________ **ID: A**

25. Use Green's Theorem to evaluate the integral

$$\int_C (y-x)\,dx + (2x-y)\,dy$$

for the path C: boundary of the region lying between the graphs of $y = x$ and $y = x^2 - 12x$.

a. $\displaystyle\int_0^{13}\int_{x^2-12x}^{x} 1x\,dydx = \frac{2,197}{6}$

b. $\displaystyle\int_0^{13}\int_{x^2-12x}^{x} 1x\,dydx = \frac{2,197}{18}$

c. $\displaystyle\int_0^{13}\int_{x}^{x^2-12x} 1x\,dydx = \frac{10,985}{18}$

d. $\displaystyle\int_0^{13}\int_{x-12x}^{x} 1x\,dydx = \frac{2,203}{6}$

e. $\displaystyle\int_0^{13}\int_{x}^{x^2-12x} 1x\,dydx = \frac{2,197}{12}$

26. Use Green's Theorem to evaluate the integral $\int_C (y-x)\,dx + (2x-y)\,dy$ for the path C defined as $x = 6\cos\theta,\ y = 7\sin\theta$.

a. 210π
b. 12π
c. 21π
d. 126π
e. 42π

27. Use Green's Theorem to evaluate the line integral $\int_C e^x \cos(13y)\, dx - 13e^x \sin(13y)\, dy$ where C is $x^2 + y^2 = a^2$.

a. 13
b. 14
c. 170
d. 26
e. 0

28. Use Green's Theorem to evaluate the line integral $\int_C \left(e^{-x^2/2} - y\right) dx + \left(e^{-y^2/2} + x\right) dy$ where C is the boundary of the region lying between the graphs of the circle $x = 8\cos\theta, y = 8\sin\theta$ and the ellipse $x = 6\cos\theta,\ y = 4\sin\theta$.

a. 120π
b. 162π
c. 172π
d. 80π
e. 91π

29. Use Green's Theorem to calculate the work done by the force $\vec{\mathbf{F}}(x,y) = \left(18x^2 + y\right)\mathbf{i} + 19xy^2\mathbf{j}$ on a particle that is moving counterclockwise around the closed path C where C is the boundary of the region lying between the graphs of $y = \sqrt{x}$, $y = 0, \text{and} x = 9$. Round your answer to two decimal places.

a. 663.60
b. 597.60
c. 618.60
d. 657.60
e. 791.60

30. Find the maximum value of $\int_{C_1} y^3\,dx + \left(27x - x^3\right)dy$, where C is any closed curve in the xy-plane, oriented counterclockwise.

a. $\frac{243\pi}{2}$

b. $\frac{675\pi}{2}$

c. $\frac{135\pi}{2}$

d. $\frac{189\pi}{2}$

e. $\frac{729\pi}{2}$

31. Find the rectangular equation for the surface by eliminating parameters from the vector-valued function. Identify the surface.

$\vec{\mathbf{r}}\left(u, v\right) = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + \frac{v}{10}\hat{\mathbf{k}}$

a. the plane $\mathrm{z} = \frac{x}{10}$

b. the cylinder $x^2 + y^2 = 100$

c. the plane $x = \frac{y}{10}$

d. the plane $\mathrm{z} = \frac{y}{10}$

e. the plane $\mathrm{y} = \frac{x}{10}$

32. Find the rectangular equation for the surface by eliminating the parameters from the vector-valued function $\mathbf{r}(u, v) = 9\cos v \cos u\mathbf{i} + 9\cos v \sin u\mathbf{j} + 6\sin v\mathbf{k}$.

a. $\frac{x^2}{9} + \frac{y^2}{6} + \frac{z^2}{6} = 1$

b. $\frac{x^2}{81} + \frac{y^2}{36} + \frac{z^2}{36} = 1$

c. $\frac{x^2}{36} + \frac{y^2}{81} + \frac{z^2}{81} = 1$

d. $\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{6} = 1$

e. $\frac{x^2}{81} + \frac{y^2}{81} + \frac{z^2}{36} = 1$

33. Find a vector-valued function whose graph is the plane $x + y + z = 4$.

a. $\mathbf{r}(u, v) = u\mathbf{i} - v\mathbf{j} - (4 - u - v)\mathbf{k}$

b. $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (4 - u - v)\mathbf{k}$

c. $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (4 + u + v)\mathbf{k}$

d. $\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} - (4 + u + v)\mathbf{k}$

e. $\mathbf{r}(u, v) = -u\mathbf{i} - v\mathbf{j} + (4 - u - v)\mathbf{k}$

34. Find a vector-valued function whose graph is the cone $x = \sqrt{9y^2 + z^2}$.

a. $\mathbf{r}(u, v) = u\mathbf{i} + u\cos v\mathbf{j} + \frac{1}{3}u\sin v\mathbf{k}$

b. $\mathbf{r}(u, v) = u\mathbf{i} + \frac{1}{3}u\cos v\mathbf{j} + u\sin v\mathbf{k}$

c. $\mathbf{r}(u, v) = u\mathbf{i} + u\cos v\mathbf{j} + \frac{1}{9}u\sin v\mathbf{k}$

d. $\mathbf{r}(u, v) = u\mathbf{i} - \frac{1}{9}u\cos v\mathbf{j} + u\sin v\mathbf{k}$

e. $\mathbf{r}(u, v) = u\mathbf{i} - \frac{1}{3}u\cos v\mathbf{j} - 3u\sin v\mathbf{k}$

35. Find a vector-valued function whose graph is the ellipsoid $\frac{x^2}{36} + \frac{y^2}{49} + \frac{z^2}{100} = 1$.

a. $\vec{\mathbf{r}}(u, v) = 6\sin u\cos v\hat{\mathbf{i}} + 7\sin u\sin v\hat{\mathbf{j}} + 10\cos u\sin v\hat{\mathbf{k}}$

b. $\vec{\mathbf{r}}(u, v) = 6\sin u\cos v\hat{\mathbf{i}} + 7\sin u\sin v\hat{\mathbf{j}} + 10\cos u\cos v\hat{\mathbf{k}}$

c. $\vec{\mathbf{r}}(u, v) = 6\sin u\cos v\hat{\mathbf{i}} + 7\sin u\sin v\hat{\mathbf{j}} + 10\cos u\hat{\mathbf{k}}$

d. $\vec{\mathbf{r}}(u, v) = \cos(6u)\hat{\mathbf{i}} + \sin(7v)\hat{\mathbf{j}} + \cos(10v)\sin(10v)\hat{\mathbf{k}}$

e. $\vec{\mathbf{r}}(u, v) = 6\sin u\cos v\hat{\mathbf{i}} + 7\sin u\sin v\hat{\mathbf{j}} + 10\sin u\hat{\mathbf{k}}$

Name: ________________________ ID: A

36. Find a vector-valued function whose graph is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.

a. $\mathbf{r}(u, v) = v \cos u\mathbf{i} + v \sin u\mathbf{j} + v\mathbf{k},\ v \geq 2$

b. $\mathbf{r}(u, v) = u^2 \cos v\mathbf{i} + u^2 \sin v\mathbf{j} + u^2\mathbf{k},\ v \geq 4$

c. $\mathbf{r}(u, v) = v^2 \sin u\mathbf{i} + v^2 \cos u\mathbf{j} + v^2\mathbf{k},\ 0 \leq v \leq 2$

d. $\mathbf{r}(u, v) = v \cos u\mathbf{i} + v \sin u\mathbf{j} + v^2\mathbf{k},\ 0 \leq v \leq 2$

e. $\mathbf{r}(u, v) = v \sin u\mathbf{i} + v \cos u\mathbf{j} + v^2\mathbf{k},\ 0 \leq v \leq 4$

37. Find an equation of the tangent plane to the surface represented by the vector-valued function at the given point.

$\vec{\mathbf{r}}(u, v) = (2u + v)\hat{\mathbf{i}} + (u - v)\hat{\mathbf{j}} + v\hat{\mathbf{k}},\ (4, -4, 4)$

a. $(x-4) - 2(y+4) - 3(z-4) = 0$
b. $2(x-4) + 2(y+4) - 3(z-4) = 0$
c. $z = 0$
d. $(x-4) - 2(y+4) - 3(z-4) = 0$
e. $2(x-4) - 2(y+4) + 1(z-4) = 0$

38. Find the area of the surface given by $\mathbf{r}(u, v) = 9u\mathbf{i} - v\mathbf{j} + v\mathbf{k}$, where $0 \leq u \leq 2$ and $0 \leq v \leq 4$.

a. $81\sqrt{2}$
b. $38\sqrt{2}$
c. $72\sqrt{2}$
d. $45\sqrt{2}$
e. $22\sqrt{2}$

Name: ________________________ ID: A

39. Find a vector-valued function for the hyperboloid $x^2 + y^2 - z^2 = 121$.

a. $\mathbf{r}(u, v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + \sqrt{u^2 + 121}\,\mathbf{k}$

b. $\mathbf{r}(u, v) = u\sin v\mathbf{i} + 11u\cos v\mathbf{j} + \sqrt{u^2 + 1}\,\mathbf{k}$

c. $\mathbf{r}(u, v) = 11u\sin v\mathbf{i} + u\cos v\mathbf{j} + \sqrt{u^2 - 1}\,\mathbf{k}$

d. $\mathbf{r}(u, v) = 11u\cos v\mathbf{i} + 11u\sin v\mathbf{j} + 11\sqrt{u^2 - 1}\,\mathbf{k}$

e. $\mathbf{r}(u, v) = 11u\cos v\mathbf{i} + 11u\sin v\mathbf{j} + 11\sqrt{u^2 - 121}\,\mathbf{k}$

40. Determine the tangent plane for the hyperboloid $x^2 + y^2 - z^2 = 121$ at $(11,0,0)$.

a. $x = 11$
b. $x = 1$
c. $x = 121$
d. $x = 22$
e. $x = 0$

41. Evaluate $\iint\limits_S (x - 8y + z)\,dS$, where

$S: z = 16 - x,\ 0 \le x \le 16,\ 0 \le y \le 16.$

a. $-7{,}373\sqrt{3}$
b. $-12{,}288\sqrt{2}$
c. 0
d. $-19{,}661\sqrt{2}$
e. $-15{,}974\sqrt{3}$

42. Evaluate $\iint\limits_S (x - 5y + z)\,dS$, where S is $z = 4$, $x^2 + y^2 \le 4$.

a. 32π
b. $64\pi + 80$
c. 16π
d. 32π
e. 320π

43. Evaluate $\iint\limits_S f(x,y)\,dS$, where $f(x,y) = y + 3$

$S : \mathbf{r}(u,v) = u\mathbf{i} + v\mathbf{j} + \frac{v}{10}\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le 10$.

a. $6\sqrt{101}$
b. $8\sqrt{101}$
c. $6\sqrt{201}$
d. 0
e. $8\sqrt{201}$

44. Evaluate $\iint\limits_S f(x,y,z)\,dS$, where $f(x,y,z) = x^2 + y^2 + z^2$ and S is given by $z = x + y$, $x^2 + y^2 \le 100$.

a. $10{,}000$
b. $10{,}000\pi$
c. $10{,}000\sqrt{3}\,\pi$
d. $10{,}000\sqrt{2}\,\pi$
e. $10{,}000\sqrt{3}$

45. Find the flux $\vec{\mathbf{F}}$ of through S, $\iint\limits_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}}\, dS$, where $\vec{\mathbf{N}}$ is the upward unit normal vector to S.

$\vec{\mathbf{F}}\left(x,y,z\right) = 2z\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + y\hat{\mathbf{k}}$

$S: x + y + z = 9$, first octant

a. $-\dfrac{81}{2}$

b. $\dfrac{81}{2}$

c. $\dfrac{99}{2}$

d. $\dfrac{93}{2}$

e. $-\dfrac{87}{2}$

46. Find the flux $\vec{\mathbf{F}}$ of through S, $\iint\limits_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}}\, dS$, where $\vec{\mathbf{N}}$ is the upward unit normal vector to S.

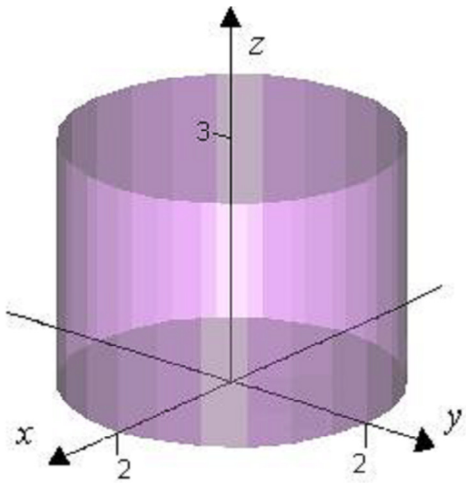
$\vec{\mathbf{F}}\left(x,y,z\right) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$S: z = 64 - x^2 - y^2$, $z \geq 0$

a. $4{,}301\pi$
b. $6{,}144\pi$
c. $7{,}987\pi$
d. 3072π
e. 1843.2π

Name: ________________________ **ID: A**

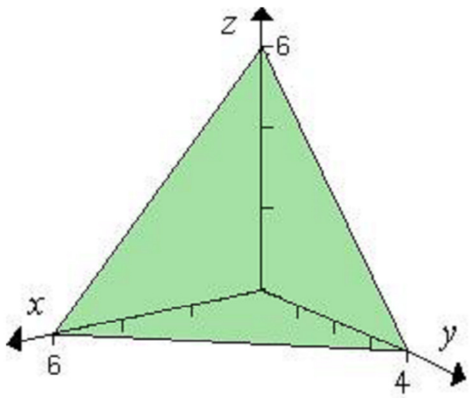
47. Let $F(x,y,z) = 2x\hat{\mathbf{i}} - 2y\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$ and let S be the cylinder $x^2 + y^2 = 4, 0 \le z \le 3.$ Verify the Divergence Theorem by evaluating $\iint_S \mathbf{F} \cdot \mathbf{N} ds$ as a surface integral and as a triple integral.



a. 18π
b. 36π
c. 12π
d. 108π
e. 54π

Name: ________________________ **ID: A**

48. Let $\mathbf{F}(x,y,z) = (2x - y)\mathbf{i} - (2y - z)\mathbf{j} + z\mathbf{k}$ and let S be the surface bounded by the plane $2x + 4y + 2z = 12$ and the coordinates planes. Verify the Divergence Theorem by evaluating $\int\int_S \mathbf{F} \cdot \mathbf{N} ds$ as a surface integral and as a triple integral.



a. 43
b. 36
c. 6
d. 18
e. 23

49. Use the Divergence Theorem to evaluate $\int\int_S \mathbf{F} \cdot \mathbf{N} ds$ and find the outward flux of $\vec{\mathbf{F}}$ through the surface of the solid bounded by the graphs of the equations. Use a computer algebra system to verify your results.

$F(x,y,z) = x^2\hat{\mathbf{i}} - 2xy\hat{\mathbf{j}} + xyz^2\hat{\mathbf{k}}$

$S: z = \sqrt{16 - x^2 - y^2}, z = 0$

a. 4π
b. 0
c. 32π
d. 16π
e. 8π

Name: ________________________ **ID: A**

50. Use Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N} ds$ and find the outward flux of $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the surface S of the solid bounded by the sphere $x^2 + y^2 + z^2 = 196$.

a. 784π
b. $10,976$
c. $10,976\pi$
d. $8,232$
e. $8,232\pi$

51. Use Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N} ds$and find the outward flux of $F(x,y,z) = x\mathbf{i} + y^2\mathbf{j} - z\mathbf{k}$ through the surface S of the solid bounded by $x^2 y^2 = 49$, $z = 0$, and z = 5.

a. 245π
b. 49
c. 35π
d. 5
e. 0

52. Use Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N} ds$ and find the outward flux of $\mathbf{F}(x,y,z) = xe^z\mathbf{i} + ye^z\mathbf{j} + e^z\mathbf{k}$ through the surface S of the solid bounded by the planes $z = 2 - y$, $z = 0, x = 0, x = 2,$ and $y = 0$.

a. $6\left(e^2 - 2\right)$
b. $6\left(e^2 - 3\right)$
c. $6\left(e^2 - 2\right)$
d. $6\left(e^2 - 3\right)$
e. $4\left(e^2 - 3\right)$

Name: ________________________ **ID: A**

53. Evaluate $\iint\limits_S \textbf{curl}\,\mathbf{F}\cdot\mathbf{N}\,dS$, where $\mathbf{F}(x, y, z) = \left(4xy + z^2\right)\mathbf{i} + \left(2x^2 + 6yz\right)\mathbf{j} + 2xz\mathbf{k}$ and S is the closed surface of the solid bounded by the graphs ,$x = 7,$ and $z = 6 - y^2$, and the coordinate planes.

a. 7
b. 0
c. 6
d. 13
e. 42

54. Let $\mathbf{F}(x,y,z) = xyz\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and let S be the plane $10x + 10y + z = 20$ in the first octant. Verify Stokes's Theorem by evaluating $\int_c \mathbf{F}.\mathbf{T}\,ds = \int_c \mathbf{F}\cdot d\mathbf{r}$ as a line integral and as a double integral.

a. 30
b. 10
c. 20
d. 0
e. 1

55. Use Stokes's Theorem to evaluate $\int_c \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$. Use a computer algebra system to verify your results. Note: C is oriented counterclockwise as viewed from above.

$\vec{\mathbf{F}}(x,y,z) = 4y\hat{\mathbf{i}} + 10z\hat{\mathbf{j}} + x\hat{\mathbf{k}}$

C: triangle with vertices $(0, 0, 0), (0,4,0), (1, 1, 1)$

a. 12
b. 6
c. 14
d. 17
e. 4

56. Let $\mathbf{F}(x,y,z) = z^2\mathbf{i} + 2x\mathbf{j} + y^2\mathbf{k}$ and let S be the graph of $z = 1 - x^2 - y^2$, $z \geq 0$,oriented counterclockwise. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

a. 0
b. 2π
c. π
d. 1
e. 2

57. Use Stokes's Theorem to evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ .Use a computer algebra system to verify your results. Note: C is oriented counterclockwise as viewed from above.

$\vec{\mathbf{F}}(x,y,z) = 36xz\hat{\mathbf{i}} + y\hat{\mathbf{j}} + 36xy\hat{\mathbf{k}}$

$S: z = 121 - x^2 - y^2$, $z \geq 0$

a. 4356π
b. 726π
c. 11π
d. 121π
e. 0

58. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot dr$ where $\mathbf{F}(x,y,z) = -\ln\sqrt{x^2+y^2}\,\mathbf{i} + \arctan\frac{x}{y}\,\mathbf{j} + \mathbf{k}$ and S is $z = 25 - 2x - 3y$ over $r = 2\sin 2\theta$ in the first octant. Use a computer algebra system to verify your result.

a. 0

b. $\frac{4}{3}$

c. $\frac{14}{3}$

d. 4

e. $\frac{8}{3}$

Name: ________________________ **ID: A**

59. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot dr$ where $\mathbf{F}(x,y,z) = yz\mathbf{i} + (2-3y)\mathbf{j} + (x^2+y^2)\mathbf{k}$, $x^2+y^2 \le 400$ and S is the first-octant portion of $x^2+y^2 \le 400$ over $x^2+y^2 = 400$. Use a computer algebra system to verify your result.

a. $-\dfrac{400}{3}$

b. 0

c. $-\dfrac{8000}{3}$

d. $\dfrac{400}{3}$

e. $\dfrac{8000}{3}$

60. The motion of a liquid in a cylindrical container of radius 1 is described by the velocity field $\mathbf{F}(x,y,z) = -8z\mathbf{i} + 2y\mathbf{k}$. Find $\iint_S (\text{curl}\,\mathbf{F}) \cdot \mathbf{N}\,dS$, where S is the upper surface of the cylindrical container.

a. -7
b. -2
c. 9
d. 0
e. 4

Chapter 15 Practice Test
Answer Section

MULTIPLE CHOICE

1. D
2. E
3. C
4. B
5. D
6. E
7. E
8. D
9. C
10. D
11. D
12. B
13. B
14. B
15. A
16. B
17. E
18. C
19. E
20. B
21. E
22. E
23. E
24. B
25. A
26. E
27. E
28. D
29. B
30. A
31. D
32. E
33. B
34. B
35. C
36. D
37. A
38. C
39. D

40. A
41. B
42. C
43. B
44. C
45. B
46. B
47. B
48. B
49. B
50. C
51. E
52. D
53. B
54. D
55. A
56. B
57. E
58. E
59. C
60. D