Chapter 14 Multivar Answer Key w/ Work

(Courtesy of Jack "The Bardic Goat" Dobbs)

1. Evaluate the following integral.

$$\int_{4x}^{x^6} \frac{-8y}{x}\, dy$$

a. $-4(x^{11} - 4x)$

b. $4(x^{11} - 4x)$

c. $-4(16x - x^{11})$

d. $-4(x^{11} - 16x)$

e. $8(x^{11} - 16x)$

Work:

$$\int_{4x}^{x^6} \frac{-8y}{x}\, dy$$

$$\frac{-8}{x}\int_{4x}^{x^6} y\, dy$$

$$\frac{-4}{x}\left[y^2\right]_{4x}^{x^6}$$

$$-4x^{11} + 64x$$

$$-4(x^{11} - 16x)$$

2. Evaluate the following integral.

$$\int_{11}^{5y} \frac{-4y}{x}\, dx$$

a. $-4\,ln\left(\frac{5y}{11}\right)$

b. $-4(ln(5y) - ln(11))$

c. $-4y(ln(5y) + ln(11))$

d. $y\,ln(\frac{5y}{11})$

e. $-4y\,ln\left(\frac{5y}{11}\right)$

Work:

$$\int_{11}^{5y} \frac{-4y}{x}\, dx$$

$$-4y\int_{11}^{5y} \frac{1}{x}\, dx$$

$$-4y[ln|x|]_{11}^{5y}$$

$$-4y\,ln|5y| + 4y\,ln(11)$$

$$-4y\,ln|\frac{5y}{11}|$$

3. Evaluate the following iterated integral.

$$\int_5^8 \int_1^{\sqrt{x}} 2ye^{-x}\, dy\, dx$$

a. $5e^{5} - 8e^{8}$

b. $5e^{-5} - 8e^{-8}$

c. $5e^{-\sqrt{5}} - 8e^{-\sqrt{8}}$

d. $5e^{-5} + 8e^{-8}$

e. $8e^{-8} - 5e^{-5}$

Work:

$$\int_5^8 \int_1^{\sqrt{x}} 2ye^{-x}\, dy\, dx$$

$$\int_5^8 2e^{-x} \int_1^{\sqrt{x}} y\, dy\, dx$$

$$\int_5^8 e^{-x} [y^2]_1^{\sqrt{x}}\, dx$$

$$\int_5^8 e^{-x}(x - 1)\, dx$$

$$\int_5^8 xe^{-x}\, dx - \int_5^8 e^{-x}\, dx$$

$$u = x$$

$$dv = e^{-x}\, dx$$

$$[-xe^{-x}]_5^8 + \int_5^8 e^{-x}\, dx - \int_5^8 e^{-x}\, dx$$

$$-8e^{-8} + 5e^{-5}$$

4. Evaluate the following improper integral.

$$\int_{10}^{\infty}\int_{0}^{\frac{11}{x}} y\,dy\,dx$$

a. $\frac{20}{11}$

b. $\frac{11}{20}$

c. $\frac{20}{121}$

d. $\frac{121}{20}$

e. The integral does not converge.

Work:

$$\int_{10}^{\infty}\int_{0}^{\frac{11}{x}} y\,dy\,dx$$

$$\frac{1}{2}\int_{10}^{\infty}[y^2]_0^{\frac{11}{x}}\,dx$$

$$\frac{121}{2}\int_{10}^{\infty}\frac{1}{x^2}\,dx$$

$$-\frac{121}{2}\left[\frac{1}{x}\right]_{10}^{\infty}$$

$$\frac{121}{20}$$

5. Evaluate the improper iterated integral $\int_0^{\infty}\int_0^{\infty} xye^{-(x^2+y^2)}\,dx\,dy$.

$$\int_0^{\infty}\int_0^{\infty} xye^{-(x^2+y^2)}\,dx\,dy$$

a. $\frac{3}{4}$

b. $\frac{5}{2}$

c. $\frac{1}{2}$

d. $\frac{1}{4}$

e. $\frac{5}{4}$

Work:

$$\int_0^{\infty}\int_0^{\infty} xye^{-(x^2+y^2)}\,dx\,dy$$

$$\int_0^{\infty} y\int_0^{\infty} xe^{-(x^2+y^2)}\,dx\,dy$$

$$\frac{1}{2}\int_0^{\infty} y\int_{x=0}^{x=\infty} e^{-u}\,du\,dy$$

$$-\frac{1}{2}\int_0^{\infty} y[e^{-u}]_{x=0}^{x=\infty}\,dy$$

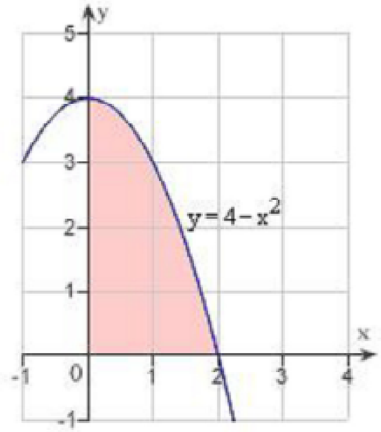
$$-\frac{1}{2}\int_0^{\infty} y[e^{-(x^2+y^2)}]_{x=0}^{x=\infty}\,dy$$

$$\frac{1}{2}\int_0^{\infty} ye^{-y^2}\,dy$$

$$-\frac{1}{4}[e^{-y^2}]_0^{\infty}$$

$$\frac{1}{4}$$

6. Use an iterated integral to find the area of the region shown in the figure below.



a. $\frac{1024}{3}$

b. $\frac{16}{3}$

c. $\frac{125}{3}$

d. $\frac{16}{3}$

e. $\frac{128}{3}$

Work:

$$\int_0^2 \int_0^{4-x^2} dy\, dx$$

$$\int_0^2 (4 - x^2)\, dx$$

$$\left[4x - \frac{1}{3}x^3\right]_0^2$$

$$8 - \frac{8}{3}$$

$$\frac{16}{3}$$

7. Use an iterated integral to find the area of the region bounded by the graphs of the equations $y = 22 - x^2$ and $y = 2x + 7$.

$$y = 22 - x^2$$
$$y = 2x + 7$$

a. $\frac{31}{3}$

b. $\frac{256}{3}$

c. $\frac{287}{6}$

d. $\frac{196}{3}$

e. $\frac{121}{3}$

Work:

$$\int_{-5}^{3}\int_{2x+7}^{22-x^2} dy\,dx$$

$$\int_{-5}^{3}(22 - x^2 - 2x - 7)\,dx$$

$$-\int_{-5}^{3}(x^2 + 2x - 15)\,dx$$

$$-\left[\frac{1}{3}x^3 + x^2 - 15x\right]_{-5}^{3}$$

$$-9 - 9 + 45 - \frac{125}{3} + 25 + 75$$

$$\frac{256}{3}$$

8. Sketch the region R of integration and then switch the order of integration for the following integral.

$$\int_0^7 \int_0^{\sqrt{49-x^2}} f(x,y)\,dy\,dx$$

a. $\int_{-7}^{7} \int_0^{\sqrt{49-y^2}} f(x,y)\,dx\,dy$

b. $\int_{-7}^{7} \int_0^{49-y^2} f(x,y)\,dx\,dy$

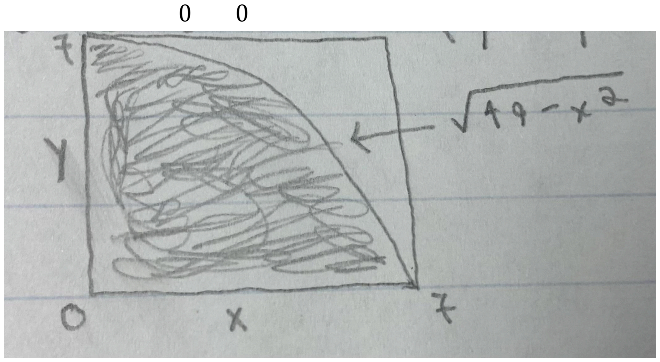
c. $\int_0^7 \int_0^{\sqrt{49-y^2}} f(x,y)\,dx\,dy$

d. $\int_0^{49} \int_0^{7-y^2} f(x,y)\,dx\,dy$

e. $\int_0^7 \int_0^{49-x^2} f(x,y)\,dx\,dy$

Work:

$$\int_0^7 \int_0^{\sqrt{49-x^2}} f(x,y)\,dy\,dx$$



$$\int_0^7 \int_0^{\sqrt{49-y^2}} f(x,y)\,dx\,dy$$

9. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^1 \int_x^1 e^{-9y^2}\, dy\, dx$$

a. $\frac{1-e^{9}}{9}$

b. $\frac{1-e^{-9}}{18}$

c. $\frac{1-e^{-9}}{9}$

d. $\frac{e^{-9}-1}{18}$

e. $\frac{1-e^{-9}}{36}$

Work:

$$\int_0^1 \int_x^1 e^{-9y^2}\, dy\, dx$$

$$\int_0^1 e^{-9y^2} \int_0^y dx\, dy$$

$$\int_0^1 ye^{-9y^2}\, dy$$

$$-\frac{1}{18}\int_{y=0}^{y=1} e^{u}\, du$$

$$-\frac{1}{18}\left[e^{u}\right]_{y=0}^{y=1}$$

$$-\frac{1}{18}\left[e^{-9y^2}\right]_0^1$$

$$-\frac{e^{-9}}{18}+\frac{1}{18}$$

$$\frac{1-e^{-9}}{18}$$

10. Evaluate the iterated integral $\int_0^2 \int_{y^2}^4 \sqrt{x}\, sin(2x)\, dx\, dy$ by switching the order of integration. Round your answer to three decimal places.

$$\int_0^2 \int_{y^2}^4 \sqrt{x}\, sin(2x)\, dx\, dy$$

a. 0.538

b. 0.558

c. 30.538

d. 1.538

e. 4.538

Work:

$$\int_0^2 \int_{y^2}^4 \sqrt{x}\, sin(2x)\, dx\, dy$$

$$\int_0^4 \sqrt{x}\, sin(2x) \int_0^{\sqrt{x}} dy\, dx$$

$$\int_0^4 x\, sin(2x)\, dx$$

$$u = x$$

$$dv = sin(2x)\, dx$$

$$-\frac{1}{2}\left[x\, cos(2x)\right]_0^4 + \frac{1}{2}\int_0^4 cos(2x)\, dx$$

$$-\,2\, cos(8) + \frac{1}{4}\, sin(8)$$

11. Set up an integral for both orders of integration, and use the more convenient order to evaluate the integral below over the region R.

$$\int_R \int \frac{y}{x^2+y^2}\, dA$$

R: triangle bounded by $y = 2x$, $y = 6x$, and $x = 5$

a. $5(log(37) - log(5))$

b. $\frac{2}{5}(log(37) - log(5))$

c. $\frac{5}{2}(log(37) - log(5))$

d. $\frac{5}{2}(log(5) - log(37))$

e. $\frac{1}{3}(log(37) - log(5))$

Work:

$$\int_R \int \frac{y}{x^2+y^2}\, dA$$

$$\int_0^5 \int_{2x}^{6x} \frac{y}{x^2+y^2}\, dy\, dx$$

$$\int_0^{30} \int_{\frac{1}{6}y}^{\frac{1}{2}y} \frac{y}{x^2+y^2}\, dx\, dy$$

$$\frac{1}{2}\int_0^5 \int_{y=2x}^{y=6x} \frac{1}{u}\, du\, dx$$

$$\frac{1}{2}\int_0^5 [ln|u|]_{y=2x}^{y=6x}\, dx$$

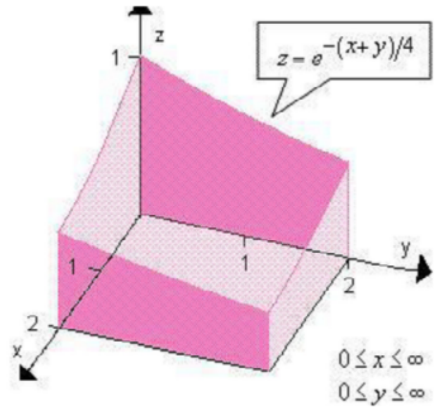
$$\frac{1}{2}\int_0^5 [ln(x^2 + y^2)]_{y=2x}^{y=6x}\, dx$$

$$\frac{1}{2}\int_0^5 (ln(37x^2) - ln(5x^2))\, dx$$

$$\frac{1}{2}\int_0^5 ln(\frac{37}{5})\, dx$$

$$\frac{5}{2}\, ln(\frac{37}{5})$$

12. Use a double integral to find the volume of the indicated solid.



a. 16

b. 9

c. 4

d. 20

e. 6

Work:

$$\int_0^{\infty}\int_0^{\infty} e^{-\frac{x+y}{4}}\,dy\,dx$$

$$-\,4\int_0^{\infty}\int_{y=0}^{y=\infty} e^{u}\,du\,dx$$

$$-\,4\int_0^{\infty}[e^{u}]_{y=0}^{y=\infty}\,dx$$

$$-\,4\int_0^{\infty}[e^{-\frac{x+y}{4}}]_{y=0}^{y=\infty}\,dx$$

$$4\int_0^{\infty} e^{-\frac{x}{4}}\,dx$$

$$-\,16\int_{x=0}^{x=\infty} e^{u}\,du$$

$$-\,16[e^{u}]_{x=0}^{x=\infty}$$

$$-\,16[e^{-\frac{x}{4}}]_0^{\infty}$$

$$16$$

13. Set up and evaluate a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$$z = xy^2$$
$$z > 0$$
$$x > 0$$
$$5x < y < 2$$

a. $\frac{7}{83}$

b. $\frac{125}{16}$

c. $\frac{83}{7}$

d. $\frac{16}{125}$

e. $\frac{32}{83}$

Work:

$$\int_0^{\frac{2}{5}} \int_{5x}^{2} xy^2 \, dy \, dx$$

$$\frac{1}{3}\int_0^{\frac{2}{5}} x[y^3]_{5x}^{2} \, dx$$

$$\frac{1}{3}\int_0^{\frac{2}{5}} (8x - 125x^4) \, dx$$

$$\frac{1}{3}[4x^2 - 25x^5]_0^{\frac{2}{5}}$$

$$\frac{1}{3}(\frac{16}{25} - \frac{32}{125})$$

$$\frac{16}{75} - \frac{32}{375}$$

$$\frac{16}{125}$$

14. Set up and evaluate a double integral to find the volume of the solid bounded by the graphs of the equations $x^2 + z^2 = 144$ and $y^2 + z^2 = 144$ in the first octant.

$$x^2 + z^2 = 144$$
$$y^2 + z^2 = 144$$

a. 5184

b. 576

c. 3456

d. 1728

e. 1152

Work:

$$\int_0^{12}\int_0^{y} 2\sqrt{144 - y^2}\, dx\, dy$$

$$2\int_0^{12} [x\sqrt{144 - y^2}]_{x=0}^{x=y}\, dy$$

$$2\int_0^{12} y\sqrt{144 - y^2}\, dy$$

$$-\int_{y=0}^{y=12} \sqrt{u}\, du$$

$$-\frac{2}{3}[u^{\frac{3}{2}}]_{y=0}^{y=12}$$

$$-\frac{2}{3}[(144 - y^2)^{\frac{3}{2}}]_0^{12}$$

$$\frac{2}{3}(144)(12)$$

$$1152$$

15. Set up and evaluate a double integral to find the volume of the solid bounded by the graphs of the equations given below.

$$z = \frac{1}{25+y^2}$$
$$x = 0$$
$$x = 2$$
$$y \geq 0$$

a. $\frac{21}{4}$

b. $\frac{4}{21}\pi$

c. $\frac{1}{5}\pi$

d. $\frac{1}{5}$

e. $\frac{21}{4}\pi$

Work:

$$\int_0^\infty \int_0^2 \frac{1}{25+y^2}\,dx\,dy$$
$$2\int_0^\infty \frac{1}{25+y^2}\,dy$$
$$2[\frac{1}{5}\,arctan(\frac{y}{5})]_0^\infty$$
$$\frac{\pi}{5}$$

16. Evaluate the iterated integral below. Note that it is necessary to switch the order of integration.

$$\int_0^4 \int_x^4 e^{-0.45y^2}\, dy\, dx$$

a. $\frac{e^{-7.2}-1}{0.9}$

b. $\frac{1-e^{-7.2}}{0.9}$

c. $\frac{1-e^{7.2}}{0.45}$

d. $\frac{1-e^{-7.2}}{0.45}$

e. $\frac{1-e^{-1.8}}{1.8}$

Work:

$$\int_0^4 \int_x^4 e^{-0.45y^2}\, dy\, dx$$

$$\int_0^4 e^{-0.45y^2} \int_0^y dx\, dy$$

$$\int_0^4 ye^{-0.45y^2}\, dy$$

$$-\frac{1}{0.9}\int_{y=0}^{y=4} e^u\, du$$

$$-\frac{1}{0.9}\left[e^u\right]_{y=0}^{y=4}$$

$$-\frac{1}{0.9}\left[e^{-0.45y^2}\right]_0^4$$

$$-\frac{1}{0.9}\left(e^{-7.2}-1\right)$$

$$\frac{1-e^{-7.2}}{0.9}$$

17. Find the average value of $f(x, y)$ over the region R where:

$$f(x, y) = xy^5$$
R: rectangle with vertices $(0, 0), (5, 0), (5, 7), (0, 7)$

a. $\frac{1}{12}$

b. $\frac{13}{84037}$

c. $\frac{1}{13}$

d. $\frac{84035}{12}$

e. $\frac{84037}{13}$

Work:

$$\frac{1}{35}\int_0^5\int_0^7 xy^5\, dy\, dx$$
$$\frac{1}{210}\int_0^5 x[y^6]_0^7\, dx$$
$$\frac{7^6}{210}\int_0^5 x\, dx$$
$$\frac{7^5}{60}[x^2]_0^5$$
$$\frac{5(7^5)}{12}$$
$$\frac{84035}{12}$$

18. Find the average value of $f(x, y) = e^{x+y}$ over the region R, where R is a triangle with vertices $(0, 0)$, $(0, 9)$, and $(9, 9)$.

$$f(x, y) = e^{x+y}$$
$$R\text{: triangle with vertices } (0, 0), (0, 9), (9, 9)$$

a. $\frac{1}{81}(e^9 - 1)^2$

b. $\frac{1}{81}(e^9 + 1)^2$

c. $(e^9 - 1)^2$

d. $(e^9 + 1)^2$

e. $\frac{1}{2}(e^9 - 1)^2$

Work:

$$\frac{2}{81}\int_0^9\int_x^9 e^{x+y}\,dy\,dx$$

$$\frac{2}{81}\int_0^9[e^{x+y}]_{y=x}^{y=9}\,dx$$

$$\frac{2}{81}\int_0^9(e^{x+9} - e^{2x})\,dx$$

$$\frac{2}{81}(\int_0^9 e^x e^9\,dx - \int_0^9 e^{2x}\,dx)$$

$$\frac{2}{81}(e^9\int_0^9 e^x\,dx - \frac{1}{2}\int_{x=0}^{x=9} e^u\,du)$$

$$\frac{2}{81}(e^9[e^x]_0^9 - \frac{1}{2}[e^u]_{x=0}^{x=9})$$

$$\frac{2}{81}(e^9(e^9 - 1) - \frac{1}{2}[e^{2x}]_0^9)$$

$$\frac{2}{81}(e^{18} - e^9 - \frac{e^{18}}{2} + \frac{1}{2})$$

$$\frac{2}{81}(\frac{e^{18}}{2} - e^9 + \frac{1}{2})$$

$$\frac{1}{81}(e^{18} - 2e^9 + 1)$$

$$\frac{1}{81}(e^9 - 1)^2$$

19. Suppose the Cobb-Douglas production function for an automobile manufacturer is $f(x, y) = 100x^{0.6}y^{0.4}$, where x is the number of units of labor and y is the number of units of capital. Estimate the average production level if the number of units of labor x varies between 150 and 200 and the number of units of capital y varies between 375 and 450. Round your answer to two decimal places.

$$f(x, y) = 100x^{0.6}y^{0.4}$$
$$150 \le x \le 200$$
$$375 \le y \le 450$$

a. 229894.96

b. 15558.45

c. 55174.79

d. 24631.60

e. 5971.30

Work:

$$\frac{1}{3750}\int_{150}^{200}\int_{375}^{450} 100x^{0.6}y^{0.4}\,dy\,dx$$

$$\frac{100}{3750(1.4)}\int_{150}^{200} x^{0.6}\left[y^{1.4}\right]_{375}^{450} dx$$

$$\frac{100(450^{1.4}-375^{1.4})}{3750(1.4)}\int_{150}^{200} x^{0.6}\,dx$$

$$\frac{100(450^{1.4}-375^{1.4})}{3750(1.4)(1.6)}\left[x^{1.6}\right]_{150}^{200}$$

$$\frac{100(450^{1.4}-375^{1.4})(200^{1.6}-150^{1.6})}{3750(1.4)(1.6)}$$

20. Suppose the temperature in degrees Celsius on the surface of a metal plate is $T(x, y) = 60 - 4x^2 - y^2$, where x and y are measured in centimeters. Estimate the average temperature if x varies between 0 and 2 centimeters and y varies between 0 and 7 centimeters.

$$T(x, y) = 60 - 4x^2 - y^2$$
$$0 \le x \le 2$$
$$0 \le y \le 7$$

a. $\frac{181}{3}$ degrees Celsius

b. $\frac{119}{3}$ degrees Celsius

c. $\frac{245}{3}$ degrees Celsius

d. $\frac{241}{3}$ degrees Celsius

e. $\frac{115}{3}$ degrees Celsius

Work:

$$\frac{1}{14}\int_0^2\int_0^7 (60 - 4x^2 - y^2)\, dy\, dx$$

$$\frac{1}{14}\int_0^2 [60y - 4x^2y - \frac{1}{3}y^3]_{y=0}^{y=7}\, dx$$

$$\frac{1}{2}\int_0^2 (60 - 4x^2 - \frac{49}{3})\, dx$$

$$\frac{1}{2}\int_0^2 (\frac{131}{3} - 4x^2)\, dx$$

$$\frac{1}{6}[131x - 4x^3]_0^2$$

$$\frac{115}{3}$$

21. Evaluate the double integral below.

$$\int_0^{2\pi}\int_0^{3} 2r^5\, sin(\theta)\, dr\, d\theta$$

a. 3π

b. 0

c. π

d. 2π

e. $\frac{\pi}{2}$

Work:

$$\int_0^{2\pi}\int_0^{3} 2r^5\, sin(\theta)\, dr\, d\theta$$

$$2\int_0^{2\pi} sin(\theta) \int_0^{3} r^5\, dr\, d\theta$$

$$\frac{1}{3}\int_0^{2\pi} sin(\theta)\, [r^6]_0^3\, d\theta$$

$$243\int_0^{2\pi} sin(\theta)\, d\theta$$

$$-\ 243[cos(\theta)]_0^{2\pi}$$

$$0$$

22. Evaluate the following iterated integral by converting to polar coordinates.

$$\int_0^{10} \int_0^{\sqrt{100-x^2}} y \, dy \, dx$$

a. $\frac{1000}{3}$

b. $\frac{100}{3}$

c. $\frac{100}{3}\pi$

d. $\frac{1000}{3}\pi$

e. $\frac{100}{5}\pi$

Work:

$$\int_0^{10} \int_0^{\sqrt{100-x^2}} y \, dy \, dx$$

$$\int_0^{\frac{\pi}{2}} \int_0^{10} r^2 \sin(\theta) \, dr \, d\theta$$

$$\frac{1}{3}\int_0^{\frac{\pi}{2}} \sin(\theta) \, [r^3]_0^{10} \, d\theta$$

$$\frac{1000}{3}\int_0^{\frac{\pi}{2}} \sin(\theta) \, d\theta$$

$$-\frac{1000}{3}[\cos(\theta)]_0^{\frac{\pi}{2}}$$

$$\frac{1000}{3}$$

23. Evaluate the iterated integral $\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} 5(x^2 + y^2)\, dy\, dx$ by converting to polar coordinates.

$$\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} 5(x^2 + y^2)\, dy\, dx$$

a. $\frac{\pi}{32}$

b. $\frac{15\pi}{64}$

c. $\frac{25\pi}{32}$

d. $\frac{25\pi}{64}$

e. $\frac{15\pi}{32}$

Work:

$$\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} 5(x^2 + y^2)\, dy\, dx$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\sqrt{\frac{1}{4}-x^2}}^{\sqrt{\frac{1}{4}-x^2}} 5((x + \tfrac{1}{2})^2 + y^2)\, dy\, dx$$

$$5\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\sqrt{\frac{1}{4}-x^2}}^{\sqrt{\frac{1}{4}-x^2}} (x^2 + y^2 + x + \tfrac{1}{4})\, dy\, dx$$

$$5\int_0^{2\pi} \int_0^{\frac{1}{2}} (r^3 + r^2 \cos(\theta) + \tfrac{1}{4} r)\, dr\, d\theta$$

$$5\int_0^{2\pi} [\tfrac{1}{4} r^4 + \tfrac{1}{3} r^3 \cos(\theta) + \tfrac{1}{8} r^2]_{r=0}^{r=\frac{1}{2}}\, d\theta$$

$$5\int_0^{2\pi} (\tfrac{3}{64} + \tfrac{1}{24}\cos(\theta))\, d\theta$$

$$5[\tfrac{3}{64}\theta + \tfrac{1}{24}\sin(\theta)]_0^{2\pi}$$

$$\frac{15\pi}{32}$$

24. Evaluate the following iterated integral by converting to polar coordinates.

$$\int_0^5 \int_0^{\sqrt{8x-x^2}} \frac{1}{5}xy\,dy\,dx$$

a. $\frac{1275}{74}$

b. $\frac{425}{24}$

c. $\frac{49}{850}$

d. $\frac{24}{425}$

e. $\frac{74}{1275}$

Work:

$$\int_0^5 \int_0^{\sqrt{8x-x^2}} \frac{1}{5}xy\,dy\,dx$$

$$\int_{-4}^1 \int_0^{\sqrt{16-x^2}} \frac{1}{5}(x+4)y\,dy\,dx$$

$$\int_{arccos(\frac{1}{4})}^{\pi} \int_0^4 \frac{1}{5}r^2(r\,cos(\theta)+4)\,sin(\theta)\,dr\,d\theta + \int_0^{arccos(\frac{1}{4})} \int_0^{sec(\theta)} \frac{1}{5}r^2(r\,cos(\theta)+4)\,sin(\theta)\,dr\,d\theta$$

$$\frac{1}{5}\left(\int_{arccos(\frac{1}{4})}^{\pi} \int_0^4 (r^3\,cos(\theta)\,sin(\theta)+4r^2\,sin(\theta))\,dr\,d\theta + \int_0^{arccos(\frac{1}{4})} \int_0^{sec(\theta)} (r^3\,cos(\theta)\,sin(\theta)+4r^2\,sin(\theta))\,dr\,d\theta\right)$$

$$\frac{1}{5}\left(\int_{arccos(\frac{1}{4})}^{\pi} \left[\frac{1}{4}r^4\,cos(\theta)\,sin(\theta)+\frac{4}{3}r^3\,sin(\theta)\right]_{r=0}^{r=4} d\theta + \int_0^{arccos(\frac{1}{4})} \left[\frac{1}{4}r^4\,cos(\theta)\,sin(\theta)+\frac{4}{3}r^3\,sin(\theta)\right]_{r=0}^{r=sec(\theta)} d\theta\right)$$

$$\frac{1}{5}\left(256\int_{arccos(\frac{1}{4})}^{\pi} \left(\frac{1}{4}cos(\theta)\,sin(\theta)+\frac{1}{3}sin(\theta)\right)d\theta + \int_0^{arccos(\frac{1}{4})} \left(\frac{1}{4}sec^2(\theta)\,tan(\theta)+\frac{4}{3}sec^2(\theta)\,tan(\theta)\right)d\theta\right)$$

$$\frac{1}{5}\left(256\left(\int_{arccos(\frac{1}{4})}^{\pi} \frac{1}{4}cos(\theta)\,sin(\theta)\,d\theta + \int_{arccos(\frac{1}{4})}^{\pi} \frac{1}{3}sin(\theta)\,d\theta\right) + \int_0^{arccos(\frac{1}{4})} \frac{19}{12}sec^2(\theta)\,tan(\theta)\,d\theta\right)$$

$$\frac{1}{5}\left(256\left(\frac{1}{8}\left[sin^2(\theta)\right]_{arccos(\frac{1}{4})}^{\pi} - \frac{1}{3}\left[cos(\theta)\right]_{arccos(\frac{1}{4})}^{\pi}\right) + \frac{19}{24}\left[tan^2(\theta)\right]_0^{arccos(\frac{1}{4})}\right)$$

$$\frac{1}{5}\left(256\left(-\frac{1}{8}sin^2(arccos(\frac{1}{4}))+\frac{1}{3}+\frac{1}{3}cos(arccos(\frac{1}{4}))\right)+\frac{19}{24}tan^2(arccos(\frac{1}{4}))\right)$$

$$\frac{1}{5}\left(256\left(-\frac{1}{8}sin^2(arcsin(\frac{\sqrt{15}}{4}))+\frac{1}{3}+\frac{1}{12}\right)+\frac{19}{24}tan^2(arctan(\sqrt{15}))\right)$$

$$\frac{1}{5}\left(256\left(-\frac{15}{128}+\frac{5}{12}\right)+\frac{285}{24}\right)$$

$$\frac{425}{24}$$

25. Evaluate the iterated integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 8\cos(x^2+y^2)\,dy\,dx$ by converting to polar coordinates. Round your answer to four decimal places.

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 8\cos(x^2+y^2)\,dy\,dx$$

a. 10.5742

b. 13.5742

c. 17.5742

d. 28.5742

e. 14.5742

Work:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 8\cos(x^2+y^2)\,dy\,dx$$

$$\int_{0}^{\pi}\int_{0}^{1} 8r\cos(r^2)\,dr\,d\theta$$

$$4\int_{0}^{\pi}\int_{r=0}^{r=1} \cos(u)\,du\,d\theta$$

$$4\int_{0}^{\pi}[\sin(u)]_{r=0}^{r=1}\,d\theta$$

$$4\sin(1)\int_{0}^{\pi} d\theta$$

$$4\pi\sin(1)$$

26. Combine the sum of the two iterated integrals into a single integral by converting to polar coordinates. Evaluate the resulting iterated integral.

$$\int_0^2\int_0^x \sqrt{x^2+y^2}\,dy\,dx + \int_2^{2\sqrt{2}}\int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$

a. $\frac{3\sqrt{2}}{8\pi}$

b. $\frac{3\pi}{8\sqrt{2}}$

c. $\frac{8\pi}{3}$

d. $\frac{4}{3}\sqrt{2}\pi$

e. $\frac{4}{3}\pi$

Work:

$$\int_0^2\int_0^x \sqrt{x^2+y^2}\,dy\,dx + \int_2^{2\sqrt{2}}\int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$

$$\int_0^{\frac{\pi}{4}}\int_0^{2\sqrt{2}} r^2\,dr\,d\theta$$

$$\frac{1}{3}\int_0^{\frac{\pi}{4}} [r^3]_0^{2\sqrt{2}}\,d\theta$$

$$\frac{1}{3}\int_0^{\frac{\pi}{4}} 16\sqrt{2}\,d\theta$$

$$\frac{4\pi\sqrt{2}}{3}$$

27. Given $f(x, y) = e^{-\frac{x^2+y^2}{2}}$, R: $x^2 + y^2 \leq 729, x \geq 0$, use polar coordinates to set up and evaluate the double integral $\int_R \int f(x, y)\, dA$.

$$f(x, y) = e^{-\frac{x^2+y^2}{2}}$$
$$R: x^2 + y^2 \leq 729, x \geq 0$$

a. $\pi(1 - e^{\frac{-19683}{2}})$

b. $(1 + e^{\frac{-729}{2}})$

c. $\pi(1 + e^{\frac{-19683}{2}})$

d. $(1 - e^{\frac{-729}{2}})$

e. $\pi(1 - e^{\frac{-729}{2}})$

Work:

$$\int_R \int e^{\frac{-(x^2+y^2)}{2}}\, dA$$

$$\int_0^{27} \int_{-\sqrt{729-x^2}}^{\sqrt{729-x^2}} e^{\frac{-(x^2+y^2)}{2}}\, dy\, dx$$

$$2\int_0^{\frac{\pi}{2}} \int_0^{27} re^{\frac{-r^2}{2}}\, dr\, d\theta$$

$$-2\int_0^{\frac{\pi}{2}} \int_{r=0}^{r=27} e^u\, du\, d\theta$$

$$-2\int_0^{\frac{\pi}{2}} [e^u]_{r=0}^{r=27}\, d\theta$$

$$-2\int_0^{\frac{\pi}{2}} [e^{\frac{-r^2}{2}}]_0^{27}\, d\theta$$

$$-2\int_0^{\frac{\pi}{2}} (e^{\frac{-729}{2}} - 1)\, d\theta$$

$$\pi(1 - e^{\frac{-729}{2}})$$

28. Use a double integral in polar coordinates to find the volume of a solid inside the hemisphere $z = \sqrt{64 - x^2 - y^2}$ but outside the cylinder $x^2 + y^2 = 25$.

$$z = \sqrt{64 - x^2 - y^2}$$
$$x^2 + y^2 = 25$$

a. $\frac{2}{3}39^{\frac{3}{2}}$

b. $\frac{2}{3}39^{\frac{3}{2}}\pi$

c. $\frac{4}{3}39^{\frac{3}{2}}\pi$

d. $\frac{2}{3}39^{\frac{1}{2}}\pi$

e. $\frac{3}{2}39^{\frac{3}{2}}$

Work:

$$\int_0^{2\pi}\int_5^8 r\sqrt{64 - r^2}\, dr\, d\theta$$

$$-\frac{1}{2}\int_0^{2\pi}\int_{r=5}^{r=8} \sqrt{u}\, du\, d\theta$$

$$-\frac{1}{3}\int_0^{2\pi} [u^{\frac{3}{2}}]_{r=5}^{r=8}\, d\theta$$

$$-\frac{1}{3}\int_0^{2\pi} [(64 - r^2)^{\frac{3}{2}}]_5^8\, d\theta$$

$$\frac{1}{3}39^{\frac{3}{2}}\int_0^{2\pi} d\theta$$

$$\frac{2}{3}39^{\frac{3}{2}}\pi$$

29. Use a double integral to find the area of the region inside the circle $r = 17\cos(\theta)$ and outside the cardioid $r = 1 + 15\cos(\theta)$. Round your answer to two decimal places.

$$r = 17\cos(\theta)$$
$$r = 1 + 15\cos(\theta)$$

a. 46.68

b. 58.34

c. 20.34

d. 55.34

e. 22.34

Work:

$$2\int_{0}^{\frac{\pi}{3}}\int_{1+15\cos(\theta)}^{17\cos(\theta)} r\,dr\,d\theta$$

$$\int_{0}^{\frac{\pi}{3}}[r^2]_{1+15\cos(\theta)}^{17\cos(\theta)}\,d\theta$$

$$\int_{0}^{\frac{\pi}{3}}(289\cos^2(\theta) - 1 - 30\cos(\theta) - 225\cos^2(\theta))\,d\theta$$

$$\int_{0}^{\frac{\pi}{3}}(64\cos^2(\theta) - 30\cos(\theta) - 1)\,d\theta$$

$$\int_{0}^{\frac{\pi}{3}}(32 + 32\cos(2\theta) - 30\cos(\theta) - 1)\,d\theta$$

$$\int_{0}^{\frac{\pi}{3}}(32\cos(2\theta) - 30\cos(\theta) + 31)\,d\theta$$

$$[16\sin(2\theta) - 30\sin(\theta) + 31\theta]_{0}^{\frac{\pi}{3}}$$

$$8\sqrt{3} - 15\sqrt{3} + \frac{31\pi}{3}$$

30. Suppose the population density of a city is approximated by the model $f(x, y) = 6000e^{-0.01(x^2+y^2)}, x^2 + y^2 \leq 25$, where x and y are measured in miles. Integrate the density function over the indicated circular region to approximate the population of the city. Round your answer to the nearest integer.

$$f(x, y) = 6000e^{-0.01(x^2+y^2)}$$
$$x^2 + y^2 \leq 25$$

a. 417127

b. 417029

c. 833901

d. 833903

e. 416951

Work:

$$\int_{-5}^{5} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} 6000e^{-0.01(x^2+y^2)} \, dy \, dx$$

$$\int_{0}^{2\pi} \int_{0}^{5} 6000re^{-0.01r^2} \, dr \, d\theta$$

$$-300000 \int_{0}^{2\pi} \int_{r=0}^{r=5} e^{u} \, du \, d\theta$$

$$-300000 \int_{0}^{2\pi} (e^{-\frac{1}{4}} - 1) \, d\theta$$

$$-600000\pi e^{-\frac{1}{4}} + 600000\pi$$

31. Find the area of the portion of the surface $z = 8x + 4y$ that lies above the triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 2)$.

$$z = 8x + 4y$$

triangle with vertices $(0, 0), (2, 0), (0, 2)$

a. 4

b. 36

c. 18

d. 162

e. 324

Work:

$$\int_0^2 \int_0^x \sqrt{1 + \left(\frac{\vartheta}{\vartheta x}(8x + 4y)\right)^2 + \left(\frac{\vartheta}{\vartheta y}(8x + 4y)\right)^2}\, dy\, dx$$

$$\int_0^2 \int_0^x \sqrt{1 + 64 + 16}\, dy\, dx$$

$$9 \int_0^2 \int_0^x dy\, dx$$

$$9 \int_0^2 x\, dx$$

$$\frac{9}{2} [x^2]_0^2$$

$$18$$

32. Find the area of the portion of the surface $z = 8x + 4y$ that lies above the region $R = \{(x, y): x^2 + y^2 \leq 4\}$.

$$z = 8x + 4y$$
$$R = \{(x, y): x^2 + y^2 \leq 4\}$$

a. 72

b. 36

c. 4π

d. 36π

e. 72π

Work:

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{1 + \left(\frac{\vartheta}{\vartheta x}(8x + 4y)\right)^2 + \left(\frac{\vartheta}{\vartheta y}(8x + 4y)\right)^2}\, dy\, dx$$

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{1 + 64 + 16}\, dy\, dx$$

$$9\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy\, dx$$

$$9\int_{0}^{2\pi}\int_{0}^{2} r\, dr\, d\theta$$

$$18\int_{0}^{2\pi} d\theta$$

$$36\pi$$

33. Find the area of the portion of the surface $f(x, y) = 7 + \frac{2}{3}y^{\frac{3}{2}}$ that lies above the region $R = \{(x, y): 0 \leq x \leq 4, 0 \leq y \leq 4 - x\}$.

$$f(x, y) = 7 + \frac{2}{3}y^{\frac{3}{2}}$$
$$R = \{(x, y): 0 \leq x \leq 4, 0 \leq y \leq 4 - x\}$$

a. 0.05

b. 2.18

c. 11.97

d. 1.22

e. 12.51

Work:

$$\int_0^4 \int_0^{4-x} \sqrt{1 + \left(\frac{\vartheta}{\vartheta x}\left(7 + \frac{2}{3}y^{\frac{3}{2}}\right)\right)^2 + \left(\frac{\vartheta}{\vartheta y}\left(7 + \frac{2}{3}y^{\frac{3}{2}}\right)\right)^2}\, dy\, dx$$

$$\int_0^4 \int_0^{4-x} \sqrt{1 + y}\, dy\, dx$$

$$\frac{2}{3}\int_0^4 [(1 + y)^{\frac{3}{2}}]_0^{4-x}\, dx$$

$$\frac{2}{3}\int_0^4 ((5 - x)^{\frac{3}{2}} - 1)\, dx$$

$$\frac{2}{3}[-\frac{2}{5}(5 - x)^{\frac{5}{2}} - x]_0^4$$

$$-\frac{4}{15} - \frac{8}{3} + \frac{20\sqrt{5}}{3}$$

34. Find the area of the surface given by $z = f(x, y)$ over the region R.

$$f(x, y) = xy$$
$$R = \{(x, y): x^2 + y^2 \leq 100\}$$

a. $\frac{2}{3}(101\sqrt{101} - 1)\pi$

b. $\frac{2}{3}(1 - 101\sqrt{101})\pi$

c. $\frac{2}{3}(101\sqrt{101} - 1)$

d. $\frac{3}{4}(101\sqrt{101} - 1)\pi$

e. $\frac{2}{3}(100\sqrt{101} - 1)\pi$

Work:

$$\int_{-10}^{10}\int_{-\sqrt{100-x^2}}^{\sqrt{100-x^2}} \sqrt{1 + \left(\frac{\vartheta}{\vartheta x}xy\right)^2 + \left(\frac{\vartheta}{\vartheta y}xy\right)^2}\, dy\, dx$$

$$\int_{-10}^{10}\int_{-\sqrt{100-x^2}}^{\sqrt{100-x^2}} \sqrt{1 + y^2 + x^2}\, dy\, dx$$

$$\int_{0}^{2\pi}\int_{0}^{10} r\sqrt{1 + r^2}\, dr\, d\theta$$

$$\frac{1}{2}\int_{0}^{2\pi}\int_{r=0}^{r=10} \sqrt{u}\, du\, d\theta$$

$$\frac{1}{3}\int_{0}^{2\pi} [u^{\frac{3}{2}}]_{r=0}^{r=10}\, d\theta$$

$$\frac{1}{3}\int_{0}^{2\pi} [(1 + r^2)^{\frac{3}{2}}]_{0}^{10}\, d\theta$$

$$\frac{1}{3}\int_{0}^{2\pi} (101\sqrt{101} - 1)\, d\theta$$

$$\frac{2\pi}{3}(101\sqrt{101} - 1)$$

35. Find the area of the surface for the portion of the sphere $x^2 + y^2 + z^2 = 225$ inside the cylinder $x^2 + y^2 = 144$.

$$x^2 + y^2 + z^2 = 225$$
$$x^2 + y^2 = 144$$

a. 540π

b. 720π

c. 360π

d. 60π

e. 40π

Work:

$$2\int_{-12}^{12}\int_{-\sqrt{144-x^2}}^{\sqrt{144-x^2}} \sqrt{1 + (\frac{\vartheta}{\vartheta x}\sqrt{225 - x^2 - y^2})^2 + (\frac{\vartheta}{\vartheta y}\sqrt{225 - x^2 - y^2})^2}\, dy\, dx$$

$$2\int_{-12}^{12}\int_{-\sqrt{144-x^2}}^{\sqrt{144-x^2}} \sqrt{1 + (\frac{-y}{\sqrt{225-x^2-y^2}})^2 + (\frac{-x}{\sqrt{225-x^2-y^2}})^2}\, dy\, dx$$

$$2\int_{-12}^{12}\int_{-\sqrt{144-x^2}}^{\sqrt{144-x^2}} \sqrt{1 + \frac{x^2+y^2}{225-x^2-y^2}}\, dy\, dx$$

$$2\int_{0}^{2\pi}\int_{0}^{12} r\sqrt{1 + \frac{r^2}{225-r^2}}\, dr\, d\theta$$

$$2\int_{0}^{2\pi}\int_{0}^{12} r\sqrt{\frac{225}{225-r^2}}\, dr\, d\theta$$

$$30\int_{0}^{2\pi}\int_{0}^{12} \frac{r}{\sqrt{225-r^2}}\, dr\, d\theta$$

$$-15\int_{0}^{2\pi}\int_{r=0}^{r=12} u^{-\frac{1}{2}}\, du\, d\theta$$

$$-30\int_{0}^{2\pi} [u^{\frac{1}{2}}]_{r=0}^{r=12}\, d\theta$$

$$-30\int_{0}^{2\pi} [(225 - r^2)^{\frac{1}{2}}]_0^{12}\, d\theta$$

$$180\int_{0}^{2\pi} d\theta$$

$$360\pi$$

36. Set up a double integral that gives the area of the surface of the graph of f over the region R.

$$f(x, y) = x^2 - 9xy + 9y^2$$
$$R = \{(x, y): -6 \le x \le 6, -8 \le y \le 8\}$$

a. $\int_{-6}^{6}\int_{-8}^{8} \sqrt{1 + (2x - 9y)^2 + (18y - 9x)^2}\, dx\, dy$

b. $\int_{-8}^{8}\int_{-6}^{6} \sqrt{1 + (2x - 9y)^2 + (18y - 9x)^2}\, dy\, dx$

c. $\int_{-6}^{6}\int_{-8}^{8} \sqrt{1 + (2x^2 - 9y)^2 + (18y^2 - 9x)^2}\, dy\, dx$

d. $\int_{-6}^{6}\int_{-8}^{8} \sqrt{1 + (2x - 9y)^2 + (18y - 9x)^2}\, dy\, dx$

e. $\int_{-6}^{6}\int_{-8}^{8} \sqrt{1 + (2x + 9y)^2 + (18y + 9x)^2}\, dy\, dx$

Work:

$$\int_{-6}^{6}\int_{-8}^{8} \sqrt{1 + \left(\frac{\partial}{\partial x}(x^2 - 9xy + 9y^2)\right)^2 + \left(\frac{\partial}{\partial y}(x^2 - 9xy + 9y^2)\right)^2}\, dy\, dx$$

$$\int_{-6}^{6}\int_{-8}^{8} \sqrt{1 + (2x - 9y)^2 + (18y - 9x)^2}\, dy\, dx$$

37. Set up a double integral that gives the area of the surface area on the graph of $f(x, y) = 10\cos(x^2 + y^2)$ over the region $R = \{(x, y): x^2 + y^2 \le \frac{\pi}{4}\}$.

$$f(x, y) = 10\cos(x^2 + y^2)$$
$$R = \{(x, y): x^2 + y^2 \le \frac{\pi}{4}\}$$

a. $S = \int_{-\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{4}}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + 400(x^2 + y^2)\sin^2(x^2 + y^2)}\, dy\, dx$

b. $S = \int_{-\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{4}}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + 400(x^2 + y^2)\cos^2(x^2 + y^2)}\, dy\, dx$

c. $S = \int_{-\sqrt{\frac{\pi}{4}-y^2}}^{\sqrt{\frac{\pi}{4}-y^2}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{3 + 400(x^2 + y^2)\cos^2(x^2 + y^2)}\, dy\, dx$

d. $S = \int_{-\sqrt{\frac{\pi}{4}-y^2}}^{\sqrt{\frac{\pi}{4}-y^2}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{3 + 400(x^2 + y^2)\sin^2(x^2 + y^2)}\, dy\, dx$

e. $S = \int_{-\sqrt{\frac{\pi}{4}-y^2}}^{\sqrt{\frac{\pi}{4}-y^2}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + 400(x^2 + y^2)\cos^2(x^2 + y^2)}\, dy\, dx$

Work:

$$\int_{-\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{4}}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + \left(\frac{\vartheta}{\vartheta x}(10\cos(x^2 + y^2))\right)^2 + \left(\frac{\vartheta}{\vartheta y}(10\cos(x^2 + y^2))\right)^2}\, dy\, dx$$

$$\int_{-\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{4}}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + (-20x\sin(x^2 + y^2))^2 + (-20y\sin(x^2 + y^2))^2}\, dy\, dx$$

$$\int_{-\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{4}}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + 400x^2\sin^2(x^2 + y^2) + 400y^2\sin^2(x^2 + y^2)}\, dy\, dx$$

$$\int_{-\sqrt{\frac{\pi}{4}}}^{\sqrt{\frac{\pi}{4}}} \int_{-\sqrt{\frac{\pi}{4}-x^2}}^{\sqrt{\frac{\pi}{4}-x^2}} \sqrt{1 + 400(x^2 + y^2)\sin^2(x^2 + y^2)}\, dy\, dx$$

38. Set up a double integral that gives the area of the surface of the graph of f over the region R.

$$f(x, y) = e^{9xy}$$
$$R = \{(x, y): 0 \le x \le 6, 0 \le y \le 2\}$$

a. $\int_0^2 \int_0^6 \sqrt{1 + 81e^{18xy}(x^2 + y^2)}\, dx\, dy$

b. $\int_0^2 \int_0^6 \sqrt{1 + 81e^{18xy}(x^2 + y^2)}\, dy\, dx$

c. $\int_0^2 \int_0^6 \sqrt{1 + 81e^{9xy}(x^2 + y^2)}\, dx\, dy$

d. $\int_0^2 \int_0^6 \sqrt{1 + 9e^{9xy}(x^2 + y^2)}\, dx\, dy$

e. $\int_0^6 \int_0^2 \sqrt{1 + 81e^{18xy}(x^2 + y^2)}\, dx\, dy$

Work:

$$\int_0^6 \int_0^2 \sqrt{1 + \left(\frac{\partial}{\partial x}(e^{9xy})\right)^2 + \left(\frac{\partial}{\partial y}(e^{9xy})\right)^2}\, dy\, dx$$

$$\int_0^6 \int_0^2 \sqrt{1 + (9ye^{9xy})^2 + (9xe^{9xy})^2}\, dy\, dx$$

$$\int_0^6 \int_0^2 \sqrt{1 + 81y^2e^{18xy} + 81x^2e^{18xy}}\, dy\, dx$$

$$\int_0^6 \int_0^2 \sqrt{1 + 81e^{18xy}(x^2 + y^2)}\, dy\, dx$$

$$\int_0^2 \int_0^6 \sqrt{1 + 81e^{18xy}(x^2 + y^2)}\, dx\, dy$$

39. A company produces a spherical object of radius 17 centimeters. A hole of radius 7 centimeters is drilled through the center of the object. Find the volume of the object.

a. $1344\pi\sqrt{15}\ cm^3$

b. $1680\pi\sqrt{15}\ cm^3$

c. $1280\pi\sqrt{15}\ cm^3$

d. $3360\pi\sqrt{15}\ cm^3$

e. $1120\pi\sqrt{15}\ cm^3$

Work:

$$\frac{4\pi 17^3}{3} - 2\int_0^{2\pi}\int_0^{7} r\sqrt{17^2 - r^2}\, dr\, d\theta$$

$$\frac{4\pi 17^3}{3} + \int_0^{2\pi}\int_{r=0}^{r=7} \sqrt{u}\, du\, d\theta$$

$$\frac{4\pi 17^3}{3} + \frac{2}{3}\int_0^{2\pi} [u^{\frac{3}{2}}]_{r=0}^{r=7}\, d\theta$$

$$\frac{4\pi 17^3}{3} + \frac{2}{3}\int_0^{2\pi} [(17^2 - r^2)^{\frac{3}{2}}]_0^7\, d\theta$$

$$\frac{4\pi 17^3}{3} + \frac{2}{3}\int_0^{2\pi} (240^{\frac{3}{2}} - 289^{\frac{3}{2}})\, d\theta$$

$$\frac{4\pi 17^3}{3} + \frac{2}{3}\int_0^{2\pi} (960\sqrt{15} - 17^3)\, d\theta$$

$$\frac{4\pi 17^3}{3} + 1280\pi\sqrt{15} - \frac{4\pi 17^3}{3}$$

$$1280\pi\sqrt{15}$$

40. A company produces a spherical object of radius 24 centimeters. A hole of radius 5 centimeters is drilled through the center of the object. Find the outer surface area of the object.

a. $73\pi\sqrt{551}\ cm^3$

b. $5\pi\sqrt{551}\ cm^3$

c. $24\pi\sqrt{551}\ cm^3$

d. $20\pi\sqrt{551}\ cm^3$

e. $96\pi\sqrt{551}\ cm^3$

Work:

$$4\pi 24^2 - 2\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{1 + \left(\frac{\vartheta}{\vartheta x}\left(\sqrt{24^2 - x^2 - y^2}\right)\right)^2 + \left(\frac{\vartheta}{\vartheta y}\left(\sqrt{24^2 - x^2 - y^2}\right)\right)^2}\, dy\, dx$$

$$4\pi 24^2 - 2\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{1 + \left(\frac{-x}{\sqrt{24^2-x^2-y^2}}\right)^2 + \left(\frac{-y}{\sqrt{24^2-x^2-y^2}}\right)^2}\, dy\, dx$$

$$4\pi 24^2 - 2\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{1 + \frac{x^2}{24^2-x^2-y^2} + \frac{y^2}{24^2-x^2-y^2}}\, dy\, dx$$

$$4\pi 24^2 - 2\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \sqrt{1 + \frac{x^2+y^2}{24^2-x^2-y^2}}\, dy\, dx$$

$$4\pi 24^2 - 2\int_{0}^{2\pi}\int_{0}^{5} r\sqrt{1 + \frac{r^2}{24^2-r^2}}\, dr\, d\theta$$

$$4\pi 24^2 - 2\int_{0}^{2\pi}\int_{0}^{5} r\sqrt{\frac{24^2}{24^2-r^2}}\, dr\, d\theta$$

$$4\pi 24^2 - 48\int_{0}^{2\pi}\int_{0}^{5} \frac{r}{\sqrt{24^2-r^2}}\, dr\, d\theta$$

$$4\pi 24^2 + 24\int_{0}^{2\pi}\int_{r=0}^{r=5} u^{-\frac{1}{2}}\, du\, d\theta$$

$$4\pi 24^2 + 48\int_{0}^{2\pi} \left[u^{\frac{1}{2}}\right]_{r=0}^{r=5} d\theta$$

$$4\pi 24^2 + 48\int_{0}^{2\pi} \left[(24^2 - r^2)^{\frac{1}{2}}\right]_0^5 d\theta$$

$$4\pi 24^2 + 48\int_{0}^{2\pi} (\sqrt{551} - 24)\, d\theta$$

$$4\pi 24^2 + 96\pi\sqrt{551} - 4\pi 24^2$$

$$96\pi\sqrt{551}$$

41. Evaluate the iterated integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{y}{7}} \int_0^{\frac{2}{y}} 3\, sin(y)\, dz\, dx\, dy$.

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{y}{7}} \int_0^{\frac{2}{y}} 3\, sin(y)\, dz\, dx\, dy$$

a. $\frac{3}{14}$

b. $\frac{6}{7}$

c. $\frac{2}{7}\pi$

d. $\frac{3}{7}\pi$

e. $\frac{4}{7}$

Work:

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{y}{7}} \int_0^{\frac{2}{y}} 3\, sin(y)\, dz\, dx\, dy$$

$$6 \int_0^{\frac{\pi}{2}} \int_0^{\frac{y}{7}} \frac{sin(y)}{y}\, dx\, dy$$

$$\frac{6}{7} \int_0^{\frac{\pi}{2}} sin(y)\, dy$$

$$-\frac{6}{7}\left[cos(y)\right]_0^{\frac{\pi}{2}}$$

$$\frac{6}{7}$$

42. Set up a triple integral for the volume of the solid bounded by the coordinate planes and the plane given below.

$$z = 12 - 6x - 8y$$

a. $\int_0^2 \int_0^{\frac{12-6x}{8}} \int_0^{12-6x-8y} dz\, dy\, dx$

b. $\int_0^2 \int_0^{\frac{12-6x}{8}} \int_0^{12-6x-8y} dy\, dz\, dx$

c. $\int_0^{12} \int_0^{\frac{12-6x}{8}} \int_0^{12-6x-8y} dz\, dy\, dx$

d. $\int_0^2 \int_0^{\frac{12-6x}{8}} \int_0^{12-6x-8y} dx\, dy\, dz$

e. $\int_0^{\frac{12-6x}{8}} \int_0^2 \int_0^{12-6x-8y} dz\, dy\, dx$

Work:

$$\int_0^2 \int_0^{\frac{12-6x}{8}} \int_0^{12-6x-8y} dz\, dy\, dx$$

43. Set up a triple integral for the volume of the solid bounded by $z = 7 - x^2 - y^2$ and $z = 0$.

$$z = 7 - x^2 - y^2$$
$$z = 0$$

a. $\int_{-\sqrt{7}}^{\sqrt{7}} \int_{-7}^{\sqrt{7-x^2}} \int_{0}^{7-x^2-y^2} dz\, dx\, dy$

b. $\int_{-7}^{7} \int_{-7}^{7-x^2} \int_{0}^{7-x^2-y^2} dz\, dx\, dy$

c. $\int_{-\sqrt{7}}^{\sqrt{7}} \int_{-\sqrt{7-x^2}}^{\sqrt{7-x^2}} \int_{0}^{\sqrt{7-x^2-y^2}} dz\, dx\, dy$

d. $\int_{-\sqrt{7}}^{\sqrt{7}} \int_{-\sqrt{7-x^2}}^{\sqrt{7-x^2}} \int_{0}^{7-x^2-y^2} dz\, dy\, dx$

e. $\int_{-7}^{7} \int_{-\sqrt{7+x^2}}^{7-x^2} \int_{0}^{7-x^2-y^2} dz\, dy\, dx$

Work:

$$\int_{-\sqrt{7}}^{\sqrt{7}} \int_{-\sqrt{7-x^2}}^{\sqrt{7-x^2}} \int_{0}^{7-x^2-y^2} dz\, dy\, dx$$

44. Rewrite the iterated integral $\int_0^6 \int_0^{\frac{6-x}{2}} \int_0^{\frac{24-4x-8y}{8}} dz\, dy\, dx$ using the order $dy\, dx\, dz$.

$$\int_0^6 \int_0^{\frac{6-x}{2}} \int_0^{\frac{24-4x-8y}{8}} dz\, dy\, dx$$

a. $\int_0^{24} \int_0^{\frac{24-z}{8}} \int_0^{\frac{24-3z-x}{4}} dy\, dx\, dz$*

b. $\int_0^{8} \int_0^{\frac{24-3z}{3}} \int_0^{\frac{24-3z-4x}{8}} dy\, dx\, dz$*

c. $\int_0^{8} \int_0^{\frac{24-3z}{3}} \int_0^{\frac{24-3z-x}{4}} dy\, dx\, dz$*

d. $\int_0^{6} \int_0^{\frac{24-z}{3}} \int_0^{\frac{24-3z-4x}{8}} dy\, dx\, dz$*

e. $\int_0^{24} \int_0^{\frac{24-3z}{4}} \int_0^{\frac{24-3z-4x}{8}} dy\, dx\, dz$*

Work:

$$\int_0^6 \int_0^{\frac{6-x}{2}} \int_0^{\frac{24-4x-8y}{8}} dz\, dy\, dx$$

$$\int_0^6 \int_0^{3-\frac{x}{2}} \int_0^{3-\frac{x}{2}-y} dz\, dy\, dx$$

$$\int_0^3 \int_0^{6-2z} \int_0^{3-\frac{x}{2}-z} dy\, dx\, dz^{**}$$

*correct answer is not among answer options
**correct answer

45. Sketch the solid whose volume is given by the iterated integral given below and use the sketch to rewrite the integral using the indicated order of integration.

$$\int_0^2\int_y^2\int_0^{\sqrt{4-y^2}} dz\,dx\,dy$$

Rewrite the integral using the order $dz\,dy\,dx$.

a. $\int_0^2\int_2^x\int_0^{\sqrt{4-z^2}} dz\,dy\,dx$

b. $\int_{-2}^2\int_0^x\int_0^{\sqrt{4-y^2}} dz\,dy\,dx$

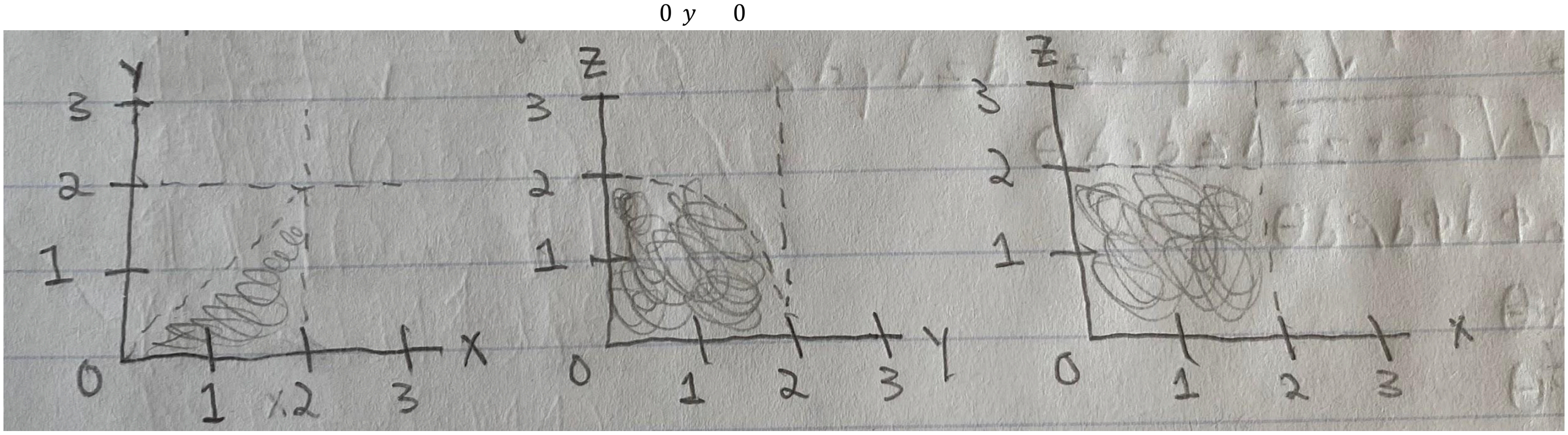
c. $\int_0^2\int_x^2\int_0^{\sqrt{2-y^2}} dz\,dy\,dx$

d. $\int_0^2\int_0^x\int_0^{\sqrt{4-y^2}} dz\,dy\,dx$

e. $\int_0^2\int_x^4\int_0^{\sqrt{4-y^2}} dz\,dy\,dx$

Work:

$$\int_0^2\int_y^2\int_0^{\sqrt{4-y^2}} dz\,dx\,dy$$



$$\int_0^2\int_0^x\int_0^{\sqrt{4-y^2}} dz\,dy\,dx$$

46. Evaluate the following iterated integral.

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos^2(\theta)} \int_0^{1-r^2} r \sin(\theta)\, dz\, dr\, d\theta$$

a. $\frac{11}{180}$

b. $\frac{13}{180}$

c. $\frac{1}{12}$

d. $\frac{180}{11}$

e. $\frac{12}{1}$

Work:

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos^2(\theta)} \int_0^{1-r^2} r \sin(\theta)\, dz\, dr\, d\theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos^2(\theta)} (1 - r^2)(r \sin(\theta))\, dr\, d\theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\cos^2(\theta)} (r \sin(\theta) - r^3 \sin(\theta))\, dr\, d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} [r^2 \sin(\theta) - \frac{1}{2} r^4 \sin(\theta)]_{r=0}^{r=\cos^2(\theta)}\, d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos^4(\theta) \sin(\theta) - \frac{1}{2} \cos^8(\theta) \sin(\theta))\, d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \sin(\theta) \cos^4(\theta)\, (1 - \frac{1}{2} \cos^4(\theta))\, d\theta$$

$$-\frac{1}{2}\int_{\theta=0}^{\theta=\frac{\pi}{2}} u^4 (1 - \frac{1}{2} u^4)\, du$$

$$-\frac{1}{2}\int_{\theta=0}^{\theta=\frac{\pi}{2}} (u^4 - \frac{1}{2} u^8)\, du$$

$$-\frac{1}{2}[\frac{1}{5} u^5 - \frac{1}{18} u^9]_{\theta=0}^{\theta=\frac{\pi}{2}}$$

$$-\frac{1}{2}[\frac{1}{5} \cos^5(\theta) - \frac{1}{18} \cos^9(\theta)]_0^{\frac{\pi}{2}}$$

$$\frac{1}{10} - \frac{1}{36}$$

$$\frac{13}{180}$$

47. Evaluate the following iterated integral.

$$\int_0^{\frac{\pi}{15}} \int_0^{\frac{\pi}{15}} \int_0^{\cos(\theta)} \rho^2 \sin(\phi)\cos(\phi)\, d\rho\, d\theta\, d\phi$$

a. $\frac{1}{36}(5 + \cos(\frac{2\pi}{15}))\sin^3(\frac{\pi}{15})$

b. $\frac{1}{6}(5 + \cos(\frac{2\pi}{15}))\sin^3(\frac{\pi}{15})$

c. $\frac{1}{6}(3 + \cos(\frac{2\pi}{15}))\sin^2(\frac{\pi}{15})$

d. $\frac{1}{36}(5 + \cos(\frac{\pi}{15}))\sin^3(\frac{\pi}{15})$

e. $\frac{1}{36}(5 + \cos(\frac{2\pi}{15}))\sin^2(\frac{\pi}{15})$

Work:

$$\int_0^{\frac{\pi}{15}} \int_0^{\frac{\pi}{15}} \int_0^{\cos(\theta)} \rho^2 \sin(\phi)\cos(\phi)\, d\rho\, d\theta\, d\phi$$

$$\frac{1}{3}\int_0^{\frac{\pi}{15}} \int_0^{\frac{\pi}{15}} \cos^3(\theta)\sin(\phi)\cos(\phi)\, d\theta\, d\phi$$

$$\frac{1}{3}\int_0^{\frac{\pi}{15}} \int_{\phi=0}^{\phi=\frac{\pi}{15}} u\cos^3(\theta)\, d\phi\, d\theta$$

$$\frac{1}{6}\int_0^{\frac{\pi}{15}} [u^2]_{\phi=0}^{\phi=\frac{\pi}{15}} \cos^3(\theta)\, d\theta$$

$$\frac{1}{6}\int_0^{\frac{\pi}{15}} [\sin^2(\phi)]_0^{\frac{\pi}{15}} \cos^3(\theta)\, d\theta$$

$$\frac{1}{6}\int_0^{\frac{\pi}{15}} \sin^2(\tfrac{\pi}{15})\cos^3(\theta)\, d\theta$$

$$\frac{\sin^2(\frac{\pi}{15})}{6}\int_0^{\frac{\pi}{15}} (\cos(\theta) - \cos(\theta)\sin^2(\theta))\, d\theta$$

$$\frac{\sin^2(\frac{\pi}{15})}{6}\int_{\theta=0}^{\theta=\frac{\pi}{15}} (1 - u^2)\, du$$

$$\frac{\sin^2(\frac{\pi}{15})}{6}(\sin(\tfrac{\pi}{15}) - \tfrac{1}{3}\sin^3(\tfrac{\pi}{15}))$$

48. Convert the integral below from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integral.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{9} x \, dz \, dy \, dx$$

a. 8π

b. $\frac{1}{4}$

c. $\frac{1}{4}\pi$

d. 2π

e. 0

Work:

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{9} x \, dz \, dy \, dx$$

$$\int_{0}^{2\pi} \int_{0}^{3} \int_{r^2}^{9} r^2 \cos(\theta) \, dz \, dr \, d\theta$$

$$\int_{0}^{2\pi} \int_{\arctan(3)}^{\frac{\pi}{2}} \int_{-3\csc(\phi)}^{3\csc(\phi)} \rho^3 \sin^2(\phi) \cos(\theta) \, d\rho \, d\phi \, d\theta$$

$$\frac{1}{4} \int_{0}^{2\pi} \int_{\arctan(3)}^{\frac{\pi}{2}} [\rho^4]_{-3\csc(\phi)}^{3\csc(\phi)} \sin^2(\phi) \cos(\theta) \, d\phi \, d\theta$$

$$\frac{1}{4} \int_{0}^{2\pi} \int_{\arctan(3)}^{\frac{\pi}{2}} (81\csc^4(\phi)\sin^2(\phi)\cos(\theta) - 81\csc^4(\phi)\sin^2(\phi)\cos(\theta)) \, d\phi \, d\theta$$

$$\frac{1}{4} \int_{0}^{2\pi} \int_{\arctan(3)}^{\frac{\pi}{2}} 0 \, d\phi \, d\theta$$

$$0$$

49. Convert the integral below from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integral.

$$\int_0^9 \int_0^{\sqrt{81-x^2}} \int_0^{\sqrt{81-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

a. $\frac{729}{8}\pi$

b. $\frac{6561}{4}\pi$

c. 6561π

d. $\frac{6561}{2}\pi$

e. $\frac{6561}{8}\pi$

Work:

$$\int_0^9 \int_0^{\sqrt{81-x^2}} \int_0^{\sqrt{81-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

$$\int_0^{\frac{\pi}{2}} \int_0^9 \int_0^{\sqrt{81-r^2}} r\sqrt{r^2+z^2}\, dz\, dr\, d\theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^9 \int_0^{\frac{\pi}{2}} \rho^3 \sin(\phi)\, d\phi\, d\rho\, d\theta$$

$$-\int_0^{\frac{\pi}{2}} \int_0^9 [\cos(\phi)]_0^{\frac{\pi}{2}} \rho^3\, d\rho\, d\theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^9 \rho^3\, d\rho\, d\theta$$

$$\frac{1}{4}\int_0^{\frac{\pi}{2}} [\rho^4]_0^9\, d\theta$$

$$\frac{6561}{4}\int_0^{\frac{\pi}{2}} d\theta$$

$$\frac{6561}{8}\pi$$

50. Use cylindrical coordinates to find the volume of the solid bounded above by $z = 15x$ and below by $z = 15x^2 + 15y^2$.

$$z = 15x$$
$$z = 15x^2 + 15y^2$$

a. $\frac{15}{32}\pi$

b. $\frac{15}{22}\pi$

c. $\frac{3}{10}\pi$

d. $\frac{15}{46}\pi$

e. $\frac{15}{64}\pi$

Work:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{cos(\theta)} \int_{15r^2}^{15r\, cos(\theta)} r\, dz\, dr\, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{cos(\theta)} (15r^2\, cos(\theta) - 15r^3)\, dr\, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (5\, cos^4(\theta) - \frac{15}{4}\, cos^4(\theta))\, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{5}{4}\, cos^4(\theta)\, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{5}{16} (1 + cos(2\theta))^2\, d\theta$$

$$\frac{5}{16} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 2\, cos(2\theta) + \frac{1}{2} + \frac{1}{2}\, cos(4\theta))\, d\theta$$

$$\frac{5}{16} [\frac{3}{2}\theta + sin(2\theta) + \frac{1}{8}\, sin(4\theta)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\frac{15}{32}\pi$$

51. Use spherical coordinates to find the volume of the solid inside $x^2 + y^2 + z^2 = 121$ and outside $z = \sqrt{x^2 + y^2}$, and above the xy-plane.

$$x^2 + y^2 + z^2 = 121$$
$$z = \sqrt{x^2 + y^2}$$

a. $242\sqrt{2}\pi$

b. $11\sqrt{3}\pi$

c. $\frac{1331}{3}\sqrt{2}\pi$

d. $11\sqrt{2}\pi$

e. $\frac{1331}{3}\sqrt{3}\pi$

Work:

$$\int_0^{2\pi}\int_0^{11}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \rho^2 \sin(\phi)\, d\phi\, d\rho\, d\theta$$
$$-\int_0^{2\pi}\int_0^{11} \rho^2 [\cos(\phi)]_{\frac{\pi}{4}}^{\frac{\pi}{2}}\, d\rho\, d\theta$$
$$\int_0^{2\pi}\int_0^{11} \frac{\rho^2}{\sqrt{2}}\, d\rho\, d\theta$$
$$\frac{1}{3\sqrt{2}}\int_0^{2\pi} [\rho^3]_0^{11}\, d\theta$$
$$\frac{1}{3\sqrt{2}}\int_0^{2\pi} 1331\, d\theta$$
$$\frac{1331}{3\sqrt{2}}[\theta]_0^{2\pi}$$
$$\frac{1331}{3}\sqrt{2}\pi$$

52. Use spherical coordinates to find the volume of the solid inside the torus given by $\rho = 11\, sin(\phi)$.

$$\rho = 11\, sin(\phi)$$

a. $\frac{671}{2}\pi^2$

b. $\frac{671}{2}$

c. $\frac{1331}{4}\pi$

d. $\frac{1331}{8}\pi$

e. $\frac{1331}{4}\pi^2$

Work:

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{11\, sin(\phi)} \rho^2\, sin(\phi)\, d\rho\, d\phi\, d\theta$$

$$\frac{1}{3}\int_0^{2\pi}\int_0^{\pi}[\rho^3]_0^{11\, sin(\phi)}\, sin(\phi)\, d\phi\, d\theta$$

$$\frac{1331}{3}\int_0^{2\pi}\int_0^{\pi} sin^4(\phi)\, d\phi\, d\theta$$

$$\frac{1331}{3}\int_0^{2\pi}\int_0^{\pi}\left(\frac{1-cos(2\phi)}{2}\right)^2 d\phi\, d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\int_0^{\pi}(1 - cos(2\phi))^2\, d\phi\, d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\int_0^{\pi}(1 - 2\, cos(2\phi) + cos^2(2\phi))\, d\phi\, d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\int_0^{\pi}\left(1 - 2\, cos(2\phi) + \frac{1-cos(4\phi)}{2}\right) d\phi\, d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\int_0^{\pi}\left(1 - 2\, cos(2\phi) + \frac{1}{2} - \frac{1}{2}\, cos(4\phi)\right) d\phi\, d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\int_0^{\pi}\left(\frac{3}{2} - 2\, cos(2\phi) - \frac{1}{2}\, cos(4\phi)\right) d\phi\, d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\left[\frac{3}{2}\phi - sin(2\phi) - \frac{1}{8}\, sin(4\phi)\right]_0^{\pi} d\theta$$

$$\frac{1331}{12}\int_0^{2\pi}\frac{3\pi}{2}\, d\theta$$

$$\frac{1331}{8}\pi[\theta]_0^{2\pi}$$

$$\frac{1331}{4}\pi^2$$

53. Find the Jacobian $\frac{\vartheta(x,y)}{\vartheta(u,v)}$ for the following change of variables:

$$x = 3u\,cos(\theta) - v\,sin(\theta)$$
$$y = 3u\,sin(\theta) + v\,cos(\theta)$$

a. 3

b. -6

c. 8

d. -3

e. 6

Work:

$$\det\begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

$$\det\begin{bmatrix} \frac{\partial}{\partial u}(3u\,cos(\theta) - v\,sin(\theta)) & \frac{\partial}{\partial v}(3u\,cos(\theta) - v\,sin(\theta)) \\ \frac{\partial}{\partial u}(3u\,sin(\theta) + v\,cos(\theta)) & \frac{\partial}{\partial v}(3u\,sin(\theta) + v\,cos(\theta)) \end{bmatrix}$$

$$\det\begin{bmatrix} 3\,cos(\theta) & -sin(\theta) \\ 3\,sin(\theta) & cos(\theta) \end{bmatrix}$$

$$3\,cos^2(\theta) + 3\,sin^2(\theta)$$

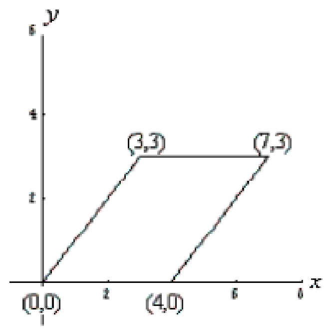
$$3(cos^2(\theta) + sin^2(\theta))$$

$$3$$

54. Use the indicated change of variables to evaluate the following double integral.

$$x = u + v$$
$$y = u$$

$$\iint_R y(x - y)\, dA$$



a. 18

b. 36

c. − 36

d. 32

e. 56

Work:

$$\int_0^3 \int_y^{y+4} y(x - y)\, dx\, dy$$

$$\int_0^3 \int_0^4 uv \left| \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} \right| dv\, du$$

$$\int_0^3 \int_0^4 uv \left| \det \begin{bmatrix} \frac{\partial}{\partial u}(u+v) & \frac{\partial}{\partial v}(u+v) \\ \frac{\partial}{\partial u} u & \frac{\partial}{\partial v} u \end{bmatrix} \right| dv\, du$$

$$\int_0^3 \int_0^4 uv \left| \det \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right| dv\, du$$

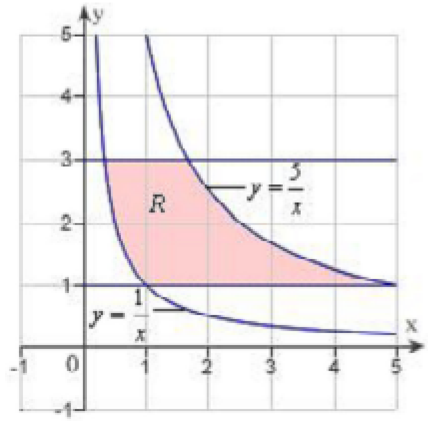
$$\int_0^3 \int_0^4 uv|-1|\, dv\, du$$

$$\int_0^3 \int_0^4 uv\, dv\, du$$

$$8 \int_0^3 u\, du$$

$$36$$

55. Use the following change of variables to evaluate the double integral $\int\int_R y\, sin(xy)\, dA$. Round your answer to four decimal places.



$$x = \frac{u}{v}$$
$$y = v$$

a. 3.6008

b. 1.2688

c. 2.0133

d. 0.5133

e. 2.2633

Work:

$$\int_1^3 \int_{\frac{1}{y}}^{\frac{5}{y}} y\, sin(xy)\, dx\, dy$$

$$\int_1^5 \int_1^3 v\, sin(u) \left| \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} \right| dv\, du$$

$$\int_1^5 \int_1^3 v\, sin(u) \left| \det \begin{bmatrix} \frac{\partial}{\partial u}\left(\frac{u}{v}\right) & \frac{\partial}{\partial v}\left(\frac{u}{v}\right) \\ \frac{\partial}{\partial u} v & \frac{\partial}{\partial v} v \end{bmatrix} \right| dv\, du$$

$$\int_1^5 \int_1^3 v\, sin(u) \left| \det \begin{bmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 0 & 1 \end{bmatrix} \right| dv\, du$$

$$\int_1^5 \int_1^3 sin(u)\, dv\, du$$

$$2 \int_1^5 sin(u)\, du$$

$$-\ 2[cos(u)]_1^5$$

$$-\ 2(cos(5) - cos(1))$$