Name: ________________________ **Class:** ___________________ **Date:** __________ **ID: A**

Chapter 13 Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Find and simplify the function $f\left(x,y\right)=5x\sin y$ at the given value $\left(4,\pi/2\right)$.

a. –8
b. 8
c. 0
d. 20
e. –20

____ 2. Use polar coordinates and L'Hopital's Rule to find the limit.

$$\lim_{(x,y)\to(0,0)}\frac{2\sin\left(x^2+y^2\right)}{x^2+y^2}$$

a. 0

b. 2

c. $\dfrac{\sqrt{2}}{2}$

d. $\dfrac{1}{2}$

e. $\dfrac{\sqrt{3}}{2}$

____ 3. Find the partial derivative f_y for the function $f\left(x,y\right)=x^2-2y^2+7$.

a. $f_y\left(x,y\right)=-8y$
b. $f_y\left(x,y\right)=-5y$
c. $f_y\left(x,y\right)=9y$
d. $f_y\left(x,y\right)=-4y$
e. $f_y\left(x,y\right)=10y$

____ 4. Find the partial derivative $\frac{\partial z}{\partial x}$ for the function $z = 10y^2\sqrt{x}$.

a. $\frac{\partial z}{\partial x} = 5y^2\sqrt{x}$

b. $\frac{\partial z}{\partial x} = \frac{10}{\sqrt{x}}$

c. $\frac{\partial z}{\partial x} = 5y^2$

d. $\frac{\partial z}{\partial x} = \frac{5}{x}$

e. $\frac{\partial z}{\partial x} = \frac{5y^2}{\sqrt{x}}$

____ 5. Find the partial derivative $\frac{\partial z}{\partial y}$ for the function $z = 10y^2\sqrt{x}$.

a. $\frac{\partial z}{\partial y} = 75y\sqrt{x}$

b. $\frac{\partial z}{\partial y} = 51y\sqrt{x}$

c. $\frac{\partial z}{\partial y} = 10y\sqrt{x}$

d. $\frac{\partial z}{\partial y} = 76y\sqrt{x}$

e. $\frac{\partial z}{\partial y} = 20y\sqrt{x}$

____ 6. Find the partial derivative $\frac{\partial z}{\partial x}$ for the function $z = \cos\left(x^8 + y^8\right)$.

a. $\frac{\partial z}{\partial x} = -8x^7 \sin\left(x^8 + y^8\right)$

b. $\frac{\partial z}{\partial x} = 8x^9 \sin\left(x^9 + y^9\right)$

c. $\frac{\partial z}{\partial x} = -8x^9 \sin\left(x^8 + y^8\right)$

d. $\frac{\partial z}{\partial x} = 8x^9 \cos\left(x^9 + y^9\right)$

e. $\frac{\partial z}{\partial x} = 8x^7 \cos\left(x^8 + y^8\right)$

____ 7. Find the partial derivative $\frac{\partial z}{\partial y}$ for the function $z = \cos\left(x^5 + y^5\right)$.

a. $\frac{\partial z}{\partial y} = -5y^4 \sin\left(x^5 + y^5\right)$

b. $\frac{\partial z}{\partial y} = -5y^6 \sin\left(x^5 + y^5\right)$

c. $\frac{\partial z}{\partial y} = 5y^6 \sin\left(x^6 + y^6\right)$

d. $\frac{\partial z}{\partial y} = 5y^6 \cos\left(x^6 + y^6\right)$

e. $\frac{\partial z}{\partial y} = 5y^4 \cos\left(x^5 + y^5\right)$

____ 8. For $f(x,y) = e^y \sin 3x$, evaluate f_x at the point $(\pi, 0)$.

a. $f_x(\pi,0) = 6$
b. $f_x(\pi,0) = -3$
c. $f_x(\pi,0) = 3$
d. $f_x(\pi,0) = -11$
e. $f_x(\pi,0) = -6$

____ 9. Find the second partial derivative for the function $z = x^8 - 8xy + 6y^2$ with respect to x.

a. $\dfrac{\partial^2 z}{\partial x^2} = 56x^6$

b. $\dfrac{\partial^2 z}{\partial x^2} = 9x^{10}$

c. $\dfrac{\partial^2 z}{\partial x^2} = 8x^7 - 8y$

d. $\dfrac{\partial^2 z}{\partial x^2} = 72x^{10}$

e. $\dfrac{\partial^2 z}{\partial x^2} = 8x^{10}$

____ 10. For $f(x,y) = x^2 - xy + y^2 - 5x + y$ find all values of x and y such that $f_x(x,y) = 0$ and $f_y(x,y) = 0$ simultaneously.

a. $x = 4;\ y = 2$
b. $x = 3;\ y = 1$
c. $x = 5;\ y = 1$
d. $x = 7;\ y = 3$
e. $x = 2;\ y = 0$

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____ 11. Find $\frac{dw}{dt}$ using the appropriate Chain Rule for $w = x^2 + y^2$ where $x = 7t$ and $y = 3t$.

a. $116t$
b. $58t$
c. $104t$
d. $32t$
e. $174t$

____ 12. Let $w = \cos(4x - 4y)$, where $x = t^9$ and $y = 3$. Find $\frac{dw}{dt}$.

a. $\frac{dw}{dt} = 12\sin\left(4t^9 - 12\right)t^8$

b. $\frac{dw}{dt} = -36\sin\left(4t^9 - 12\right)t^8$

c. $\frac{dw}{dt} = 9\sin\left(4t^9 - 12\right)t^8$

d. $\frac{dw}{dt} = 36\sin\left(4t^9 - 12\right)t^8$

e. $\frac{dw}{dt} = -12\sin\left(4t^9 - 12\right)t^8$

____ 13. Let $w = xy\cos z$, where $x = t^3$, $y = t^5$, and $z = \arccos t$. Find $\frac{dw}{dt}$.

a. $\frac{dw}{dt} = 8t^8 - \sqrt{1 - t^2}$

b. $\frac{dw}{dt} = 8t^8 + \sqrt{1 - t^2}$

c. $\frac{dw}{dt} = t^8\left(8 + \sqrt{1 - t^2}\right)$

d. $\frac{dw}{dt} = 9t^8$

e. $\frac{dw}{dt} = t^8\left(8 - \sqrt{1 - t^2}\right)$

____ 14. The parametric equations for the paths of two projectiles are given below. At what rate is the distance between the two objects changing at $t = \frac{\pi}{2}$? Round your answer to two decimal places.

$x_1 = 12\cos 2t,\ y_1 = 6\sin 2t$

$x_2 = 6\cos t,\ y_2 = 7\sin t$

a. 0.86
b. 1.54
c. 0.58
d. –1.73
e. –0.65

____ 15. Find $\frac{\partial w}{\partial s}$ using the appropriate Chain Rule for $w = y^3 - 8x^2 y$ where $x = e^s$ and $y = e^t$, and evaluate the partial derivative at $s = -2$ and $t = 3$. Round your answer to two decimal places.

a. –8.83
b. –5.89
c. –6.62
d. –2.72
e. –8.09

____ 16. Find $\frac{\partial w}{\partial t}$ using the appropriate Chain Rule for $w = y^3 - 8x^2 y$ where $x = e^s$ and $y = e^t$, and evaluate the partial derivative at $s = -3$ and $t = 2$. Round your answer to two decimal places.

a. 1,209.99
b. 806.71
c. 1,210.14
d. 806.69
e. 1,209.20

____ 17. Find $\frac{\partial w}{\partial s}$ using the appropriate Chain Rule for $w = x^2 + y^2 + z^2$ where $x = 4t\sin s$, $y = 4t\cos s$, and $z = 8st^2$.

a. $16s^4 t$
b. $128s^4 t$
c. $16st^4$
d. $64st^4$
e. $128st^4$

____ 18. Find $\frac{\partial w}{\partial t}$ using the appropriate Chain Rule for $w = x^2 + y^2 + z^2$ where $x = 4t\sin s$, $y = 4t\cos s$, and $z = 8st^2$.

a. $32t + 64s^3t^2$
b. $32t^2 + 128s^2t^3$
c. $32t^2 + 128s^3t^2$
d. $32t + 256s^2t^3$
e. $128t + 64s^2t^3$

____ 19. Differentiate implicitly to find $\frac{dy}{dx}$.

$x^2 - 4xy + y^2 - 10x + y - 9 = 0$

a. $\frac{dy}{dx} = -\frac{2x - 4y - 10}{2y - 4x + 1}$

b. $\frac{dy}{dx} = \frac{2x + 4y + 10}{2y + 4x + 1}$

c. $\frac{dy}{dx} = \frac{2x - 4y - 10}{2y - 4x + 1}$

d. $\frac{dy}{dx} = \frac{2x + 4y - 10}{2y + 4x - 1}$

e. $\frac{dy}{dx} = -\frac{2x + 4y + 10}{2y + 4x + 1}$

____ 20. Differentiate implicitly to find $\frac{\partial z}{\partial y}$, given $3x + \sin(10y + z) = 0$.

a. 3

b. 10

c. $-\frac{3}{\cos(y+z)}$

d. $-\frac{10}{\cos(y+z)}$

e. -10

____ 21. Differentiate implicitly to find $\frac{\partial w}{\partial x}$, given $x^2 + y^2 + z^2 - 7yw + 2w^2 = 4$.

a. $\frac{x}{4w - 7y}$

b. $\frac{2x}{7w + 2y}$

c. $\frac{2x}{7y - 4w}$

d. $\frac{x}{7y - 2w}$

e. $\frac{2x}{7w + 4y}$

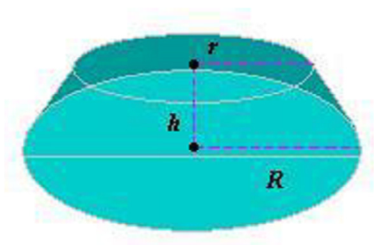
____ 22. The radius of a right circular cylinder is increasing at a rate of 8 inches per minute, and the height is decreasing at a rate of 3 inches per minute. What is the rate of change of the volume when the radius is 12 inches and the height is 32 inches?

a. 5640π in.^{3} / min
b. 6480π in.^{3} / min
c. 5712π in.^{3} / min
d. 6096π in.^{3} / min
e. 6188π in.^{3} / min

____ 23. \The radius of a right circular cylinder is increasing at a rate of 9 inches per minute, and the height is decreasing at a rate of 7 inches per minute. What is the rate of change of the surface area when the radius is 12 inches and the height is 32 inches?

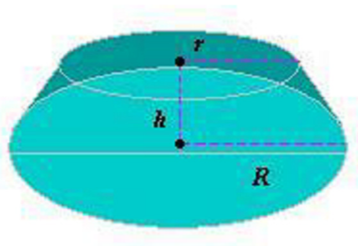
a. 936π in.^{2} / min
b. 984π in.^{2} / min
c. 876π in.^{2} / min
d. 840π in.^{2} / min
e. 968π in.^{2} / min

____ 24. The two radii of the frustum of a right circular cone are increasing at a rate of 4 centimeters per minute, and the height is increasing at a rate of 12 centimeters per minute (see figure). Find the rate at which the volume is changing when the two radii are 15 centimeters and 30 centimeters, and the height is 10 centimeters.



a. $7{,}275\pi$ cm^3 / min
b. $8{,}100\pi$ cm^3 / min
c. $8{,}250\pi$ cm^3 / min
d. $4{,}800\pi$ cm^3 / min
e. $7{,}650\pi$ cm^3 / min

____ 25. The two radii of the frustum of a right circular cone are increasing at a rate of 7 centimeters per minute, and the height is increasing at a rate of 11 centimeters per minute (see figure). Find the rate at which the surface area is changing when the two radii are 16 centimeters and 26 centimeters, and the height is 13 centimeters. [*Note:* The surface area does not include the top and bottom circles.] Round your answer to two decimal places.



a. $1871.79\ \text{cm}^2/\text{min}$
b. $2077.89\ \text{cm}^2/\text{min}$
c. $1511.11\ \text{cm}^2/\text{min}$
d. $2185.54\ \text{cm}^2/\text{min}$
e. $1445.22\ \text{cm}^2/\text{min}$

____ 26. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x,y) = 3x - 2xy + 7y,\ P(1,9),\ \vec{\mathbf{v}} = \frac{1}{2}\left(\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}}\right)$$

a. $\dfrac{-15+5\sqrt{3}}{4}$

b. $\dfrac{-15+5\sqrt{3}}{2}$

c. $\dfrac{21+5\sqrt{3}}{2}$

d. $\dfrac{21+5\sqrt{3}}{4}$

e. $\dfrac{-15+9\sqrt{3}}{2}$

____ 27. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$f(x,y,z) = xy + yz + xz,\ P(1,1,1),\ \vec{\mathbf{v}} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

a. $4/5\sqrt{2}$
b. $-8/5\sqrt{2}$
c. $12/5\sqrt{2}$
d. $24/5\sqrt{2}$
e. $-24/5\sqrt{2}$

____ 28. Find the gradient of the function at the given point.

$w = 4x^2y - 3yz + z^2,\ (1,1,-2)$

a. $8\hat{\mathbf{i}} + 10\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

b. $8\hat{\mathbf{i}} + 10\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

c. $4\hat{\mathbf{i}} + 10\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

d. $4\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

e. $12\hat{\mathbf{i}} + 13\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

____ 29. Use the gradient to find the directional derivative of the function at P in the direction of Q.

$g(x,y) = x^2 + y^2 + 1,\quad P(4,8),\quad Q(12,24)$

a. $-4\sqrt{5}$
b. $8\sqrt{5}$
c. $16\sqrt{5}$
d. $4\sqrt{5}$
e. $-8\sqrt{5}$

____ 30. Use the gradient to find the directional derivative of the function at P in the direction of Q.

$$f(x,y) = \sin(7x)\cos y,\quad P(0,0),\quad Q\left(\frac{\pi}{7},\pi\right)$$

a. π

b. $\pi\left(\sin(7x)\cos y + \cos(7x)\sin y\right)$

c. $\pi\left(\cos(7x)\cos y - \sin(7x)\sin y\right)$

d. $\dfrac{1}{\sqrt{50}}$

e. $\dfrac{7}{\sqrt{50}}$

____ 31. Find $\nabla f(x,y)$ for function $f(x,y) = 8 - \frac{x}{8} - \frac{y}{2}$.

a. $\nabla f(x,y) = \frac{1}{8}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}$

b. $\nabla f(x,y) = \frac{1}{8}\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}}$

c. $\nabla f(x,y) = -8\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

d. $\nabla f(x,y) = 8\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

e. $\nabla f(x,y) = -\frac{1}{8}\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}}$

____ 32. For function $f(x,y) = 8 - \frac{x}{8} - \frac{y}{7}$, find the maximum value of the directional derivative at (3,2).

a. $\sqrt{15}/8$

b. $\sqrt{113}/56$

c. $\sqrt{113}/224$

d. $\sqrt{113}/112$

e. $\sqrt{113}/168$

____ 33. Use the gradient to find a normal vector to the graph of the equation at the given point.

$2x^2 - y = 8, \quad (6,64)$

a. $24\hat{\mathbf{i}} + \hat{\mathbf{j}}$

b. $24\hat{\mathbf{i}} - \hat{\mathbf{j}}$

c. $-24\hat{\mathbf{i}} - 64\hat{\mathbf{j}}$

d. $-24\hat{\mathbf{i}} - \hat{\mathbf{j}}$

e. $24\hat{\mathbf{i}} + 64\hat{\mathbf{j}}$

____ 34. Find the path of a heat-seeking particle placed at point $P(16,16)$ on a metal plate with a temperature field $T(x,y) = 401 - 6x^2 - 3y^2$.

a. $y^2 = 32x$
b. $y^2 = 32x^2$
c. $y = 256x^2$
d. $y^2 = 16x$
e. $y = 16x^2$

____ 35. The temperature at the point (x, y) on a metal plate is modeled by $T(x, y) = 300e^{-(x^2+y)/2}$, $x \geq 0,\ y \geq 0$. Find the directions of no change in heat on the plate from the point $(5, 7)$.

a. There will be no change in directions perpendicular to the gradient $\pm(15\mathbf{i} - \mathbf{j})$.

b. There will be no change in directions parallel to the gradient $\pm(10\mathbf{i} - \mathbf{j})$.

c. There will be no change in directions parallel to the gradient $\pm(\mathbf{i} + 10\mathbf{j})$.

d. There will be no change in directions perpendicular to the gradient $\pm(\mathbf{i} + 15\mathbf{j})$.

e. There will be no change in directions perpendicular to the gradient $\pm(\mathbf{i} - 10\mathbf{j})$.

____ 36. The temperature at the point (x, y) on a metal plate is modeled by $T(x, y) = 300e^{-(x^2+y)/2}$, $x \geq 0,\ y \geq 0$. Find the direction of greatest increase in heat from the point $(4, 18)$.

a. The greatest increase is in the direction of the gradient $-4\mathbf{i} - \frac{1}{4}\mathbf{j}$.

b. The greatest increase is in the direction of the gradient $-6\mathbf{i} - \frac{1}{2}\mathbf{j}$.

c. The greatest increase is in the direction of the gradient $-4\mathbf{i} - \frac{1}{2}\mathbf{j}$.

d. The greatest increase is in the direction of the gradient $-8\mathbf{i} - \frac{1}{2}\mathbf{j}$.

e. The greatest increase is in the direction of the gradient $-6\mathbf{i} - \frac{1}{4}\mathbf{j}$.

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____ 37. For the function given by $F(x,y,z) = x^2 + y^2 + z^2 - 16$ describe the level surface given by $F(x,y,z) = 0$.

a. sphere of radius 4 centered at the origin

b. circle of radius 4 centered at the origin

c. elliptic cone centered at the origin

d. sphere of radius 16 centered at the origin

e. right circular cone centered at the origin

____ 38. Find the unit normal vector to the surface $10x + 10y + 5z = 0$ at the point $(0,0,0)$.

a. $10\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$

b. $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}$

c. $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

d. $\frac{2}{45}\mathbf{i} + \frac{2}{45}\mathbf{j} + \frac{1}{80}\mathbf{k}$

e. $\frac{2}{45}\mathbf{i} + \frac{2}{45}\mathbf{j} + \frac{1}{45}\mathbf{k}$

____ 39. Find a unit normal vector to the surface $x+y+z=20$ at the point $\left(10,0,10\right)$.

a. $\frac{1}{\sqrt{2}}\left\langle 1,1,0\right\rangle$

b. $-\left\langle 1,1,1\right\rangle$

c. $-\frac{1}{\sqrt{2}}\left\langle 1,1,0\right\rangle$

d. $\left\langle 1,1,1\right\rangle$

e. $\frac{1}{\sqrt{3}}\left\langle 1,1,1\right\rangle$

____ 40. Find an equation of the tangent plane to the surface $f\left(x,y\right)=6-\frac{6}{5}x-y$ at the point $\left(5,-1,1\right)$.

a. $\frac{6}{5}(x-5)-\left(y+1\right)-(z-1)=0$

b. $-\frac{6}{5}(x-5)+\left(y+1\right)-(z-1)=0$

c. $-\frac{6}{5}(x-5)-\left(y+1\right)-(z-1)=0$

d. $-\frac{6}{5}(x+5)-\left(y-1\right)-(z+1)=0$

e. $\frac{6}{5}(x+5)-\left(y-1\right)-(z+1)=0$

____ 41. Find symmetric equations of the normal line to the surface $3xy - z = 0$ at the point $\left(-2,-4,24\right)$.

a. $\dfrac{x+2}{12} = \dfrac{y+4}{6} = \dfrac{z-24}{1}$

b. $-\dfrac{x+2}{12} = -\dfrac{y+4}{6} = z-24$

c. $-\dfrac{x+2}{6} = -\dfrac{y+4}{12} = z-24$

d. $3(x-2) = 3\left(y-4\right) = -(z-24)$

e. $3(x-2) = 3\left(y-4\right) = (z-24)$

____ 42. Find symmetric equations of the tangent line to the curve of intersection of the surfaces $z = x^2 + y^2$, $z = 13 - y$ at the point $\left(1,-3,10\right)$.

a. $\dfrac{x-1}{-7} = \dfrac{y+3}{2} = \dfrac{z-10}{-2}$

b. $\dfrac{x-1}{5} = \dfrac{y+3}{2} = \dfrac{z-10}{-2}$

c. $\dfrac{x+5}{1} = \dfrac{y+2}{3} = \dfrac{z-2}{10}$

d. $\dfrac{x+5}{1} = \dfrac{y-2}{-3} = \dfrac{z-2}{10}$

e. $\dfrac{x-1}{5} = \dfrac{y-3}{2} = \dfrac{z-10}{-2}$

____ 43. Identify the point(s) on the surface $z = 5x^2 + 5y^2 - 4x + 30y - 7$ where the tangent plane is horizontal.

a. $\left(0, 0, -7\right)$

b. $\left(-4, 30, -1\right)$

c. $\left(\frac{2}{5}, 3, \frac{636}{5}\right)$

d. $\left(\frac{2}{5}, 3, -\frac{264}{5}\right)$

e. $\left(\frac{2}{5}, -3, -\frac{264}{5}\right)$

____ 44. Find the point(s) on the hyperboloid $x^2 + 4y^2 + z^2 = 84$ where the tangent plane is perpendicular to the line with parametric equations $x = 5 - 8t$, $y = 1 + 32t$, and $z = 2 - 4t$.

a. $\left(-4, -4, -2\right), \left(4, 4, 2\right)$

b. $\left(-4, 4, 2\right)$

c. $\left(4, -2, -2\right)$

d. $\left(4, -4, -2\right), \left(4, 4, 2\right)$

e. $\left(-4, 4, -2\right), \left(4, -4, 2\right)$

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____ 45. Examine the function $f(x,y) = \left(-2x^2 - 2y^2 + 2x - 4y + 1\right)$ for relative extrema.

a. relative minimum: $\left(\frac{1}{2}, -1, \frac{5}{2}\right)$

b. relative maximum: $\left(\frac{1}{2}, -1, \frac{3}{2}\right)$

c. relative maximum: $(0,0,1)$

d. relative maximum: $\left(\frac{1}{2}, -1, \frac{7}{2}\right)$

e. relative minimum: $(0,0,1)$

____ 46. Examine the function $f(x,y) = 8\left(x^2 + y^2\right)^{1/3} + 5$ for relative extrema.

a. relative maximum: $(8,8,5)$

b. relative minimum: $(8,8,5)$

c. relative maximum: $(0,0,5)$

d. relative minimum: $(0,0,5)$

e. no relative extrema

____ 47. Examine the function $f(x,y) = 5x^2 - 6y^2 - 30x - 36y + 9$ for relative extrema and saddle points.

a. saddle point: $(3, 3, -198)$

b. relative minimum: $(3, -3, 18)$

c. relative minimum: $(-3, -3, 198)$

d. saddle point: $(3, -3, 18)$

e. saddle point: $(-3, -3, 198)$

____ 48. Examine the function $f(x,y) = x^3 - 18xy + y^3 + 7$ for relative extrema and saddle points.

a. saddle point: $(0,0,7)$; relative minimum: $(6,6,-209)$
b. saddle point: $(6,6,-209)$; relative minimum: $(0,0,7)$
c. relative minimum: $(0,0,7)$; relative maximum: $(6,6,-209)$
d. relative minimum: $(6,6,-209)$; relative maximum: $(0,0,7)$
e. saddle points: $(0,0,7)$, $(6,6,-209)$

____ 49. Determine whether there is a relative maximum, a relative minimum, a saddle point, or insufficient information to determine the nature of the function $f(x,y)$ at the critical point (x_0, y_0), if $f_{xx}(x_0,y_0) = 16, f_{yy}(x_0,y_0) = -2, f_{xy}(x_0,y_0) = -1$.

a. relative maximum
b. relative minimum
c. saddle point
d. insufficient information

____ 50. Find the absolute extrema of $f(x,y) = 4x^2 + 2y^2 + 24x - 8$ on the region $R = \{(x,y): x^2 + y^2 \le 49\}$.

a. absolute minimum: $(-3,0,-44)$
absolute maximum: $(-6,-\sqrt{13},306)$

b. absolute minimum: $(-3,0,-44)$
absolute maximum: $(0,0,-8)$

c. absolute minimum: $(0,0,-8)$
absolute maximum: $(3,0,100)$

d. absolute minimum: $(-3,0,-44)$
absolute maximum: $(7,0,356)$

e. absolute minimum: $(-3,0,-44)$
absolute maximum: $(-6,\sqrt{13},18)$

____ 51. Find the minimum distance from the point $(-5,-5,0)$ to the surface $z = \sqrt{7-8x-8y}$.

a. $\sqrt{429}$

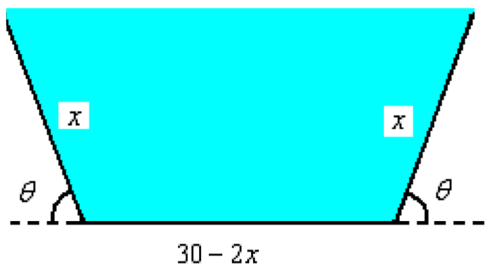
b. $\sqrt{415}$

c. $\sqrt{55}$

d. $\sqrt{607}$

e. $3\sqrt{7}$

____ 52. Suppose a trough with trapezoidal cross sections is formed by turning up the edges of a 30-inch-wide sheet of aluminum (see figure). Find the cross section of maximum area.



a. $x = 12$; $\theta = 45°$
b. $x = 20$; $\theta = 55°$
c. $x = 10$; $\theta = 45°$
d. $x = 10$; $\theta = 60°$
e. $x = 20$; $\theta = 60°$

____ 53. Find three positive numbers x, y, and z whose sum is 48 and $P = xy^2z$ is a maximum.

a. $x = 10, y = 7, z = 31$
b. $x = 7, y = 31, z = 10$
c. $x = 31, y = 10, z = 7$
d. $x = 12, y = 24, z = 12$
e. $x = 12, y = 12, z = 24$

____ 54. Use Lagrange multipliers to minimize the function $f(x,y) = x^2 - y^2$ subject to the following constraint:

$x - 6y + 45 = 0$

Assume that x and y are positive.

a. $-\frac{405}{7}$

b. 0

c. $-\frac{7}{405}$

d. $\frac{405}{7}$

e. no absolute minimum

____ 55. Use Lagrange multipliers to maximize the function $f(x,y) = \sqrt{77 - x^2 - y^2}$ subject to the following constraint.

$x + y - 12 = 0$

Assume that x and y are positive.

a. $\sqrt{5}$

b. 5

c. 149

d. $\sqrt{149}$

e. no absolute maximum

____ 56. Use Lagrange multipliers to minimize the function $f(x,y,z) = x^2 + y^2 + z^2$ subject to the following two constraints.

$x + 8z = 6$

$x + y = 12$

Assume that x, y, and z are nonnegative.

a. 108
b. 72
c. 324
d. 216
e. 36

____ 57. Use Lagrange multipliers to find the minimum distance from the line $6x + 7y = -1$ to the point $(0,0)$.

a. $\frac{1}{85}$

b. $\frac{\sqrt{13}}{13}$

c. $\frac{\sqrt{13}}{85}$

d. $\frac{1}{13}$

e. $\frac{\sqrt{85}}{85}$

Name: ________________________ ID: A

____ 58. Find the highest point on the curve of intersection of the following surfaces.

Cone: $x^2 + y^2 - z^2 = 0$, Plane: $x + 16z = 4$

a. $\frac{4}{15}$

b. 64

c. $\frac{15}{4}$

d. 68

e. 60

____ 59. Let $T(x,y,z) = 200 + x^2 + y^2$ represent the temperature at each point on the sphere $x^2 + y^2 + z^2 = 40$. Find the maximum temperature on the curve formed by the intersection of the sphere and the plane $x - z = 0$.

a. 440
b. 100
c. 220
d. 240
e. 120

____ 60. Find the minimum cost of producing 55,000 units of a product $P = 100x^{0.16}y^{0.84}$, where x is the number of units of labor (at $76 per unit) and y is the number of units of capital (at $56 per unit). Round your answer to the nearest cent.

a. $68,837.29
b. $50,201.54
c. $42,169.29
d. $57,823.32
e. $54,500.66

Chapter 13 Practice Test
Answer Section

MULTIPLE CHOICE

1. D
2. B
3. D
4. E
5. E
6. A
7. A
8. B
9. A
10. B
11. A
12. B
13. D
14. A
15. B
16. C
17. E
18. D
19. A
20. E
21. C
22. C
23. D
24. B
25. A
26. B
27. C
28. B
29. B
30. E
31. E
32. B
33. B
34. D
35. E
36. C
37. A
38. C
39. E

40. C
41. A
42. B
43. E
44. E
45. D
46. D
47. D
48. A
49. C
50. D
51. C
52. D
53. D
54. A
55. A
56. B
57. E
58. A
59. D
60. B