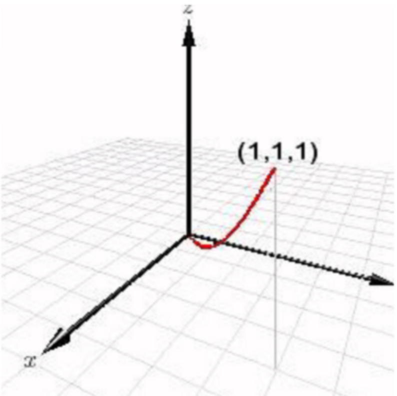
Name: ________________________ Class: ___________________ Date: __________ ID: A

Chapter 12 Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Match the equation with the graph shown in red below.



$x = t$
$y = t$
$z = t^2$
$0 \le t \le 1$

or

$x = t$
$y = t$
$z = t^3$
$0 \le t \le 1$
this could have been an option

a. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k} \quad 0 \le t \le 1$

b. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$

c. $\mathbf{r}(t) = t\mathbf{i} + t\,\mathbf{j} + t^2\mathbf{k}, \quad 0 \le t \le 1$

d. $\mathbf{r}(t) = \ \mathbf{i} + t\,\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 4$

e. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$

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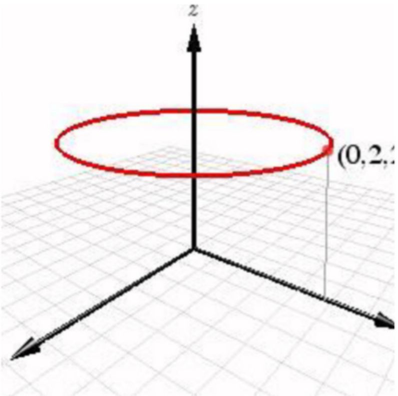
____ 2. The graph below is most likely the graph of which of the following equations?



$x = 2\cos t$
$y = 2\sin t$ } or
$z = 2$

a. $\mathbf{r}(t) = 2t\cos(t)\mathbf{i} + 2t\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

b. $\mathbf{r}(t) = 2\sin(t)\mathbf{i} + 2\cos t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

c. $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2t\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

d. $\mathbf{r}(t) = 2t\cos(t)\mathbf{i} + 2\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

e. $\mathbf{r}(t) = 2\sin(t)\mathbf{i} + 2\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

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____ 3. Sketch the curve represented by the vector-valued function $\mathbf{r}(t) = (4-t)\mathbf{i} + \sqrt{t}\,\mathbf{j}$ and give the orientation of the curve.

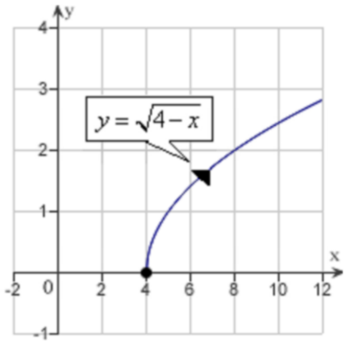
$x = 4 - t$

$y = \sqrt{t}$

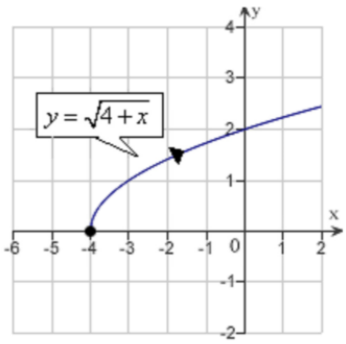
$y = \sqrt{4-x}$

$x \le 4$

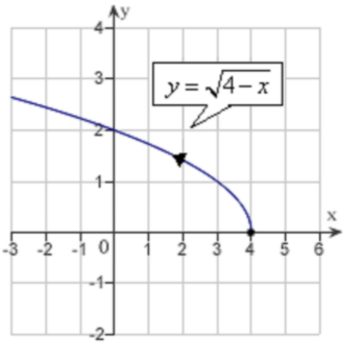
a.



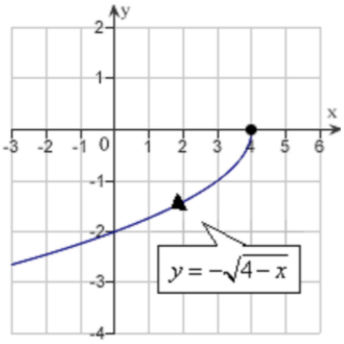
b.



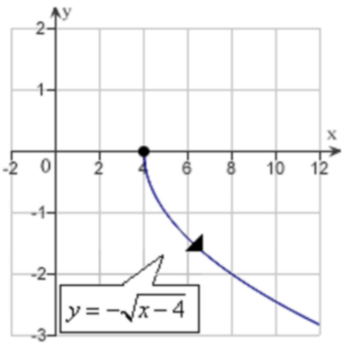
c.



d.



e.



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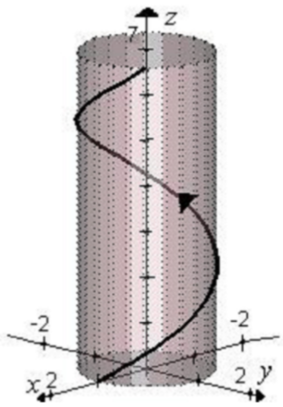
____ 4. Sketch the curve represented by the vector-valued function $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\mathbf{k}$ and give the orientation of the curve.

$x = \cos t$

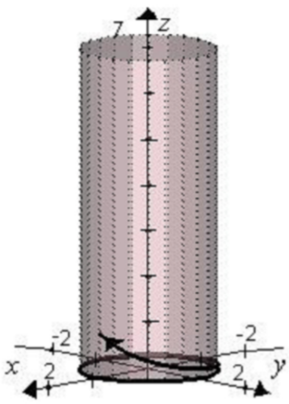
$y = \sin t$

$z = t$

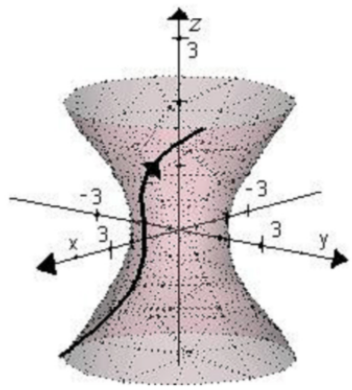
a.



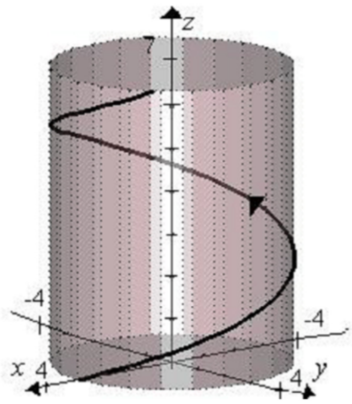
c.



b.



d.



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____ 5. Represent the following curve by a vector-valued function.

$$\frac{x^2}{121}+\frac{y^2}{64}=1,\ x\geq 0$$

a. $\mathbf{r}(t)=\frac{11}{8}\sqrt{64-t^2}\,\mathbf{i}+t\mathbf{j};\ 0\leq t\leq 16$

b. $\mathbf{r}(t)=\frac{11}{8}\sqrt{64-t^2}\,\mathbf{i}+t\mathbf{j};\ 0\leq t\leq 8$

c. $\mathbf{r}(t)=11\cos 2\pi t\,\mathbf{i}+8\sin 2\pi t\,\mathbf{j};\ \frac{-1}{4}\leq t\leq\frac{1}{4}$

d. $\mathbf{r}(t)=11\cos t\,\mathbf{i}+8\sin t\,\mathbf{j};\ -\pi\leq t\leq\pi$

e. $\mathbf{r}(t)=11\cos \pi t\,\mathbf{i}-8\sin \pi t\,\mathbf{j};\ \frac{-1}{4}\leq t\leq\frac{1}{4}$

$r(t)=f(t)i+g(t)j$

$\left(\frac{x}{11}\right)^2+\left(\frac{y}{8}\right)^2=1$

let $x=11\cos t$, $y=8\sin t$

$\underbrace{\cos^2 t}_{x^2}+\underbrace{\sin^2 t}_{y^2}=1$

$x=\frac{\cos t}{11}$ $y=\frac{\sin t}{8}$

bad! not on test

____ 6. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces

$z=\frac{x^2}{25}+\frac{y^2}{4}$, $y-8x=0$

Parameter

$x=t$

a. $\mathbf{r}(t)=t\mathbf{i}+\frac{401}{25}t^2\mathbf{j}+8t\mathbf{k}$

b. $\mathbf{r}(t)=t\mathbf{i}+8t\mathbf{j}+\frac{401}{25}t^2\mathbf{k}$

c. $\mathbf{r}(t)=t\mathbf{i}-8t\mathbf{j}+\frac{401}{25}t^2\mathbf{k}$

d. $\mathbf{r}(t)=t\mathbf{i}+8t\mathbf{j}+\frac{25}{401}t^2\mathbf{k}$

e. $\mathbf{r}(t)=t\mathbf{i}+\frac{25}{401}t^2\mathbf{j}+8t\mathbf{k}$

$y-8t=0$

$y=8t$

$z=\frac{t^2}{25}+\frac{64t^2}{4}$

$z=\frac{t^2}{25}+\frac{16t^2}{1}\cdot\frac{25}{25}$

$z=\frac{401t^2}{25}$

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____ 7. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces: $x^2+y^2=25,\ z=x^2$ Parameter: $x=5\sin 2\pi t$

so $y = 5\cos 2\pi t$

$z = 25\sin^2 2\pi t$

a. $\mathbf{r}(t) = 25\sin 2\pi t\,\mathbf{i} + 25\cos 2\pi t\,\mathbf{j} - 5\sin^2 2\pi t\,\mathbf{k}$

b. $\mathbf{r}(t) = 5\sin 2\pi t\,\mathbf{j} + 5\cos 2\pi t\,\mathbf{i} + 25\sin^2 2\pi t\,\mathbf{k}$

c. $\mathbf{r}(t) = 5\sin 2\pi t\,\mathbf{i} + 5\cos 2\pi t\,\mathbf{j} - 25\sin^2 2\pi t\,\mathbf{k}$

d. $\mathbf{r}(t) = 5\sin 2\pi t\,\mathbf{i} + 5\cos 2\pi t\,\mathbf{j} + 25\sin^2 2\pi t\,\mathbf{k}$

e. $\mathbf{r}(t) = 25\sin 2\pi t\,\mathbf{i} + 25\cos 2\pi t\,\mathbf{j} + 5\sin^2 2\pi t\,\mathbf{k}$

____ 8. Suppose the two particles travel along the space curves $\mathbf{r}(t) = t^2\mathbf{i} + (8t-16)\mathbf{j} + t^2\mathbf{k}$ and use s → $\mathbf{u}(t) = (3t+4)\mathbf{i} + t^2\mathbf{j} + (5t-4)\mathbf{k}$. A collision will occur at the point of intersection P if both particles are at P at the same time. Find the point of collision.

$t^2 = 3s+4$ $8t-16 = s^2$ $t^2 = 5s-4$

$3s+4 = 5s-4$

$2s = 8$

$s = 4 \rightarrow \begin{cases} 3(4)+4 = 16 \\ 4^2 = 16 \\ 5(4)-4 = 16 \end{cases}$

a. $(9,9,9)$

b. $(9,16,9)$

c. $(16,9,16)$

d. $(16,16,16)$

e. $(8,8,8)$

____ 9. Find the vectors $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ for the following vector function.

$\mathbf{r}(t) = (1+2t)\mathbf{i} + \left(2+5t^2\right)\mathbf{j} + 2\mathbf{k}$ $t_0 = 3$

$r'(t) = 2i + 10tj + 0k$

$r'(3) = 2i + 30j$

a. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 2\mathbf{k},\ \mathbf{r}'(3) = 18\mathbf{i} + 30\mathbf{j}$

b. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 2\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 30\mathbf{j}$

c. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 4\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 30\mathbf{j}$

d. $\mathbf{r}(3) = 7\mathbf{i} + 2\mathbf{j} + 47\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 30\mathbf{j}$

e. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 2\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 90\mathbf{j}$

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____ 10. Find $\mathbf{r}'(t)$ given the following vector function.

$\mathbf{r}(t) = 2t^2\mathbf{i} + 4t^4\mathbf{j} + 2t^3\mathbf{k}$

$r'(t) = 4t\,i + 16t^3 j + 6t^2 k$

a. $\mathbf{r}'(t) = 2t\mathbf{i} + 4t^3\mathbf{j} + 2t^2\mathbf{k}$

b. $\mathbf{r}'(t) = 4t^2\mathbf{i} + 16t^4\mathbf{j} + 6t^3\mathbf{k}$

c. $\mathbf{r}'(t) = 2t\mathbf{i} + 4t^2\mathbf{j} + 2t^3\mathbf{k}$

d. $\mathbf{r}'(t) = 4t\mathbf{i} + 4t^3\mathbf{j} + 2t^2\mathbf{k}$

e. $\mathbf{r}'(t) = 4t\mathbf{i} + 16t^3\mathbf{j} + 6t^2\mathbf{k}$

____ 11. Find $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ given the following vector function.

$\mathbf{r}(t) = \left(2t^2 + 2t\right)\mathbf{i} + \left(3t^2 + 4t\right)\mathbf{j}$

$r'(t) = (4t + 2)i + (6t + 4)j \qquad r''(t) = 4i + 6j$

$r'(t) \cdot r''(t) = (16t + 8) + (36t + 24)$

$= 52t + 32$

a. $32 + 52t$
b. $16 + 52t$
c. $32 + 26t$
d. $16 + 26t$
e. $32 + 52t^2$

____ 12. Find $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ given the following vector function.

$\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 5\sin t\,\mathbf{j}$

$r'(t) = -3\sin t\,i + 5\cos t\,j$

$r''(t) = -3\cos t\,i - 5\sin t\,j$

$r'(t) \cdot r''(t) = 9\sin t\cos t - 25\sin t\cos t$

$= -16\sin t\cos t$

a. cost sint
b. $-34\cos t\sin t$
c. $34\cos t\sin t$
d. $-16\sin t\cos t$
e. $3\cos t\sin t$

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____ 13. Use the properties of the derivative to find $D_t\left[\mathbf{r}(t) \times \mathbf{u}(t)\right]$ given the following vector-valued functions.

$\mathbf{r}(t) = 2t\mathbf{i} - 3t^3\mathbf{j} + 3t^2\mathbf{k}$

$\mathbf{u}(t) = 3\mathbf{i} + 5t^2\mathbf{j} + 4t^3\mathbf{k}$

a. $\left(-60t^3 - 72t^5\right)\mathbf{i} + \left(18t^2 - 32t^3\right)\mathbf{j} + 57t^2\mathbf{k}$

b. $\left(-60t^3 - 72t^5\right)\mathbf{i} + \left(32t + 18t^3\right)\mathbf{j} + 57t^2\mathbf{k}$

c. $\left(-60t^3 - 72t^5\right)\mathbf{i} + \left(18t - 32t^3\right)\mathbf{j} - 3t^2\mathbf{k}$

d. $\left(-60t^3 - 72t^5\right)\mathbf{i} + \left(18t - 32t^3\right)\mathbf{j} + 57t^2\mathbf{k}$

e. $\left(-60t^3 - 72t^5\right)\mathbf{i} + \left(18t + 32t^3\right)\mathbf{j} + 57t^3\mathbf{k}$

$r'(t) \times u(t) + r(t) \times u'(t)$

$r'(t) = 2i - 9t^2 j + 6t\,k$

$u'(t) = 0i + 10t\,j + 12t^2 k$

$r'(t) \times u(t)$

$$\begin{vmatrix} i & j & k \\ 2 & -9t^2 & 6t \\ 3 & 5t^2 & 4t^3 \end{vmatrix} = \begin{cases} (-36t^5 - 30t^3)i \\ -(8t^3 - 18t)j \\ (10t^2 + 27t^2)k \end{cases}$$

$r(t) \times u'(t)$

$$\begin{vmatrix} i & j & k \\ 2t & -3t^3 & 3t^2 \\ 0 & 10t & 12t^2 \end{vmatrix} = \begin{cases} (-36t^5 - 30t^3)i \\ -(24t^3 - 0)j \\ (20t^2 - 0)k \end{cases}$$

____ 14. Use the properties of the derivative to find $D_t\left[\mathbf{r}(t) \cdot \mathbf{u}(t)\right]$ given the following vector-valued functions.

$\mathbf{r}(t) = t\mathbf{i} + 2\cos 4t\mathbf{j} + 2\sin 4t\mathbf{k}$

$\mathbf{u}(t) = \frac{5}{t}\mathbf{i} + 2\cos 4t\mathbf{j} + 2\sin 4t\mathbf{k}$

a. 2

b. $\frac{2}{t}$

c. $2t$

d. 5

e. 0

$r'(t) = i - 8\sin 4t\,j + 8\cos 4t\,k$

$u'(t) = -\frac{5}{t^2} i - 8\sin 4t\,j + 8\cos 4t\,k$

$r'(t) \cdot u(t) + r(t) \cdot u'(t)$

$\frac{5}{t} \cancel{- 16\sin 4t\cos 4t} \cancel{+ 16\sin 4t\cos 4t}$

$+ \quad -\frac{5}{t} \cancel{- 16\sin 4t\cos 4t} \cancel{+ 16\sin 4t\cos 4t}$

0

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____ 15. Find the indefinite integral below.

$$\int \left(5e^{5t}\,\mathbf{i} - 4\sin 4t\,\mathbf{j} + 6\cos 3t\,\mathbf{k}\right)dt$$

Do not include an arbitrary constant vector.

a. $e^{5t}\mathbf{i} - \cos 4t\,\mathbf{j} + 2\sin 3t\,\mathbf{k}$

b. $e^{5t}\mathbf{i} + \cos 4t\,\mathbf{j} - 2\sin 3t\,\mathbf{k}$

c. $e^{5t}\mathbf{i} + \cos 4t\,\mathbf{j} + 6\sin 3t\,\mathbf{k}$

d. $e^{5t}\mathbf{i} + \cos 4t\,\mathbf{j} + 2\sin 3t\,\mathbf{k}$

e. $\dfrac{e^{5t}}{5}\mathbf{i} + \cos 4t\,\mathbf{j} + 2\sin 3t\,\mathbf{k}$

____ 16. Find $\mathbf{r}(t)$ given the following.

$\mathbf{r}'(t) = 18t^5\mathbf{j} + 6t\mathbf{k}$, $\mathbf{r}(0) = 2\mathbf{i} + 18\mathbf{j}$

a. $\mathbf{r}(t) = 2\mathbf{i} + \left(18 + 3t^6\right)\mathbf{j} + 3t^2\mathbf{k}$

b. $\mathbf{r}(t) = 2\mathbf{i} + 3t^2\mathbf{j} + \left(18 + 3t^6\right)\mathbf{k}$

c. $\mathbf{r}(t) = \left(18 + 3t^6\right)\mathbf{j} + 3t^2\mathbf{k}$

d. $\mathbf{r}(t) = 2\mathbf{i} + \left(18 - 3t^6\right)\mathbf{j} - 3t^2\mathbf{k}$

e. $\mathbf{r}(t) = 18\mathbf{i} + \left(2 + 3t^6\right)\mathbf{j} + 3t^2\mathbf{k}$

$r(t) = \int (18t^5 j + 6t k)\,dt$

$r(t) = (0 + C_1)i + (3t^6 + C_2)j + (3t^2 + C_3)k$

$r(0) = 2i + 18j + 0k$

$0 + C_1 = 2 \quad C_1 = 2$

$0 + C_2 = 18 \quad C_2 = 18$

$0 + C_3 = 0 \quad C_3 = 0$

$r(t) = 2i + (3t^6 + 18)j + 3t^2 k$

____ 17. A particle moves in the yz-plane along the curve represented by the vector-valued function $\mathbf{r}(t) = (4\cos t)\,\mathbf{j} + (9\sin t)\,\mathbf{k}$. Find the minimum value of $\|\mathbf{r}'\|$.

a. 6
b. 4
c. 7
d. 9
e. 8

use a graphing utility to help

$$\textbf{Speed} = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$$

$\|r'(t)\| = \sqrt{(-4\sin t)^2 + (9\cos t)^2}$

minimum at $\frac{\pi}{2} + \pi k$

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____ 18. A particle moves in the yz-plane along the curve represented by the vector-valued function $\mathbf{r}(t) = (5\cos t)\,\mathbf{j} + (7\sin t)\,\mathbf{k}$. Find the maximum value of $\|\mathbf{r}'\|$.

a. 4
b. 3
c. 7
d. 8
e. 5

$\|r'(t)\| = \sqrt{(-5\sin t)^2 + (7\cos t)^2}$ max at $t = 0 + \pi k$

____ 19. The position vector $r(t) = \langle 7t, 6\cos t, 6\sin t\rangle$ describes the path of an object moving in space. Find the speed $s(t)$ of the object.

a. 21
b. $3\sqrt{7}$
c. $\sqrt{93}$
d. 149
e. $\sqrt{85}$

$s(t) = \sqrt{7^2 + (-6\sin t)^2 + (6\cos t)^2}$

$= \sqrt{49 + 36(\sin^2 t + \cos^2 t)}$

$= \sqrt{49+36} = \sqrt{85}$

____ 20. The position vector $\mathbf{r}(t) = \langle 2\cos t, 4\sin t, t^2\rangle$ describes the path of an object moving in space. Find the velocity $\mathbf{v}(t)$ of the object.

a. $\mathbf{v}(t) = 2\sin t\,\mathbf{i} - 4\cos t\,\mathbf{j} + \mathbf{k}$
b. $\mathbf{v}(t) = -2\cos t\,\mathbf{i} - 4\sin t\,\mathbf{j} + 2\mathbf{k}$
c. $\mathbf{v}(t) = -2\sin t\,\mathbf{i} - 4\cos t\,\mathbf{j} + \mathbf{k}$
d. $\mathbf{v}(t) = -2\sin t\,\mathbf{i} + 4\cos t\,\mathbf{j} + 2t\mathbf{k}$
e. $\mathbf{v}(t) = -2\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + \mathbf{k}$

$v(t) = \langle -2\sin t, 4\cos t, 2t\rangle$

____ 21. Use the given acceleration function and initial conditions to find the position at time t = 3.

$\mathbf{a}(t) = 5\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j}$, $\mathbf{v}(0) = 8\mathbf{j} + 5\mathbf{k}$, $\mathbf{r}(0) = -5\mathbf{i}$

a. $\mathbf{r}(3) = -5\cos 3\mathbf{i} - (3\sin 3 + 15)\mathbf{j} + 15\mathbf{k}$
b. $\mathbf{r}(3) = -5\cos 3\mathbf{i} + (3\sin 3 - 15)\mathbf{j} + 15\mathbf{k}$
c. $\mathbf{r}(3) = -5\cos 3\mathbf{i} + (3\sin 3 + 15)\mathbf{j} + 15\mathbf{k}$
d. $\mathbf{r}(3) = 5\cos 3\mathbf{i} + (3\sin 3 + 15)\mathbf{j} + 15\mathbf{k}$
e. $\mathbf{r}(3) = -5\cos 3\mathbf{i} + 15\mathbf{j} + (3\sin 3 + 15)\mathbf{k}$

method 1

$v(t) = (5\sin t + C_1)i + (3\cos t + C_2)j + (0 + C_3)k$

when $t=0$: 0 $c=0$ 8 $C_2=5$ 5 $C_3=5$

$v(t) = 5\sin t\,i + (3\cos t + 5)j + 5k$

$r(t) = (-5\cos t + C_4)i + (3\sin t + 5t + C_5)j + (5t + C_6)k$

when $t=0$: -5 $C_4=0$ 0 $C_5=0$ 0 $C_6=0$

$r(t) = (-5\cos t)i + (3\sin t + 5t)j + 5tk$

$r(3) = (-5\cos 3)i + (3\sin 3 + 15)j + 15k$

method 2

$v(t) = 5\sin t\,i + 3\cos t\,j + C_1$

$v(0) = 0 + 3j + C = 8j + 5k$

$C_1 = 5j + 5k$

$v(t) = 5\sin t\,i + (3\cos t + 5)j + 5k$

$r(t) = -5\cos t\,i + (3\sin t + 5t)j + 5k + C_2$

$r(0) = -5i + 0j + 0k + C_2 = -5i \rightarrow C_2 = 0$

$r(t) = -5\cos t\,i + (3\sin t + 5t)j + 5k$

$r(3) = -5\cos 3\,i + (3\sin 3 + 15)j + 15k$

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____ 22. Find the unit tangent vector $\mathbf{T}(t)$ for the line tangent to the space curve $\mathbf{r}(t) = \left\langle 12\cos t, 12\sin t, 3 \right\rangle$ at point $P\left(6\sqrt{2}, 6\sqrt{2}, 3\right)$. $t = \frac{\pi}{4}$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}, \quad \mathbf{r}'(t) \neq \mathbf{0}$$

a. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{4}\left\langle -\sqrt{2}, -\sqrt{2}, 3 \right\rangle$

b. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{4}\left\langle -\sqrt{2}, \sqrt{2}, 0 \right\rangle$

c. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{2}\left\langle \sqrt{2}, -\sqrt{2}, 3 \right\rangle$

d. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{2}\left\langle -\sqrt{2}, \sqrt{2}, 0 \right\rangle$

e. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{2}\left\langle \sqrt{2}, -\sqrt{2}, 0 \right\rangle$

$r'(t) = \langle -12\sin t, 12\cos t, 0 \rangle$

$\|r'(t)\| = \sqrt{(-12\sin t)^2 + (12\cos t)^2 + 0^2} = 12$

$T\left(\frac{\pi}{4}\right) = \frac{\langle -6\sqrt{2}, 6\sqrt{2}, 0 \rangle}{12}$

____ 23. Find the principle unit normal vector to the curve given below at the specified point.

$\mathbf{r}(t) = t\mathbf{i} + 4t^2\mathbf{j}, \ t = 3$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

first find $T(t)$

a. $\mathbf{N}(3) = \frac{-24}{\sqrt{145}}\mathbf{i} - \frac{1}{\sqrt{145}}\mathbf{j}$

b. $\mathbf{N}(3) = \frac{-24}{\sqrt{145}}\mathbf{i} + \frac{1}{\sqrt{145}}\mathbf{j}$

c. $\mathbf{N}(3) = \frac{-24}{\sqrt{577}}\mathbf{i} - \frac{1}{\sqrt{577}}\mathbf{j}$

d. $\mathbf{N}(3) = \frac{-24}{\sqrt{577}}\mathbf{i} + \frac{1}{\sqrt{577}}\mathbf{j}$

e. $\mathbf{N}(3) = \frac{24}{\sqrt{577}}\mathbf{i} - \frac{1}{\sqrt{577}}\mathbf{j}$

$r'(t) = i + 8tj$

$\|r'(t)\| = \sqrt{1^2 + (8t)^2}$

$T(t) = \frac{1}{\sqrt{1+(8t)^2}}(i + 8tj)$

$T'(t) = \frac{-64t(i + 8tj)}{(1+64t^2)^{3/2}} + \frac{8j}{(1+64t^2)^{1/2}} \frac{(1+64t^2)}{(1+64t^2)}$

$= \frac{-64ti - \cancel{512t^2j} + 8j + \cancel{512t^2j}}{(1+64t^2)^{3/2}} = \frac{8(-8ti + j)}{(1+64t^2)^{3/2}}$

$\|T'(t)\| = \sqrt{\left(\frac{8((-8t)^2 + 1^2)}{(1+64t^2)^{3/2}}\right)^2} = \frac{8}{1+64t^2}$

$N(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{8(-8ti + j)}{(1+64t^2)^{3/2}} \cdot \frac{1+64t^2}{8} = \frac{-8ti + j}{\sqrt{1+64t^2}}$

$N(3) = \frac{-24}{\sqrt{577}}i + \frac{1}{\sqrt{577}}j$

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____ 24. Find the principle unit normal vector to the curve given below at the specified point.

$\mathbf{r}(t) = 5\cos t\,\mathbf{i} + 5\sin t\,\mathbf{j}, \quad t = \frac{5\pi}{3}$

a. $\mathbf{N} = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

b. $\mathbf{N} = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

c. $\mathbf{N} = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$

d. $\mathbf{N} = \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

e. $\mathbf{N} = \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$

$r'(t) = -5\sin t\, i + 5\cos t\, j$

$\|r'(t)\| = \sqrt{(-5\sin t)^2 + (5\cos t)^2} = 5$

$T(t) = \dfrac{-5\sin t\, i + 5\cos t\, j}{5} = -\sin t\, i + \cos t\, j$

$T'(t) = -\cos t\, i - \sin t\, j$

$\|T'(t)\| = \sqrt{(-\cos t)^2 + (-\sin t)^2} = 1$

$N(t) = -\cos t\, i - \sin t\, j$

$N\left(\frac{5\pi}{3}\right) = -\frac{1}{2} i - \left(\frac{-\sqrt{3}}{2}\right) = \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

____ 25. Find a_N at time $t = \frac{\pi}{3}$ for the space curve $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + 5t\,\mathbf{k}$.

a. 16
b. 0
c. 5
d. 25
e. 4

$a_T = \dfrac{v \cdot a}{\|v\|} = 0$

$v(t) = -4\sin t\, i + 4\cos t\, j + 5k$

$a(t) = -4\cos t\, i - 4\sin t\, j$

$\|v(t)\| = \sqrt{(-4\sin t)^2 + (4\cos t)^2} = 4$

$\|a\| = 4$

$a_N = \sqrt{\|a\|^2 - a_T^2} = \sqrt{16} = 4$

____ 26. Find the length of the space curve given below.

$\mathbf{r}(t) = 2t\,\mathbf{i} + 5\cos t\,\mathbf{j} + 5\sin t\,\mathbf{k}, [0,3]$

a. $\sqrt{29}$
b. 3
c. $3\sqrt{29}$
d. $6\sqrt{29}$
e. 29

$\int_0^3 \sqrt{(2)^2 + (-5\sin t)^2 + (5\cos t)^2}\, dt$

$\int_0^3 \sqrt{4 + 25}\, dt = \left[\sqrt{29}\, t\right]_0^3 = \boxed{3\sqrt{29}}$

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$r'(t) = i + 4tj + 2k$

$r''(t) = 4j$

$\|r'(t)\| = \sqrt{1^2 + (4t)^2 + 2^2} = \sqrt{5+16t^2}$

____ 27. Find the curvature K of the curve given below.

$\mathbf{r}(t) = t\,\mathbf{i} + 2t^2\,\mathbf{j} + 2t\,\mathbf{k}$ $\qquad K = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}.$

a. $\dfrac{5}{\sqrt{\left(5+16t^2\right)^3}}$

b. $\dfrac{4\sqrt{5}}{\left(5+4t^2\right)^{3/2}}$

c. $\dfrac{2}{\sqrt{\left(5+16t^2\right)^3}}$

d. $\dfrac{4}{\sqrt{\left(5+16t^2\right)^3}}$

e. $4\sqrt{\dfrac{5}{\left(5+16t^2\right)^3}}$

$T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{i+4tj+2k}{\sqrt{5+16t^2}}$

$T'(t) = \dfrac{\frac{\sqrt{5+16t^2}}{\sqrt{5+16t^2}} \cdot \frac{4j\sqrt{5+16t^2}}{1} - \frac{16t(i+4tj+2k)}{\sqrt{5+16t^2}}}{\left(\sqrt{5+16t^2}\right)^2}$

$= \dfrac{4j(5+16t^2) - 16ti - 64t^2j - 32tk}{\sqrt{5+16t^2}^{\,3}}$

$= \dfrac{20j + \cancel{64t^2}j - 16ti - \cancel{64t^2}j - 32tk}{\sqrt{(5+16t^2)^3}}$

$= \dfrac{-16ti + 20j - 32tk}{\sqrt{(5+16t^2)^3}}$

$\|T'(t)\| = \dfrac{\sqrt{(-16t)^2 + (20)^2 + (-32t)^2}}{(5+16t^2)^3}$

____ 28. Find the radius of curvature of the plane curve $y = 3x^2 + 2$ at $x = -1$. Round your answer to three decimal places.

a. 24.694
b. 85.333
c. 28.133
d. 34.510
e. 37.510

____ 29. Find the point on the curve $y = (x-8)^2 + 9$ at which the curvature K is a maximum.

a. $(8,9)$
b. $(0,-55)$
c. $(-8,-9)$
d. $(0,73)$
e. $(-8,265)$

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____ 30. Find the point on the curve given below at which the curvature ***K*** is zero.

$y = 10x^3 + 31x^2 + 5x$

a. $x = \frac{30}{31}$

b. $x = \frac{31}{30}$

c. $x = -\frac{31}{30}$

d. $x = -\frac{30}{31}$

e. $x = -\frac{31}{10}$

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Chapter 12 Practice Test
Answer Section

MULTIPLE CHOICE

1. C
2. B
3. C
4. A
5. C
6. B
7. D
8. D
9. B
10. E
11. A
12. D
13. D
14. E
15. D
16. A
17. B
18. C
19. E
20. D
21. C
22. D
23. D
24. D
25. E
26. C
27. E
28. E
29. A
30. C