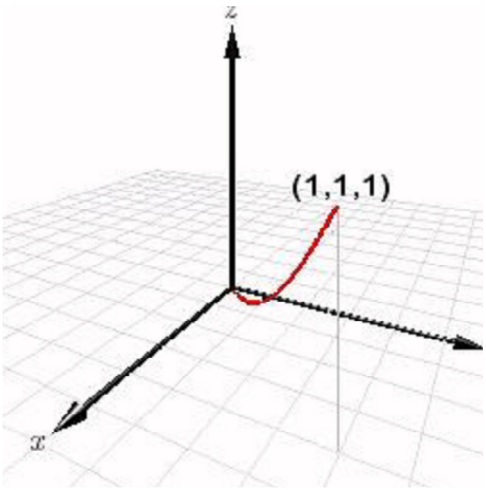
Name: _____ **Class:** _____ **Date:** _____ **ID: A**

Chapter 12 Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Match the equation with the graph shown in red below.



a. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k} \quad 0 \le t \le 1$

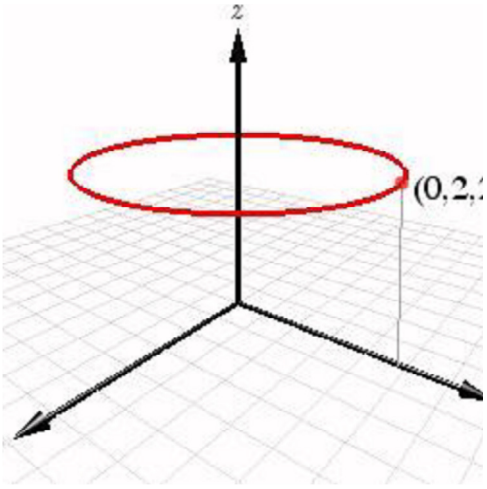
b. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$

c. $\mathbf{r}(t) = t\mathbf{i} + t\,\mathbf{j} + t^2\mathbf{k}, \quad 0 \le t \le 1$

d. $\mathbf{r}(t) = \mathbf{i} + t\,\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 4$

e. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$

____ 2. The graph below is most likely the graph of which of the following equations?



a. $\mathbf{r}(t) = 2t\cos(t)\mathbf{i} + 2t\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

b. $\mathbf{r}(t) = 2\sin(t)\mathbf{i} + 2\cos t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

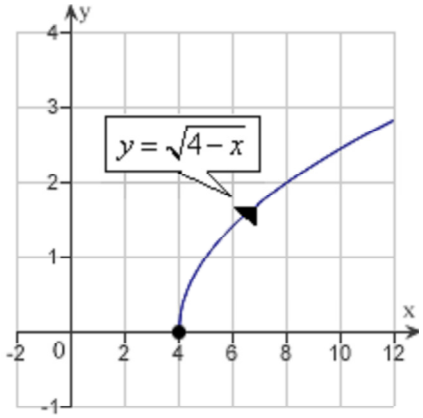
c. $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2t\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

d. $\mathbf{r}(t) = 2t\cos(t)\mathbf{i} + 2\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

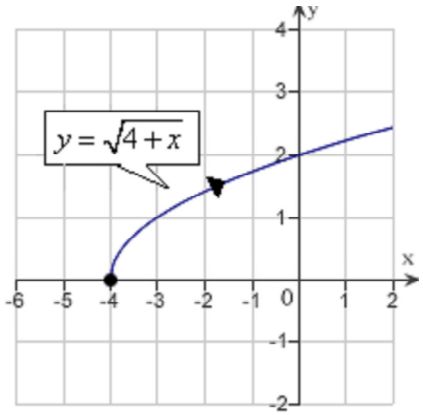
e. $\mathbf{r}(t) = 2\sin(t)\mathbf{i} + 2\sin t\mathbf{j} + 2\mathbf{k},\ 0 \le t \le 2\pi$

____ 3. Sketch the curve represented by the vector-valued function $\mathbf{r}(t) = (4-t)\mathbf{i} + \sqrt{t}\,\mathbf{j}$ and give the orientation of the curve.

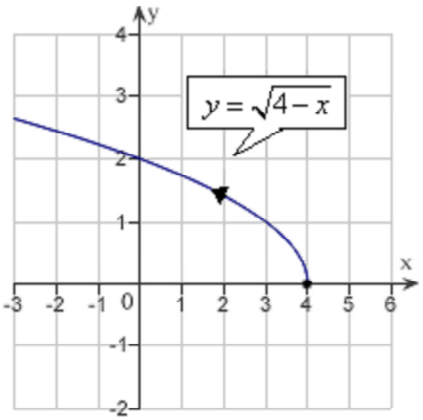
a.



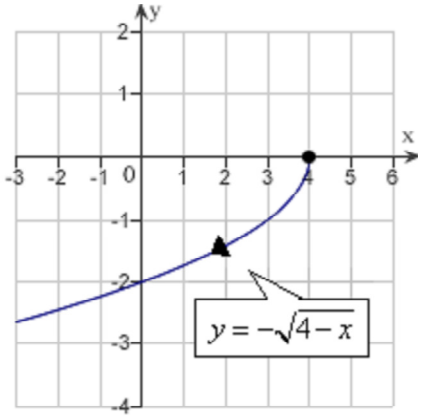
b.



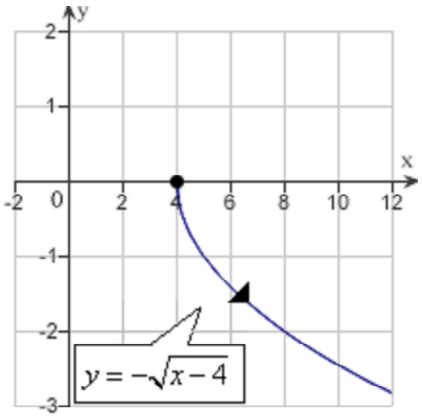
c.



d.

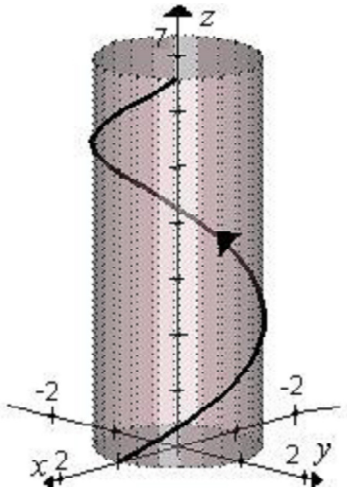


e.

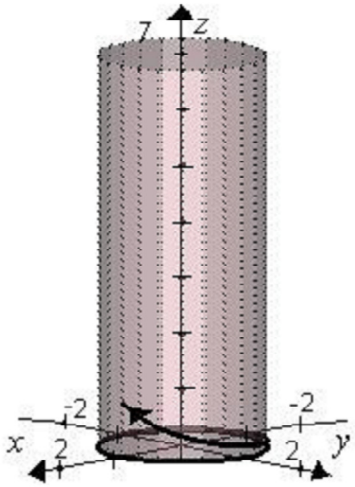


____ 4. Sketch the curve represented by the vector-valued function $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\mathbf{k}$ and give the orientation of the curve.

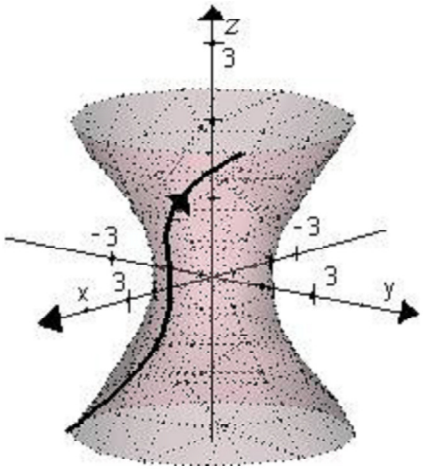
a.



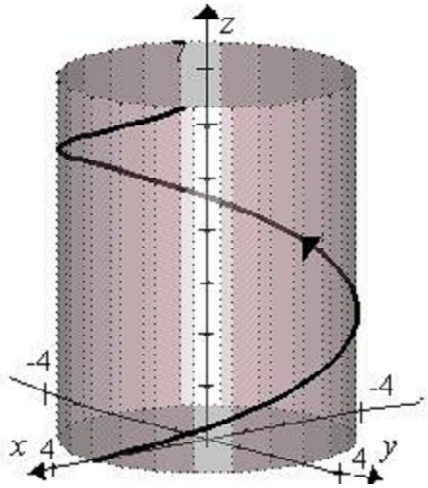
c.



b.



d.



____ 5. Represent the following curve by a vector-valued function.

$\frac{x^2}{121} + \frac{y^2}{64} = 1,\ x \geq 0$

a. $\mathbf{r}(t) = \frac{11}{8}\sqrt{64 - t^2}\,\mathbf{i} + t\,\mathbf{j};\ 0 \leq t \leq 16$

b. $\mathbf{r}(t) = \frac{11}{8}\sqrt{64 - t^2}\,\mathbf{i} + t\,\mathbf{j};\ 0 \leq t \leq 8$

c. $\mathbf{r}(t) = 11\cos 2\pi t\,\mathbf{i} + 8\sin 2\pi t\,\mathbf{j};\ \frac{-1}{4} \leq t \leq \frac{1}{4}$

d. $\mathbf{r}(t) = 11\cos t\,\mathbf{i} + 8\sin t\,\mathbf{j};\ -\pi \leq t \leq \pi$

e. $\mathbf{r}(t) = 11\cos \pi t\,\mathbf{i} - 8\sin \pi t\,\mathbf{j};\ \frac{-1}{4} \leq t \leq \frac{1}{4}$

____ 6. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$z = \frac{x^2}{25} + \frac{y^2}{4},\ y - 8x = 0$	$x = t$

a. $\mathbf{r}(t) = t\,\mathbf{i} + \frac{401}{25}t^2\,\mathbf{j} + 8t\,\mathbf{k}$

b. $\mathbf{r}(t) = t\,\mathbf{i} + 8t\,\mathbf{j} + \frac{401}{25}t^2\,\mathbf{k}$

c. $\mathbf{r}(t) = t\,\mathbf{i} - 8t\,\mathbf{j} + \frac{401}{25}t^2\,\mathbf{k}$

d. $\mathbf{r}(t) = t\,\mathbf{i} + 8t\,\mathbf{j} + \frac{25}{401}t^2\,\mathbf{k}$

e. $\mathbf{r}(t) = t\,\mathbf{i} + \frac{25}{401}t^2\,\mathbf{j} + 8t\,\mathbf{k}$

____ 7. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$x^2 + y^2 = 25,\ z = x^2$	$x = 5\sin 2\pi t$

a. $\mathbf{r}(t) = 25\sin 2\pi t\,\mathbf{i} + 25\cos 2\pi t\,\mathbf{j} - 5\sin^2 2\pi t\mathbf{k}$

b. $\mathbf{r}(t) = 5\sin 2\pi t\,\mathbf{j} + 5\cos 2\pi t\,\mathbf{i} + 25\sin^2 2\pi t\mathbf{k}$

c. $\mathbf{r}(t) = 5\sin 2\pi t\,\mathbf{i} + 5\cos 2\pi t\,\mathbf{j} - 25\sin^2 2\pi t\mathbf{k}$

d. $\mathbf{r}(t) = 5\sin 2\pi t\,\mathbf{i} + 5\cos 2\pi t\,\mathbf{j} + 25\sin^2 2\pi t\mathbf{k}$

e. $\mathbf{r}(t) = 25\sin 2\pi t\,\mathbf{i} + 25\cos 2\pi t\,\mathbf{j} + 5\sin^2 2\pi t\mathbf{k}$

____ 8. Suppose the two particles travel along the space curves $\mathbf{r}(t) = t^2\mathbf{i} + (8t - 16)\mathbf{j} + t^2\mathbf{k}$ and $\mathbf{u}(t) = (3t + 4)\mathbf{i} + t^2\mathbf{j} + (5t - 4)\mathbf{k}$. A collision will occur at the point of intersection *P* if both particles are at *P* at the same time. Find the point of collision.

a. $(9, 9, 9)$
b. $(9, 16, 9)$
c. $(16, 9, 16)$
d. $(16, 16, 16)$
e. $(8, 8, 8)$

____ 9. Find the vectors $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ for the following vector function.

$\mathbf{r}(t) = (1 + 2t)\mathbf{i} + \left(2 + 5t^2\right)\mathbf{j} + 2\mathbf{k}$

a. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 2\mathbf{k},\ \mathbf{r}'(3) = 18\mathbf{i} + 30\mathbf{j}$

b. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 2\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 30\mathbf{j}$

c. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 4\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 30\mathbf{j}$

d. $\mathbf{r}(3) = 7\mathbf{i} + 2\mathbf{j} + 47\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 30\mathbf{j}$

e. $\mathbf{r}(3) = 7\mathbf{i} + 47\mathbf{j} + 2\mathbf{k},\ \mathbf{r}'(3) = 2\mathbf{i} + 90\mathbf{j}$

____ 10. Find $\mathbf{r}'(t)$ given the following vector function.

$\mathbf{r}(t) = 2t^2\mathbf{i} + 4t^4\mathbf{j} + 2t^3\mathbf{k}$

a. $\mathbf{r}'(t) = 2t\mathbf{i} + 4t^3\mathbf{j} + 2t^2\mathbf{k}$

b. $\mathbf{r}'(t) = 4t^2\mathbf{i} + 16t^4\mathbf{j} + 6t^3\mathbf{k}$

c. $\mathbf{r}'(t) = 2t\mathbf{i} + 4t^2\mathbf{j} + 2t^3\mathbf{k}$

d. $\mathbf{r}'(t) = 4t\mathbf{i} + 4t^3\mathbf{j} + 2t^2\mathbf{k}$

e. $\mathbf{r}'(t) = 4t\mathbf{i} + 16t^3\mathbf{j} + 6t^2\mathbf{k}$

____ 11. Find $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ given the following vector function.

$\mathbf{r}(t) = \left(2t^2 + 2t\right)\mathbf{i} + \left(3t^2 + 4t\right)\mathbf{j}$

a. $32 + 52t$
b. $16 + 52t$
c. $32 + 26t$
d. $16 + 26t$
e. $32 + 52t^2$

____ 12. Find $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$ given the following vector function.

$\mathbf{r}(t) = 3\cos t\mathbf{i} + 5\sin t\mathbf{j}$

a. cost sint
b. $-34\cos t \sin t$
c. $34\cos t \sin t$
d. –16 sin t cos t
e. $3\cos t \sin t$

____ 13. Use the properties of the derivative to find $D_t\left[\mathbf{r}(t)\times\mathbf{u}(t)\right]$ given the following vector-valued functions.

$\mathbf{r}(t)=2t\mathbf{i}-3t^3\mathbf{j}+3t^2\mathbf{k}$

$\mathbf{u}(t)=3\mathbf{i}+5t^2\mathbf{j}+4t^3\mathbf{k}$

a. $\left(-60t^3-72t^5\right)\mathbf{i}+\left(18t^2-32t^3\right)\mathbf{j}+57t^2\mathbf{k}$

b. $\left(-60t^3-72t^5\right)\mathbf{i}+\left(32t+18t^3\right)\mathbf{j}+57t^2\mathbf{k}$

c. $\left(-60t^3-72t^5\right)\mathbf{i}+\left(18t-32t^3\right)\mathbf{j}-3t^2\mathbf{k}$

d. $\left(-60t^3-72t^5\right)\mathbf{i}+\left(18t-32t^3\right)\mathbf{j}+57t^2\mathbf{k}$

e. $\left(-60t^3-72t^5\right)\mathbf{i}+\left(18t+32t^3\right)\mathbf{j}+57t^3\mathbf{k}$

____ 14. Use the properties of the derivative to find $D_t\left[\mathbf{r}(t)\cdot\mathbf{u}(t)\right]$ given the following vector-valued functions.

$\mathbf{r}(t)=t\mathbf{i}+2\cos 4t\mathbf{j}+2\sin 4t\mathbf{k}$

$\mathbf{u}(t)=\frac{5}{t}\mathbf{i}+2\cos 4t\mathbf{j}+2\sin 4t\mathbf{k}$

a. 2

b. $\frac{2}{t}$

c. $2t$

d. 5

e. 0

____ 15. Find the indefinite integral below.

$$\int \left(5e^{5t}\,\mathbf{i} - 4\sin 4t\,\mathbf{j} + 6\cos 3t\,\mathbf{k}\right) dt$$

Do not include an arbitrary constant vector.

a. $e^{5t}\,\mathbf{i} - \cos 4t\,\mathbf{j} + 2\sin 3t\,\mathbf{k}$

b. $e^{5t}\,\mathbf{i} + \cos 4t\,\mathbf{j} - 2\sin 3t\,\mathbf{k}$

c. $e^{5t}\,\mathbf{i} + \cos 4t\,\mathbf{j} + 6\sin 3t\,\mathbf{k}$

d. $e^{5t}\,\mathbf{i} + \cos 4t\,\mathbf{j} + 2\sin 3t\,\mathbf{k}$

e. $\frac{e^{5t}}{5}\,\mathbf{i} + \cos 4t\,\mathbf{j} + 2\sin 3t\,\mathbf{k}$

____ 16. Find $\mathbf{r}(t)$ given the following.

$\mathbf{r}'(t) = 18t^5\,\mathbf{j} + 6t\,\mathbf{k}$, $\mathbf{r}(0) = 2\mathbf{i} + 18\mathbf{j}$

a. $\mathbf{r}(t) = 2\mathbf{i} + \left(18 + 3t^6\right)\mathbf{j} + 3t^2\,\mathbf{k}$

b. $\mathbf{r}(t) = 2\mathbf{i} + 3t^2\,\mathbf{j} + \left(18 + 3t^6\right)\mathbf{k}$

c. $\mathbf{r}(t) = \left(18 + 3t^6\right)\mathbf{j} + 3t^2\,\mathbf{k}$

d. $\mathbf{r}(t) = 2\mathbf{i} + \left(18 - 3t^6\right)\mathbf{j} - 3t^2\,\mathbf{k}$

e. $\mathbf{r}(t) = 18\,\mathbf{i} + \left(2 + 3t^6\right)\mathbf{j} + 3t^2\,\mathbf{k}$

____ 17. A particle moves in the *yz*-plane along the curve represented by the vector-valued function $\mathbf{r}(t) = (4\cos t)\,\mathbf{j} + (9\sin t)\,\mathbf{k}$. Find the minimum value of $\|\mathbf{r}'\|$.

a. 6
b. 4
c. 7
d. 9
e. 8

____ 18. A particle moves in the *yz*-plane along the curve represented by the vector-valued function $\mathbf{r}(t) = (5\cos t)\,\mathbf{j} + (7\sin t)\,\mathbf{k}$. Find the maximum value of $\|\mathbf{r}'\|$.

a. 4
b. 3
c. 7
d. 8
e. 5

____ 19. The position vector $r(t) = \langle 7t, 6\cos t, 6\sin t \rangle$ describes the path of an object moving in space. Find the speed $s(t)$ of the object.

a. 21
b. $3\sqrt{7}$
c. $\sqrt{93}$
d. 149
e. $\sqrt{85}$

____ 20. The position vector $\mathbf{r}(t) = \langle 2\cos t, 4\sin t, t^2 \rangle$ describes the path of an object moving in space. Find the velocity $\mathbf{v}(t)$ of the object.

a. $\mathbf{v}(t) = 2\sin t\,\mathbf{i} - 4\cos t\,\mathbf{j} + \mathbf{k}$
b. $\mathbf{v}(t) = -2\cos t\,\mathbf{i} - 4\sin t\,\mathbf{j} + 2\mathbf{k}$
c. $\mathbf{v}(t) = -2\sin t\,\mathbf{i} - 4\cos t\,\mathbf{j} + \mathbf{k}$
d. $\mathbf{v}(t) = -2\sin t\,\mathbf{i} + 4\cos t\,\mathbf{j} + 2t\mathbf{k}$
e. $\mathbf{v}(t) = -2\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + \mathbf{k}$

____ 21. Use the given acceleration function and initial conditions to find the position at time t = 3.

$\mathbf{a}(t) = 5\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j}, \quad \mathbf{v}(0) = 8\mathbf{j} + 5\mathbf{k}, \quad \mathbf{r}(0) = -5\mathbf{i}$

a. $\mathbf{r}(3) = -5\cos 3\mathbf{i} - (3\sin 3 + 15)\mathbf{j} + 15\mathbf{k}$
b. $\mathbf{r}(3) = -5\cos 3\mathbf{i} + (3\sin 3 - 15)\mathbf{j} + 15\mathbf{k}$
c. $\mathbf{r}(3) = -5\cos 3\mathbf{i} + (3\sin 3 + 15)\mathbf{j} + 15\mathbf{k}$
d. $\mathbf{r}(3) = 5\cos 3\mathbf{i} + (3\sin 3 + 15)\mathbf{j} + 15\mathbf{k}$
e. $\mathbf{r}(3) = -5\cos 3\mathbf{i} + 15\mathbf{j} + (3\sin 3 + 15)\mathbf{k}$

____ 22. Find the unit tangent vector $\mathbf{T}(t)$ for the line tangent to the space curve $\mathbf{r}(t) = \langle 12\cos t, 12\sin t, 3\rangle$ at point $P\left(6\sqrt{2}, 6\sqrt{2}, 3\right)$.

a. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{4}\langle -\sqrt{2}, -\sqrt{2}, 3\rangle$

b. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{4}\langle -\sqrt{2}, \sqrt{2}, 0\rangle$

c. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{2}\langle \sqrt{2}, -\sqrt{2}, 3\rangle$

d. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{2}\langle -\sqrt{2}, \sqrt{2}, 0\rangle$

e. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{2}\langle \sqrt{2}, -\sqrt{2}, 0\rangle$

____ 23. Find the principle unit normal vector to the curve given below at the specified point.

$\mathbf{r}(t) = t\mathbf{i} + 4t^2\mathbf{j}, \quad t = 3$

a. $\mathbf{N}(3) = \frac{-24}{\sqrt{145}}\mathbf{i} - \frac{1}{\sqrt{145}}\mathbf{j}$

b. $\mathbf{N}(3) = \frac{-24}{\sqrt{145}}\mathbf{i} + \frac{1}{\sqrt{145}}\mathbf{j}$

c. $\mathbf{N}(3) = \frac{-24}{\sqrt{577}}\mathbf{i} - \frac{1}{\sqrt{577}}\mathbf{j}$

d. $\mathbf{N}(3) = \frac{-24}{\sqrt{577}}\mathbf{i} + \frac{1}{\sqrt{577}}\mathbf{j}$

e. $\mathbf{N}(3) = \frac{24}{\sqrt{577}}\mathbf{i} - \frac{1}{\sqrt{577}}\mathbf{j}$

____ 24. Find the principle unit normal vector to the curve given below at the specified point.

$$\mathbf{r}(t) = 5\cos t\,\mathbf{i} + 5\sin t\,\mathbf{j}, \quad t = \frac{5\pi}{3}$$

a. $\mathbf{N} = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

b. $\mathbf{N} = \left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

c. $\mathbf{N} = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$

d. $\mathbf{N} = \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

e. $\mathbf{N} = \left\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle$

____ 25. Find $a_{\mathbf{N}}$ at time $t = \frac{\pi}{3}$ for the space curve $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + 5t\,\mathbf{k}$.

a. 16
b. 0
c. 5
d. 25
e. 4

____ 26. Find the length of the space curve given below.

$$\mathbf{r}(t) = 2t\,\mathbf{i} + 5\cos t\,\mathbf{j} + 5\sin t\,\mathbf{k}, [0,3]$$

a. $\sqrt{29}$
b. 3
c. $3\sqrt{29}$
d. $6\sqrt{29}$
e. 29

____ 27. Find the curvature ***K*** of the curve given below.

$\mathbf{r}(t) = t\,\mathbf{i} + 2t^2\,\mathbf{j} + 2t\,\mathbf{k}$

a. $\dfrac{5}{\sqrt{\left(5+16t^2\right)^3}}$

b. $\dfrac{4\sqrt{5}}{\left(5+4t^2\right)^{3/2}}$

c. $\dfrac{2}{\sqrt{\left(5+16t^2\right)^3}}$

d. $\dfrac{4}{\sqrt{\left(5+16t^2\right)^3}}$

e. $4\sqrt{\dfrac{5}{\left(5+16t^2\right)^3}}$

____ 28. Find the radius of curvature of the plane curve $y = 3x^2 + 2$ at $x = -1$. Round your answer to three decimal places.

a. 24.694
b. 85.333
c. 28.133
d. 34.510
e. 37.510

____ 29. Find the point on the curve $y = (x-8)^2 + 9$ at which the curvature K is a maximum.

a. $(8,9)$
b. $(0,-55)$
c. $(-8,-9)$
d. $(0,73)$
e. $(-8,265)$

____ 30. Find the point on the curve given below at which the curvature ***K*** is zero.

$y = 10x^3 + 31x^2 + 5x$

a. $x = \frac{30}{31}$

b. $x = \frac{31}{30}$

c. $x = -\frac{31}{30}$

d. $x = -\frac{30}{31}$

e. $x = -\frac{31}{10}$

Chapter 12 Practice Test
Answer Section

MULTIPLE CHOICE

1. C
2. B
3. C
4. A
5. C
6. B
7. D
8. D
9. B
10. E
11. A
12. D
13. D
14. E
15. D
16. A
17. B
18. C
19. E
20. D
21. C
22. D
23. D
24. D
25. E
26. C
27. E
28. E
29. A
30. C