Name: _____ Class: _____ Date: _____ ID: A

Multivariable Calculus Chapter 11 Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

_____ 1. Find vectors **u** and **v** whose initial and terminal points are given. Determine whether **u** and **v** are equivalent.

u: (6,5), (9,11)   **v:** (3,–5), (6,1)

a. $\mathbf{u} = 6\mathbf{i} + 3\mathbf{j}, \mathbf{v} = 6\mathbf{i} + 3\mathbf{j}$, which are equivalent
b. $\mathbf{u} = 3\mathbf{i} + 6\mathbf{j}, \mathbf{v} = 6\mathbf{j} + 3\mathbf{i}$, which are not equivalent
c. $\mathbf{u} = 3\mathbf{i} + 6\mathbf{j}, \mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$, which are equivalent
d. $\mathbf{u} = 3\mathbf{i} - 6\mathbf{j}, \mathbf{v} = -3\mathbf{i} - 6\mathbf{j}$, which are equivalent
e. $\mathbf{u} = -3\mathbf{i} + 6\mathbf{j}, \mathbf{v} = -3\mathbf{i} + 6\mathbf{j}$, which are equivalent

Handwritten: component form
$u = (9-6, 11-5)$ $v = (6-3, 1--5)$
$u = (3,6)$ $v = (3,6)$
also $3i + 6j$

_____ 2. Find the vector **v** whose initial and terminal points are given below.

(5,5), (7,2)

a. $\mathbf{v} = 2\mathbf{i} + 7\mathbf{j}$
b. $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$
c. $\mathbf{v} = -2\mathbf{i} - 6\mathbf{j}$
d. $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j}$
e. $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j}$

Handwritten:
$(7-5, 2-5)$
$(2, -3)$
$2i - 3j$

_____ 3. Given $\mathbf{u} = \langle 6, 12 \rangle$ and $\mathbf{v} = \langle 3, -12 \rangle$, find $2\mathbf{u} + 5\mathbf{v}$.

a. $\langle 6, -36 \rangle$
b. $\langle 3, 84 \rangle$
c. $\langle 9, -52 \rangle$
d. $\langle 12, 60 \rangle$
e. $\langle 27, -36 \rangle$

Handwritten:
$2u = \langle 2(6), 2(12) \rangle = \langle 12, 24 \rangle$
$5v = \langle 5(3), 5(-12) \rangle = \langle 15, -60 \rangle$
$2u + 5v = \langle 12+15, 24-60 \rangle$
$2u + 5v = \langle 27, -36 \rangle$

11-PT solutions

_____ 4. The vector **v** and its initial point is given. Find the terminal point.

$\mathbf{v} = \langle 3, -2 \rangle$, initial point $(-6, 10)$

$(-6 + 3, \ 10 - 2)$

$(-3, 8)$

   a. $(-3, -8)$
   b. $(8, -3)$
   c. $(3, 8)$
   d. $(3, -8)$
   (e.) $(-3, 8)$

_____ 5. Find the magnitude of the vector given below. Round your answer to four decimal places.

$\mathbf{v} = \langle 5, -2 \rangle$

$\|v\| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$

   a. 0
   b. $\sqrt{21}$ or 4.5826
   (c.) $\sqrt{29}$ or 5.3852
   d. $\sqrt{7}$ or 2.6458
   e. $\sqrt{10}$ or 3.1623

_____ 6. Find the unit vector in the direction of **u**.

$\mathbf{u} = \langle 5, 4 \rangle$

$$\frac{u}{\|u\|} = \frac{\langle 5, 4 \rangle}{\sqrt{25 + 16}}$$

$$= \left\langle \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}} \right\rangle$$

   a. $\left\langle \dfrac{1}{\sqrt{41}}, \dfrac{1}{\sqrt{41}} \right\rangle$

   b. $\left\langle \dfrac{1}{\sqrt{41}}, \dfrac{-1}{\sqrt{41}} \right\rangle$

   c. $\left\langle \dfrac{-5}{\sqrt{41}}, \dfrac{-4}{\sqrt{41}} \right\rangle$

   d. $\left\langle \dfrac{-1}{\sqrt{41}}, \dfrac{-1}{\sqrt{41}} \right\rangle$

   (e.) $\left\langle \dfrac{5}{\sqrt{41}}, \dfrac{4}{\sqrt{41}} \right\rangle$

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_____ 7. Find the vector **v** with the given magnitude $\|\mathbf{v}\| = 185$ and the same direction as $\mathbf{u} = \langle -11, 8 \rangle$.

a. $\left\langle 11\sqrt{187}, 8\sqrt{187} \right\rangle$

b. $\left\langle 8\sqrt{185}, -11\sqrt{185} \right\rangle$

c. $\left\langle -11\sqrt{185}, 8\sqrt{185} \right\rangle$ ⊙

d. $\left\langle 8\sqrt{187}, 8\sqrt{187} \right\rangle$

e. $\left\langle \sqrt{2618}, 4\sqrt{187} \right\rangle$

$$\|v\| \frac{u}{\|u\|} = 185 \frac{\langle -11, 8 \rangle}{\sqrt{185}}$$

$$= 185 \left\langle \frac{-11\sqrt{185}}{185}, \frac{8\sqrt{185}}{185} \right\rangle$$

$$= \left\langle -11\sqrt{185}, 8\sqrt{185} \right\rangle$$

_____ 8. Find the component form of vector **v** given its magnitude is $\|\mathbf{v}\| = 7$ and the angle it makes with the positive *x*-axis is $\theta = 120°$.

a. $\left\langle \frac{7\sqrt{3}}{2}, \frac{7\sqrt{3}}{4} \right\rangle$

b. $\left\langle -\frac{\sqrt{5}}{3}, 7\frac{\sqrt{3}}{3} \right\rangle$

c. $\left\langle \frac{7\sqrt{3}}{4}, \frac{7\sqrt{3}}{4} \right\rangle$

d. $\left\langle -\frac{7}{2}, 7\frac{\sqrt{3}}{2} \right\rangle$ ⊙

e. $\left\langle \frac{7}{2}, \frac{7\sqrt{5}}{2} \right\rangle$

$$v = \|v\| \langle \cos\theta, \sin\theta \rangle$$

$$v = 7 \left\langle \cos\frac{2\pi}{3}, \sin\frac{2\pi}{3} \right\rangle$$

$$v = 7 \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

$$v = \left\langle -\frac{7}{2}, \frac{7\sqrt{3}}{2} \right\rangle$$

11-PT solutions

_____ 9. Find the component form of vector **u** + **v** given $\|\mathbf{u}\| = 2$ and $\|\mathbf{v}\| = 3$ and the angles that **u** and **v** make with the positive *x*-axis are $\theta_\mathbf{u} = 0°$ and $\theta_\mathbf{v} = 45°$.

a. $\left\langle \dfrac{4+3\sqrt{3}}{2}, \dfrac{9\sqrt{2}}{2} \right\rangle$

b. $\left\langle \dfrac{2+3\sqrt{2}}{2}, \dfrac{9\sqrt{2}}{2} \right\rangle$

c. $\left\langle \dfrac{22+9\sqrt{2}}{2}, \dfrac{6\sqrt{2}}{2} \right\rangle$

d. $\left\langle \dfrac{4+3\sqrt{2}}{2}, \dfrac{3\sqrt{2}}{2} \right\rangle$

e. $\left\langle \dfrac{2+9\sqrt{2}}{2}, \dfrac{9\sqrt{2}}{2} \right\rangle$

Handwritten:
$u = 2\langle \cos 0, \sin 0 \rangle = 2\langle 1, 0 \rangle = \langle 2, 0 \rangle$
$v = 3\langle \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \rangle = 3\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle = \langle \frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle$
$u + v = \langle 2 + \frac{3\sqrt{2}}{2}, 0 + \frac{3\sqrt{2}}{2} \rangle = \langle \frac{4+3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle$

_____ 10. Three forces with magnitudes 80 pounds, 110 pounds and 25 pounds act on an object at angles 30°, −70°, and 120° respectively, with the positive *x*-axis. Find the direction and magnitude of the resultant force. (The choices below are given to two decimal places.)

a. 136.12 pounds at an angle of − 66.16° with the positive *x*-axis
b. 103.21 pounds at an angle of 23.84° with the positive *x*-axis
c. 103.21 pounds at an angle of − 66.16° with the positive *x*-axis
d. 136.12 pounds at an angle of − 23.84° with the positive *x*-axis
e. 103.21 pounds at an angle of −23.84° with the positive *x*-axis
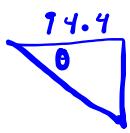
Handwritten:
$F = F_1 + F_2 + F_3$
94.4
θ 41.7
$F_1 = 80\cos 30 i + 80\sin j$
$F_2 = 110 \cos(-70)i + 110 \sin(-70)j$
$F_3 = 25\cos 120 i + 25 \sin 25 j$
$94.4 i - 41.7 j$

_____ 11. Suppose a gun with a muzzle velocity of 1200 feet per second is fired at an angle of 4° above the horizontal. Find the horizontal component of the velocity. Round your answer to two decimal places.

a. 1,240.08 ft/sec
b. 2,395.15 ft/sec
c. 1,197.08 ft/sec
d. 2,393.15 ft/sec
e. 1,209.08 ft/sec

Handwritten:
$\|v\| \cos \theta$
$1200 \cos 4°$
1200
4°

11-PT solutions
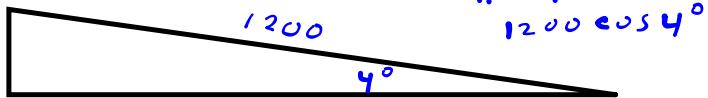
_____ 12. To carry a 100-pound cylindrical weight, two workers lift on the ends of short ropes tied to an eyelet on the top center of the cylinder. One rope makes a 20° angle away from the vertical and the other makes a 30° angle as shown in the figure below. Find each rope's tension if the resultant force is vertical. Round your answer to two decimal places.
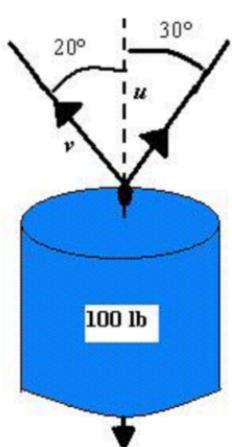
$$u = \|u\|(\cos 60° i + \sin 60° j)$$

$$v = \|v\|(\cos 110° i + \sin 110° j)$$

$$\|u\| \sin 60° + \|v\| \sin 110° = 100 \quad \text{vert. comp.}$$

$$\|u\| \cos 60° + \|v\| \cos 110° = 0 \quad \text{horiz. comp.}$$

solve this system of equations

use your calculator

a. $\|u\| = 44.65$ lb; $\|v\| = 65.27$ lb
b. $\|u\| = 144.65$ lb; $\|v\| = 165.27$ lb
c. $\|u\| = 88.30$ lb; $\|v\| = 129.54$ lb
d. $\|u\| = 55.65$ lb; $\|v\| = 76.27$ lb
e. $\|u\| = 90.30$ lb; $\|v\| = 131.54$ lb

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_____ 13. To carry a 100-pound cylindrical weight, two workers lift on the ends of short ropes tied to an eyelet on the top center of the cylinder. One rope makes a 20° angle away from the vertical and the other makes a 30° angle as shown in the figure below. Find the vertical component of each worker's force. Round your answer to two decimal places.
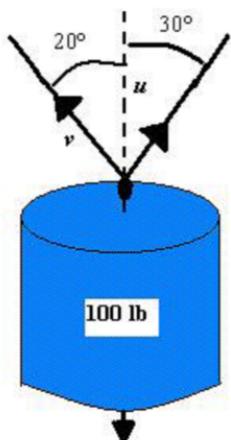
$$\|u\| \sin 60° \approx 38.67$$

$$\|v\| \sin 110° \approx 61.33$$

$$note: \text{ their } sum = 100$$

a. $\|\mathbf{u}\| \sin 60° \approx 55.67$ lb; $\|\mathbf{v}\| \sin 110° \approx 78.33$ lb
b. $\|\mathbf{u}\| \sin 60° \approx 85.67$ lb; $\|\mathbf{v}\| \sin 110° \approx 108.33$ lb
c. $\|\mathbf{u}\| \sin 60° \approx 78.33$ lb; $\|\mathbf{v}\| \sin 110° \approx 123.67$ lb
d. $\|\mathbf{u}\| \sin 60° \approx 156.67$ lb; $\|\mathbf{v}\| \sin 110° \approx 179.33$ lb
e. $\|\mathbf{u}\| \sin 60° \approx 38.67$ lb; $\|\mathbf{v}\| \sin 110° \approx 61.33$ lb

_____ 14. Suppose a plane flies at a constant groundspeed of 500 miles per hour due east and encounters a 50 mile-per-hour wind from the northwest. Both the airspeed and the compass direction must change to for the plane to maintain its groundspeed and eastward direction. Find the airspeed to maintain its groundspeed and eastward direction. Round your answer to two decimal places.

a. 526.99 mi/h
b. 507.99 mi/h
c. 485.99 mi/h
d. 643.99 mi/h
e. 465.99 mi/h

$$u = 500i \ (plane)$$

$$v = 50 \cos 135°i + \sin 135°j = -25\sqrt{2}i + 25\sqrt{2}j \ (wind)$$

$$u+v = 464i + 35j$$

$$\tan \theta = \frac{35}{464} \rightarrow \arctan \frac{35}{464} = \theta \approx 4.35°$$

speed
4.35°
35.36j
464.64i

$$\boxed{465.99}$$
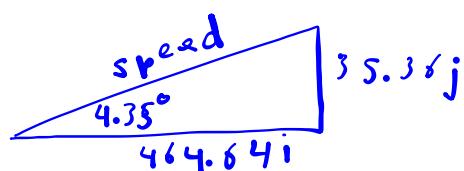
11-PT solutions

_____ 15. Find the distance between the points given below.

(−4, 2, −3), (0, 5, 1)

$$\sqrt{(0--4)^2+(5-2)^2+(1--3)^2}$$

$$\sqrt{4^2+3^2+4^2}$$

$$\sqrt{41}$$

     a. $\sqrt{41}$ or 6.4031 to four decimal places.
     b. $2\sqrt{10}$ or 6.3246 to four decimal places.
     c. $\sqrt{42}$ or 6.4807 to four decimal places.
     d. $\sqrt{43}$ or 5.9161 to four decimal places.
     e. $\sqrt{43}$ or 6.5574 to four decimal places.

_____ 16. Find the coordinates of the midpoint of the line segment joining the points given below.

(4, 8, 4), (7, 10, 7) $\left(\dfrac{4+7}{2}, \dfrac{8+10}{2}, \dfrac{4+7}{2}\right)$

     a. (5.5, 1, 5.5)
     b. (1.5, 1, 5.5)
     c. (1.5, 9, 5.5)
     d. (1.5, 9, 1.5)
     e. (5.5, 9, 5.5)

_____ 17. Find the standard equation of the sphere with center (2, 8, 4), and radius 2.

     a. $(x-2)^2+(y-8)^2+(z+4)^2=4$ $(x-2)^2+(y-8)^2+(z-4)^2=2^2$

     b. $(x+2)^2+(y-8)^2+(z-4)^2=2$

     c. $(x-2)^2+(y-8)^2+(z-4)^2=2$

     d. $(x-2)^2+(y+8)^2+(z-4)^2=4$

     e. $(x-2)^2+(y-8)^2+(z-4)^2=4$

_____ 18. Find the standard equation of a sphere that has diameter with the end points given below.

(3, −2, 4), (7, 12, 4)

center: $\left(\dfrac{3+7}{2}, \dfrac{-2+12}{2}, \dfrac{4+4}{2}\right)=(5,5,4)$

     a. $(x-5)^2+(y-5)^2+(z-4)^2=53$ radius: $\sqrt{(7-5)^2+(12-5)^2+(4-4)^2}$

     b. $(x-5)^2+(y-5)^2+(z-4)^2=9$

     c. $(x+5)^2+(y-5)^2+(z-4)^2=53$ $=\sqrt{2^2+7^2+0^2}=\sqrt{53}$

     d. $(x-5)^2+(y+5)^2+(z+4)^2=53$

     e. $(x-5)^2+(y+5)^2+(z-4)^2=53$ $(x-5)^2+(y-5)^2+(z-4)^2=53$

11-PT solutions

_____ 19. Find the standard equation of the sphere with center at $\left(-2,3,6\right)$ and tangent to the yz-plane.

a. $(x+2)^2 + (y-3)^2 + (z-6)^2 = 4$

b. $(x+2)^2 + (y-3)^2 + (z-6)^2 = 9$

c. $(x+2)^2 + (y-3)^2 + (z-6)^2 = 45$

d. $(x-2)^2 + (y+3)^2 + (z+6)^2 = 4$

e. $(x-2)^2 + (y+3)^2 + (z+6)^2 = 45$

radius = 2

$(x+2)^2 + (y-3)^2 + (z-6)^2 = 4$

_____ 20. Find the component form of the vector **u** with the given initial and terminal points.

Initial point: (5, 7, 8)
Terminal point: (–2 , 10 , 10)

$(-2-5, \ 10-7, \ 10-8)$

$(-7, 3, 2)$

a. $\mathbf{u} = 7\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$
b. $\mathbf{u} = 3\mathbf{i} + 17\mathbf{j} + 18\mathbf{k}$
c. $\mathbf{u} = -7\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
d. $\mathbf{u} = -7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
e. $\mathbf{u} = 7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

$-7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

_____ 21. Given the vector **v** and its initial point find the terminal point of the vector.

$\mathbf{v} = \langle 3,2,1 \rangle$,   initial point $(2,-4,5)$

a. $(5,4,-6)$
b. $(5,-2,6)$
c. $(-5,-2,6)$
d. $(5,4,6)$
e. $(-5,4,6)$

$x - 2 = 3 \quad x = 5$
$y - 4 = 2 \quad y = -2$
$z - 5 = 1 \quad z = 6$

_____ 22. Find the magnitude of the vector given below.

$\mathbf{v} = \langle -5,-4,-3 \rangle$

a. $\sqrt{54}$ or 7.3485
b. $\sqrt{100}$ or 10.0000
c. $\sqrt{50}$ or 7.0711
d. $\sqrt{12}$ or 3.4641
e. $\sqrt{56}$ or 7.4833

$\sqrt{(-5)^2 + (-4)^2 + (-3)^2}$

$\sqrt{25 + 16 + 9}$

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11-PT solutions

_____ 23. Find the unit vector in the direction of **u**.

$$\mathbf{u} = \langle -5, -2, -1 \rangle$$

$$\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\langle -5, -2, -1 \rangle}{\sqrt{(-5)^2 + (-2)^2 + (-1)^2}} = \frac{\langle -5, -2, -1 \rangle}{\sqrt{30}}$$

   (a.) $\left\langle \dfrac{-5}{\sqrt{30}}, \dfrac{-2}{\sqrt{30}}, \dfrac{-1}{\sqrt{30}} \right\rangle$

   b. $\left\langle \dfrac{-1}{\sqrt{30}}, \dfrac{-1}{\sqrt{30}}, \dfrac{-1}{\sqrt{30}} \right\rangle$

   c. $\left\langle \dfrac{5}{\sqrt{30}}, \dfrac{2}{\sqrt{30}}, \dfrac{1}{\sqrt{30}} \right\rangle$

   d. $\left\langle \dfrac{25}{\sqrt{30}}, \dfrac{4}{\sqrt{30}}, \dfrac{1}{\sqrt{30}} \right\rangle$

   e. $\left\langle \dfrac{-150}{\sqrt{30}}, \dfrac{-60}{\sqrt{30}}, \dfrac{-30}{\sqrt{30}} \right\rangle$

_____ 24. Find a unit vector **u** in the direction opposite of $\mathbf{v} = \langle 2, 8, -3 \rangle$.

   (a.) $\mathbf{u} = (-1/\sqrt{77})\langle 2, 8, -3 \rangle$

   b. $\mathbf{u} = (1/\sqrt{59})\langle 2, 8, -3 \rangle$

   c. $\mathbf{u} = (-1/\sqrt{59})\langle 2, 8, -3 \rangle$

   d. $\mathbf{u} = (1/\sqrt{77})\langle 2, 8, -3 \rangle$

   e. $\mathbf{u} = (-1/6\sqrt{2})\langle 2, 8, -3 \rangle$

$$\mathbf{u} = -1\left(\frac{\mathbf{v}}{\|\mathbf{v}\|}\right) = \frac{\langle -2, -8, 3 \rangle}{\sqrt{(-2)^2 + (-8)^2 + 3^2}} = -\frac{1}{\sqrt{77}}\langle 2, 8, -3 \rangle$$

_____ 25. Find the vector **v** with the magnitude $\dfrac{15}{2}$ and direction of $\mathbf{u} = \langle 18, -18, 9 \rangle$.

   (a.) $\mathbf{v} = \left\langle 5, -5, \dfrac{5}{2} \right\rangle$

   b. $\mathbf{v} = \langle 5, -5, 5 \rangle$

   c. $\mathbf{v} = \left\langle 9, -9, \dfrac{9}{2} \right\rangle$

   d. $\mathbf{v} = \left\langle \dfrac{5}{2}, -\dfrac{5}{2}, \dfrac{5}{18} \right\rangle$

   e. $\mathbf{v} = \left\langle \dfrac{5}{18}, -\dfrac{5}{18}, \dfrac{5}{6} \right\rangle$

$$\mathbf{v} = \frac{15}{2}\left(\frac{\langle 18, -18, 9 \rangle}{\sqrt{18^2 + (-18)^2 + 9^2}}\right)$$

$$= \frac{15}{2}\left(\frac{\langle 18, -18, 9 \rangle}{\sqrt{729}}\right)$$

$$= \frac{15}{2}\left(\frac{\langle 18, -18, 9 \rangle}{27}\right)$$

11-PT solutions

_____ 26. Given the vector **v** lies in the *yz*-plane, has magnitude 8, and makes an angle of 150° with the positive *y*-axis. Find the component form of **v**.

a. $v = \langle 0, -4\sqrt{3}, \pm 4 \rangle$

b. $v = \langle 4\sqrt{3}, 0, 4 \rangle$

c. $v = \langle 0, -4\sqrt{3}, 4 \rangle$

d. $v = \langle -4\sqrt{3}, 0, \pm 4 \rangle$

e. $v = \langle 0, 4\sqrt{3}, \pm 4 \rangle$
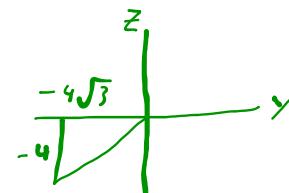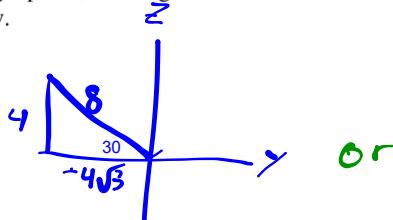
(handwritten: $\langle 0, -4\sqrt{3}, 4 \rangle$ or $\langle 0, -4\sqrt{3}, -4 \rangle$)

_____ 27. Suppose the guy wire to a 100-foot tower has a tension of 500 pounds. Using the distances shown in the figure, write the component form of the vector **F** representing the tension in the wire. Round numerical values in your answer to the nearest integer.

(handwritten annotations: starting point, ending point, labels 100, -40, 70)
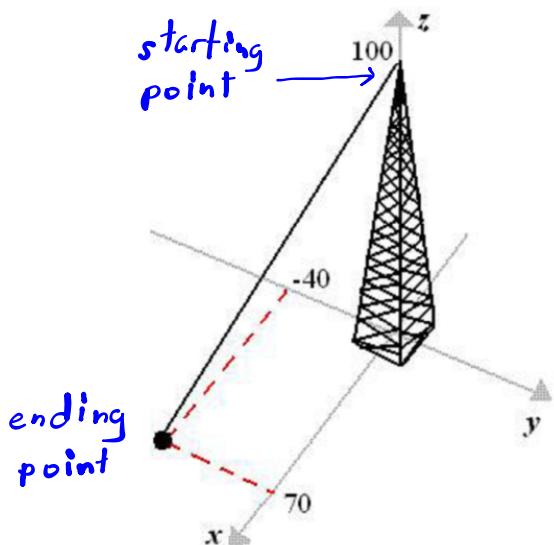
(handwritten work:
$500 = \| c(70i - 40j - 100k) \|$
$250000 = 16500 c^2$
$15.15 \approx c^2$
$c \approx 3.89$
$F \approx 3.89(70i - 40j - 100k)$
$\approx 272i - 156j - 389k$ *)*

a. $F = \langle 302, 186, 419 \rangle$

b. $F = \langle 261, 145, 378 \rangle$

c. $F = \langle 302, 186, 419 \rangle$

d. $F = \langle 272, -156, -389 \rangle$

e. $F = \langle 272, 156, 389 \rangle$

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11-PT solutions

____ 28. Given $\mathbf{u} = \langle 1,3 \rangle$ and $\mathbf{v} = \langle -2,2 \rangle$ find $\mathbf{u} \cdot \mathbf{v}$.

$(1)(-2) + (3)(2)$

$-2 + 6$

    a. −8
    b. −4
    ⓒ 4
    d. 7
    e. −1

____ 29. Given $\mathbf{u} = \langle -4,2 \rangle$, find $\mathbf{u} \cdot \mathbf{u}$.

$(-4)^2 + 2^2 = 20$

    a. −6
    b. 12
    c. −8
    d. −2
    ⓔ 20

____ 30. Given $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 7\mathbf{j} + 2\mathbf{k}$ find $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$.

    ⓐ $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = -15\mathbf{i} + 105\mathbf{j} - 30\mathbf{k}$
    b. $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = -30\mathbf{i} - 15\mathbf{j} + 75\mathbf{k}$
    c. $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 5\mathbf{i} - 35\mathbf{j} + 10\mathbf{k}$
    d. $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 30\mathbf{i} + 15\mathbf{j} - 75\mathbf{k}$
    e. $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 15\mathbf{i} - 105\mathbf{j} + 30\mathbf{k}$

$$\Big((2)(1) + (1)(-7) + (-5)(2)\Big)\Big(\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}\Big)$$

$$-15\left(\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}\right)$$

$$-15\mathbf{i} + 105\mathbf{j} - 30\mathbf{k}$$

____ 31. Find the angle between the vectors for $\mathbf{u}$ and $\mathbf{v}$ given below.

$\mathbf{u} = \langle 1,1 \rangle$, $\mathbf{v} = \langle 3,-1 \rangle$

$$= \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}\right)$$

    ⓐ 63.43°
    b. 24.09°
    c. 26.57°
    d. 69.62°
    e. 84.26°

$$= \cos^{-1}\left(\frac{3 - 1}{\sqrt{2} \ \sqrt{10}}\right) \approx 63.43°$$

____ 32. Find the angle between the vectors for $\mathbf{u}$ and $\mathbf{v}$.

$\mathbf{u} = 6\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$

$$\cos^{-1}\left(\frac{30 + 12}{\sqrt{52} \ \sqrt{34}}\right) \approx 2.73°$$

    a. 48.22°
    ⓑ 2.73°
    c. 87.27°
    d. 44.97°
    e. 88.64°

11-PT solutions

_____ 33. Find the direction cosines of the vector **u** given below.

u $= 4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

a. $\cos(\alpha) = \dfrac{16}{\sqrt{57}}, \cos(\beta) = \dfrac{16}{\sqrt{57}}, \cos(\gamma) = \dfrac{25}{\sqrt{57}}$

b. $\cos(\alpha) = \dfrac{-8}{\sqrt{57}}, \cos(\beta) = \dfrac{-8}{\sqrt{57}}, \cos(\gamma) = \dfrac{-10}{\sqrt{57}}$

c. $\cos(\alpha) = \dfrac{4}{\sqrt{57}}, \cos(\beta) = \dfrac{4}{\sqrt{57}}, \cos(\gamma) = \dfrac{5}{\sqrt{57}}$

d. $\cos(\alpha) = \dfrac{8}{\sqrt{57}}, \cos(\beta) = \dfrac{8}{\sqrt{57}}, \cos(\gamma) = \dfrac{10}{\sqrt{57}}$

e. $\cos(\alpha) = \dfrac{-4}{\sqrt{57}}, \cos(\beta) = \dfrac{-4}{\sqrt{57}}, \cos(\gamma) = \dfrac{-5}{\sqrt{57}}$

Handwritten: $\cos \alpha = \dfrac{4}{\sqrt{57}}$

$\cos \beta = \dfrac{4}{\sqrt{57}}$

$\cos \gamma = \dfrac{5}{\sqrt{57}}$

_____ 34. Given $\mathbf{u} = \langle 9,7 \rangle$ and $\mathbf{v} = \langle 2,3 \rangle$, find the projection of **u** onto **v**.

a. $\langle 27,21 \rangle$

b. $\langle 6,9 \rangle$

c. $\langle 31,25 \rangle$

d. $\langle 10,13 \rangle$

e. $\langle 27,9 \rangle$

Handwritten:
$$W_1 = proj_v u = \left(\dfrac{u \cdot v}{\|v\|^2} \right) v = \dfrac{18+21}{\sqrt{4+9}^2} \langle 2,3 \rangle$$
$$= \dfrac{39}{13} \langle 2,3 \rangle$$
$$= 3 \langle 2,3 \rangle$$

_____ 35. Given $\mathbf{u} = \langle 36,9,0 \rangle$ and $\mathbf{v} = \langle 5,1,-1 \rangle$, find the vector component of **u** orthogonal to **v**. *Handwritten:* $W_2 = u - W_1$

a. $\langle 1,16,-7 \rangle$

b. $\langle 40,8,-8 \rangle$

c. $\langle 71,16,-7 \rangle$

d. $\langle 35,2,-7 \rangle$

e. $\langle -30,-6,6 \rangle$

f. $\langle 1,2,7 \rangle$

Handwritten:
$$W_2 = \langle 36,9,0 \rangle - \dfrac{u \cdot v}{\|v\|^2} v = \langle 36,9,0 \rangle - \dfrac{189}{\sqrt{5^2+1^2+(-1)^2}^2} \langle 5,1,-1 \rangle$$
$$= \langle 36,9,0 \rangle - \dfrac{189}{27} \langle 5,1,-1 \rangle$$
$$= \langle 36,9,0 \rangle - 7 \langle 5,1,-1 \rangle$$
$$= \langle 36,9,0 \rangle - \langle 35,7,-7 \rangle$$
$$= \langle 1,2,7 \rangle$$

Name: _____

_____ 36. Suppose a 48000-pound truck is parked on a 10° slope as shown in the figure. Assume the only force to overcome is that due to gravity. Find the force required to keep the truck from rolling down the hill. Round your answer to two decimal places.

Weight = 48,000 lb
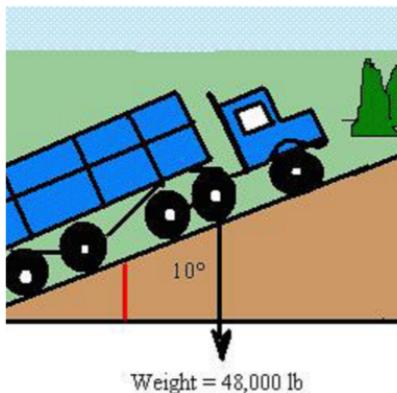
$$proj_v F = \frac{F \cdot v}{\|v\|^2} v = \|F\| \cos \theta$$

$$\|F\| = 48000$$

$$\theta = 80°$$

48000 cos 80°

= 8335.11

F = -48000j
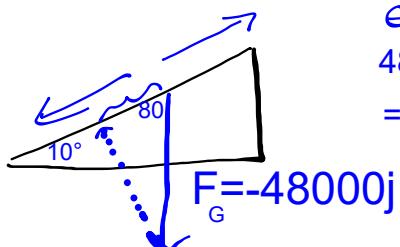
a. 8,354.11 lb
b. 8,335.11 lb
c. 7,994.11 lb
d. 8,392.11 lb
e. 8,517.11 lb

_____ 37. Suppose a 48000-pound truck is parked on a 10° slope as shown in the figure. Assume the only force to overcome is that due to gravity. Find the force perpendicular to the hill. Round your answer to one decimal place.

Weight = 48,000 lb
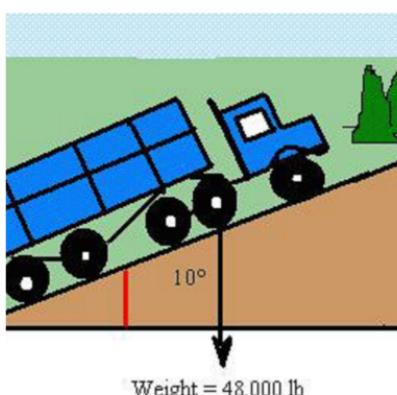
$$-48000 \cos 10°$$
$$or$$
$$-48000 \sin 80°$$

$$\approx 47{,}270 \text{ pounds}$$

a. 47,008.8 lb
b. 47,426.8 lb
c. 47,328.8 lb
d. 47,270.8 lb
e. 47,289.8 lb

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_____ 38. An object is pulled 5 feet horizontally across a floor, using a force of 95 pounds. The direction of the force is 10° above the horizontal. Find the work done. Round your answer to two decimal places.

- a. 467.78 ft-lb
- b. 398.56 ft-lb
- c. 82.48 ft-lb
- d. 6.96 ft-lb
- e. 511.94 ft-lb

The work W done by a constant force $\mathbf{F}$ as its point of application moves along the vector $\overrightarrow{PQ}$ is one of the following.

1. $W = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{F}\|\|\overrightarrow{PQ}\|$ Projection form

2. $W = \mathbf{F} \cdot \overrightarrow{PQ}$ Dot product form

(handwritten: 95, 10°, 5(95 cos 10°))
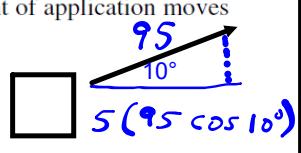
_____ 39. Find the cross product of the unit vector $\mathbf{k} \times \mathbf{i}$.

- a. $\mathbf{j}$
- b. $\mathbf{i}$
- c. $-\mathbf{k}$
- d. $-\mathbf{j}$
- e. $\mathbf{k}$

geometric interpretation using the right hand rule

_____ 40. Given $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$, find $\mathbf{v} \times \mathbf{v}$.

- a. $5\mathbf{i} + 4\mathbf{j}$
- b. 0
- c. $4\mathbf{i} + 5\mathbf{j}$
- d. $4\mathbf{i} - 5\mathbf{j}$
- e. $80\mathbf{k}$

property

_____ 41. Given $\mathbf{u} = \langle 8,3,3\rangle$ and $\mathbf{v} = \langle 2,-1,5\rangle$ find $\mathbf{u} \times \mathbf{v}$.

- a. $\langle 29,-22,6\rangle$
- b. $\langle -18,-68,-14\rangle$
- c. $\langle 18,-34,-14\rangle$
- d. $\langle -18,34,14\rangle$
- e. $\langle 37,-34,-27\rangle$

$$\begin{vmatrix} i & j & k \\ 8 & 3 & 3 \\ 2 & -1 & 5 \end{vmatrix} = (15+3)i - (40-6)j + (-8-6)k$$

$$18i - 34j - 14k$$

$$\langle 18, -34, -14\rangle$$

_____ 42. Given $\mathbf{u} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = -8\mathbf{i} + \mathbf{j} + \mathbf{k}$, find $\mathbf{u} \times \mathbf{v}$.

- a. $6\mathbf{i} - \mathbf{j} + 26\mathbf{k}$
- b. $3\mathbf{i} + \mathbf{j} + 24\mathbf{k}$
- c. $3\mathbf{i} - \mathbf{j} + 25\mathbf{k}$
- d. $4\mathbf{i} + \mathbf{j} - 25\mathbf{k}$
- e. $3\mathbf{i} - \mathbf{j} - 25\mathbf{k}$

$$\begin{vmatrix} i & j & k \\ 1 & 3 & 0 \\ -8 & 1 & 1 \end{vmatrix} = (3-0)i - (1-0)j + (1--24)k$$

$$3i - j + 25k$$

Name: _____   ID: A

_____ 43. Find a unit vector that is orthogonal to both $\mathbf{u} = \langle -16, -12, 8 \rangle$ and $\mathbf{v} = \langle 20, -24, -4 \rangle$.

a. $\left\langle \dfrac{5}{7\sqrt{22}}, \dfrac{2}{7\sqrt{22}}, \dfrac{13}{7\sqrt{22}} \right\rangle$

b. $\left\langle -\dfrac{5}{3\sqrt{22}}, -\dfrac{2}{3\sqrt{22}}, -\dfrac{13}{3\sqrt{22}} \right\rangle$

c. $\left\langle \dfrac{5}{3\sqrt{23}}, \dfrac{2}{3\sqrt{23}}, \dfrac{13}{3\sqrt{23}} \right\rangle$

d. $\left\langle -\dfrac{5}{7\sqrt{23}}, -\dfrac{2}{7\sqrt{23}}, -\dfrac{13}{7\sqrt{23}} \right\rangle$

e. $\left\langle \dfrac{5}{3\sqrt{22}}, \dfrac{2}{3\sqrt{22}}, \dfrac{13}{3\sqrt{22}} \right\rangle$

Handwritten:
$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|}$$

$$\begin{vmatrix} i & j & k \\ -16 & -12 & 8 \\ 20 & -24 & -4 \end{vmatrix} = (48+192)i - (64-160)j + (384+240)k$$

$$= \frac{\langle 240, -96, 624 \rangle \rightarrow \mathbf{u} \times \mathbf{v}}{\sqrt{240^2 + (-96)^2 + (624)^2} \rightarrow \|\mathbf{u} \times \mathbf{v}\|}$$

_____ 44. Find the area of the parallelogram that has the given vectors $\mathbf{u} = \mathbf{j}$ and $\mathbf{v} = 2\mathbf{j} + \mathbf{k}$ as adjacent sides.

a. 2
b. 0
c. 4
d. 1
e. 3

Handwritten:
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = \langle 1, 0, 0 \rangle \qquad \|\mathbf{u} \times \mathbf{v}\| = 1$$

_____ 45. Find the area of a parallelogram that has the given vectors as adjacent sides. Round your answer to two decimal places.

$\mathbf{u} = \langle -7, 9, 5 \rangle$, $\mathbf{v} = \langle -2, -2, 5 \rangle$

Handwritten:
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ -7 & 9 & 5 \\ -2 & -2 & 5 \end{vmatrix} = \langle 55, -25, 32 \rangle$$

a. $\sqrt{9,348}$ or 96.69
b. $\sqrt{4,676}$ or 68.38
c. $\sqrt{4674}$ or 68.37
d. $\sqrt{9,701}$ or 98.49
e. $\sqrt{9,349}$ or 96.69

Handwritten:
$$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{55^2 + (-25)^2 + 32^2} = \sqrt{4674}$$

_____ 46. Find the area of the parallelogram with the vertices $A = (3,1,0)$, $B = (4,3,1)$, $C = (6,4,4)$, and $D = (5,2,3)$.
Round your answer to two decimal places.

a. 35.00
b. 3.00
c. 5.92
d. 7.00
e. 2.65
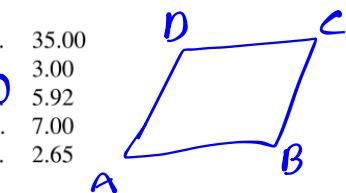
Handwritten:
$$\overrightarrow{AB} = \langle 1, 2, 1 \rangle$$
$$\overrightarrow{AD} = \langle 2, 1, 3 \rangle$$

$$A \times D = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \langle 5, -1, -3 \rangle$$

$$\|A \times B\| = \sqrt{5^2 + (-1)^2 + (-3)^2} = \sqrt{35}$$

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____ 47. Find the area of the triangle with the vertices $A = (0,0,0)$, $B = (3,1,2)$, and $C = (1,1,2)$. Round your answer to two decimal places. half the area of a parallelogram $= \frac{1}{2}\|u \times v\|$

a. 20.00
b. 8.00
c. 6.00
d. 2.24
e. 4.47

$u \times v = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix} = \langle 0, -4, 2 \rangle$

$\frac{1}{2}\|u \times v\| = \frac{1}{2}\sqrt{0^2 + (-4)^2 + 2^2} = \frac{1}{2}\sqrt{20}$

____ 48. A child applies the brakes on a bicycle by applying a downward force of 25 pounds on the pedal when the crank makes a 60° angle with the horizontal (see figure). The crank is 6 inches in length. Find the torque at P. Round your answer to two decimal places.
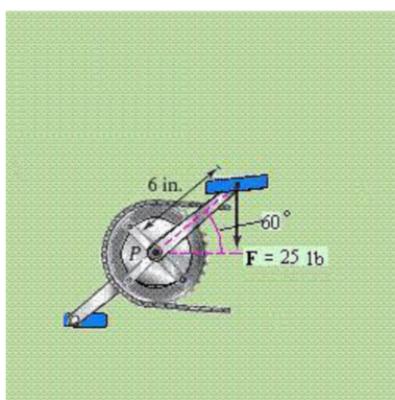
Torque $= \|\vec{PQ} \times F\|$

$F = -25k$

$\vec{PQ} = \frac{1}{2}(\cos 60° j + \sin 60° k)$

$= \frac{1}{4}j + \frac{\sqrt{3}}{4}k$

$\vec{PQ} \times F = \begin{vmatrix} i & j & k \\ 0 & \frac{1}{4} & \frac{\sqrt{3}}{4} \\ 0 & 0 & -25 \end{vmatrix} = -\frac{25}{4}i + 0j + 0k$

6 in.
60°
P
F = 25 lb

torque $= \tau = rF\sin\theta$

Point of application of the force
Lever arm = $r\sin\theta$
r
Applied force
θ
F
Lever arm is measured from the axis of rotation.
Radius from axis of rotation to point of application of the force.
Axis of rotation

a. 12.50 ft-lb
b. 6.25 ft-lb
c. 75.00 ft-lb
d. 10.83 ft-lb
e. 21.65 ft-lb

____ 49. Find the triple scalar product $(u \times v) \cdot w$.

$u = \langle -6, 7, 4 \rangle$, $v = \langle -6, 3, -2 \rangle$, $w = \langle 3, -3, 6 \rangle$

a. −46
b. 46
c. 174
d. 178
e. 348

$\begin{vmatrix} i & j & k \\ -6 & 7 & 4 \\ -6 & 3 & -2 \end{vmatrix} = \langle -26, -36, 24 \rangle$

$\langle -26, -36, 24 \rangle \cdot \langle 3, -3, 6 \rangle = 174$
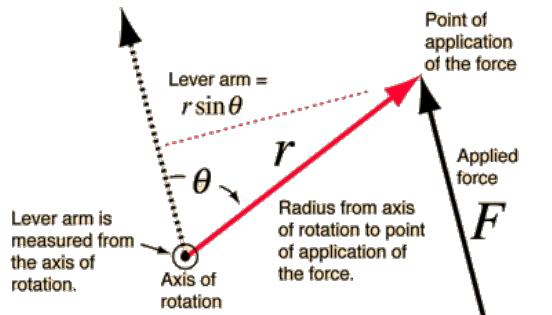
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_____ 50. Use the triple scalar product to find the volume of the parallelepiped having adjacent edges given by the vectors

$$V = |u \cdot v \times w|$$

$\mathbf{u} = \langle -4,9,5 \rangle, \quad \mathbf{v} = \langle 3,5,-1 \rangle, \quad \mathbf{w} = \langle 8,-8,7 \rangle$

a. 220
b. 689
c. 692
d. 516
e. 344

$$\begin{vmatrix} -4 & 9 & 5 \\ 3 & 5 & -1 \\ 8 & -8 & 7 \end{vmatrix} = (27)(-4) - (29)(9) + (-64)(5) = -689$$

_____ 51. Find the volume of the parallelepiped with the following vertices.

$(0,0,0), (5,0,0), (0,9,3), (5,9,3), (2,0,4), (7,0,4), (2,9,7), (7,9,7)$

a. 180
b. 167
c. 60
d. 30
e. 22

$$\begin{vmatrix} 5 & 0 & 0 \\ 0 & 9 & 3 \\ 2 & 0 & 4 \end{vmatrix} = (36)(5) - (-6)(0) + (-18)(0)$$

_____ 52. Find a set of parametric equations of the line through the point $(-7,5,4)$ parallel to the vector $\mathbf{v} = \langle 8,5,2 \rangle$.

a. $x = -7 + 8s, y = -5 + 5s, z = 4 + 2s$
b. $x = -7 - 8s, z = 5 + 3s, y = -4 + 8s$
c. $x = -7 - 8s, y = 5 + 3s, z = 4 + 8s$
d. $x = -7 - 8s, y = 5 + 5s, z = 4 - 2s$
e. $x = -7 + 8s, y = 5 + 5s, z = 4 + 2s$

$x = -7 + 8s$

$y = 5 + 5s$

$z = 4 + 2s$

_____ 53. Find a set of symmetric equations of the line through the point $(8,5,3)$ parallel to the vector $\mathbf{v} = \langle 2,4,4 \rangle$.

a. $\dfrac{x-8}{-3} = \dfrac{z-5}{4} = \dfrac{y-3}{-7}$

b. $\dfrac{x-8}{2} = \dfrac{y-5}{4} = \dfrac{z-3}{4}$

c. $\dfrac{x-8}{2} = \dfrac{y+5}{4} = \dfrac{z+3}{4}$

d. $\dfrac{x-8}{-3} = \dfrac{y-5}{4} = \dfrac{z-3}{-7}$

e. $\dfrac{x-8}{2} = \dfrac{y+5}{4} = \dfrac{z-3}{4}$

$$\frac{x-8}{2} = \frac{y-5}{4} = \frac{z-3}{4}$$

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____ 54. Find the set of parametric equations of the line through the point $(-3,6,3)$ that is parallel to the line

$x = -8 + 5t$, $y = -3 + 8t$, and $z = 4 + 4t$.

$$\langle a, b, c \rangle = \langle 5, 8, 4 \rangle$$

a. $x = -3 - 5t$, $y = 6 + 8t$, $z = 3 - 4t$

b. $x = -3 - 8t$, $z = 6 - 3t$, $y = 3 + 4t$

c. $x = -3 + 5t$, $y = -6 + 8t$, $z = 3 + 4t$

d. $x = -3 + 5t$, $y = 6 + 8t$, $z = 3 + 4t$

e. $x = -3 - 8t$, $y = 6 - 3t$, $z = 3 + 4t$

$x = -3 + 5t$
$y = 6 + 8t$
$z = 3 + 4t$

____ 55. Find a set of parametric equations of the line through the points $(7,8,3)$ and $(1,-7,-6)$.

P Q

a. $x = 7 - 2t$, $y = 8 + 5t$, $z = 3 - 3t$

b. $x = 7 + 7t$, $y = 8 + 4t$, $z = 3 + 8t$

c. $x = 7 + 2t$, $y = -8 + 5t$, $z = 3 + 3t$

d. $x = 7 + 7t$, $z = 8 + 4t$, $y = 3 + 8t$

equivalent e. $x = 7 - 6t$, $y = 8 - 15t$, $z = 3 - 9t$

$\overrightarrow{PQ} = \langle 6, 15, 9 \rangle$
  a   b   c

$\rightarrow x = 7 + 6t$, $y = 8 + 15t$, $z = 3 + 9t$

____ 56. Find a set of symmetric equations of the line through the points $(8,6,3)$ and $(0,-1,-1)$.

P Q

a. $\dfrac{x-8}{8} = \dfrac{y-6}{7} = \dfrac{z-3}{4}$

b. $\dfrac{x-8}{3} = \dfrac{y-6}{9} = \dfrac{z-3}{-5}$

c. $\dfrac{x-8}{8} = \dfrac{y+6}{7} = \dfrac{z-3}{4}$

d. $\dfrac{x+8}{8} = \dfrac{y+6}{7} = \dfrac{z-3}{4}$

e. $\dfrac{x-8}{3} = \dfrac{z-6}{9} = \dfrac{y-3}{-5}$

$\overrightarrow{PQ} = \langle 8, 7, 4 \rangle = \langle a, b, c \rangle$

$$\frac{x-8}{8} = \frac{y-6}{7} = \frac{z-3}{4}$$

____ 57. Find a set of parametric equations of the line that passes through the point $(-2,5,-1)$ and is parallel to the xy-plane and yz-plane.

a. $x = -2$, $y = 5 + t$, $z = -1$

b. $x = -2 + t$, $y = 5$, $z = -1 + t$

c. $x = -2$, $y = 5t$, $z = -1$

d. $x = -2t$, $y = 5$, $z = -t$

e. $x = -2 + t$, $y = 5 + t$, $z = -1 + t$

$\langle 0, 1, 0 \rangle$
  a b c

$x = -2 + 0t$
$y = 5 + t$
$z = -1 + 0t$
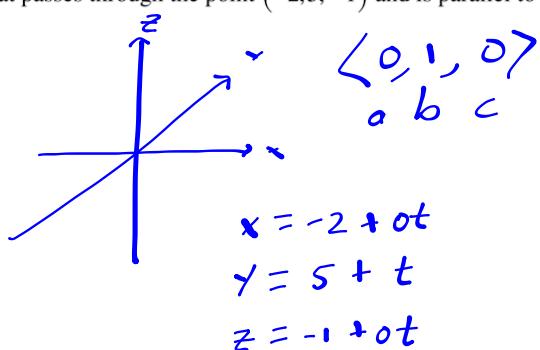
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____ 58. Determine whether the lines given below intersect, and, if so, find the point of intersection.

$x = -6 + 2t,\ y = 9 + 4t,\ z = -13 + 3t$

$x = 4 + 2s,\ y = 25 + 2s,\ z = 2 + 3s$

$-6 + 2t = 4 + 2s$

$-9 + 4t = 25 + 2s$

_____

$-15 - 2t = -21$

$-2t = -6$

$t = 3$

$s = -2$

$x = 0$

$y = -21$

$z = -4$

a. The lines intersect at $(-4, 24, -4)$.
b. The lines are parallel.
c. The lines intersect at $(3, 24, -7)$.
d. The lines intersect at $(0, 21, -4)$.
e. The lines are not parallel but do not intersect.

____ 59. Determine whether the lines given below intersect, and, if so, find the point of intersection.

$\dfrac{x-1}{5} = \dfrac{y-2}{7} = \dfrac{z+8}{5}$

$\dfrac{x-3}{1} = \dfrac{x-15}{-2} = \dfrac{z-5}{-2}$

$x = 1 + 5t \qquad y = 2 + 7t \qquad z = -8 + 5t$

$x = 3 + s \qquad y = 15 - 2s \qquad z = 5 - 2s$

$2 + 7t = 15 - 2s$

$+8 + 5t = -5 + 2s$

_____

$10 + 2t = 10$

$t = 0$

$s = \tfrac{1}{2}$

a. The lines are parallel.
b. The lines intersect at $(2, 6, -33)$.
c. The lines intersect at $(7, 11, -3)$.
?? d. The lines intersect at $(6, 9, -1)$.
e. The lines are not parallel but do not intersect.

____ 60. Find an equation of a plane passing through the given point and perpendicular to the given vector.

Point: $(3, -1, -3)$ Vector: $\mathbf{v} = \langle 6, 5, 4 \rangle$

> The plane containing the point (x_1, y_1, z_1) and having normal vector
> $\mathbf{n} = \langle a, b, c \rangle$
> can be represented by the **standard form** of the equation of a plane
> $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$

a. $6x - 5y + 4z - 1 = 0$
b. $6x + 5y - 4z + 1 = 0$
c. $6x + 5y + 4z - 1 = 0$
d. $6x + 5y + 4z + 1 = 0$
e. $6x + 5y + 4z - 3 = 0$

$6(x-3) + 5(y+1) + 4(z+3) = 0$

$6x + 5y + 4z - 1 = 0$

____ 61. Find an equation of a plane passing through the following three points.

$\quad A \qquad\quad B \qquad\quad C$

$(1, 2, -2),\ (-4, 2, 5),\ (3, 4, 5)$

$\overrightarrow{AB} \quad \langle -5, 0, 7 \rangle \qquad \overrightarrow{AC} \quad \langle 2, 2, 7 \rangle \qquad$ we need a normal vector

a. $14x + 49y + 10z + 104 = 0$
b. $14x - 49y + 10z - 104 = 0$
c. $-14x - 49y + 10z + 104 = 0$
d. $14x - 49y - 10z + 104 = 0$
e. $14x - 49y + 10z + 104 = 0$

$\begin{vmatrix} i & j & k \\ -5 & 0 & 7 \\ 2 & 2 & 7 \end{vmatrix} = \langle -14, +49, -10 \rangle$

$-14(x-3) + 49(y-4) - 10(z-5) = 0$

Name: _____

let v be the vector from (-1, -1, -7) to
(2, 3, 40)   v = 3i +4j +47k

let n be a vector normal to the plane
3x + y +5z + = 0

because v and n both lie in plane P,
the normal vector to P is v x n

____ 62. Find an equation of a plane passing through the points

$$(2,3,40), \ (-1,-1,-7)$$

and perpendicular to the plane

$$3x+y+5z+8=0.$$

a. $3x-14y+z=-4$
b. $3x-14y+2z-4=0$
c. $3x-14y+z-4=0$
d. $3x-14y+2z=-4$
e. $3x-14y+z=0$

$$v \times n = \begin{vmatrix} i & j & k \\ 3 & 4 & 47 \\ 3 & 1 & 5 \end{vmatrix} = \langle -27, 126, -9 \rangle$$

$$-27(x+1) + 126(y+1) - 9(z+7) = 0$$
$$3(x+1) - 14(y+1) + (z+7) = 0$$
$$3x + 3 - 14y - 14 + z + 7 = 0$$
$$3x - 14y + z = 4$$

____ 63. Determine whether the following planes are parallel, orthogonal, or neither. If they are neither parallel nor orthogonal, find the angle of intersection.

$$-2x - 31y + 5z + 3 = 0 \quad n_1 = \langle -2, -31, 5 \rangle$$
$$-3x + y + 5z - 4 = 0 \quad n_2 = \langle -3, 1, 5 \rangle$$

$$\cos \theta = \frac{|n_1 \cdot n_2|}{\|n_1\|\|n_2\|} = 0 \quad \text{so } \theta = 90°$$

a. The planes are neither parallel nor orthogonal, the angle of intersection is 15°.
b. The planes are parallel.
c. The planes are neither parallel nor orthogonal, the angle of intersection is 30°.
d. The planes are orthogonal.

____ 64. Find the distance between the point $(2,-2,-3)$ and the plane $2x - 3y + 4z = 8$.

a. $\dfrac{15}{\sqrt{29}}$

b. $\dfrac{-10}{\sqrt{29}}$

c. $\dfrac{10}{\sqrt{29}}$

d. $\dfrac{20}{\sqrt{29}}$

e. $\dfrac{-20}{\sqrt{29}}$

The distance between a plane and a point Q (not in the plane) is

$$D = \|\text{proj}_n \overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot n|}{\|n\|}$$

where P is a point in the plane and $\mathbf{n}$ is normal to the plane.

$$n = \langle 2, -3, 4 \rangle$$
$$Q = (2, -2, -3)$$
$$P = (4, 0, 0)$$
$$\overrightarrow{PQ} = \langle -2, -2, -3 \rangle$$

$$\frac{|\overrightarrow{PQ} \cdot n|}{\|n\|} = \frac{|-10|}{\sqrt{29}} = \frac{10}{\sqrt{29}}$$

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Name: _____

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

____ 65. Find the distance between the planes given below.

$4x - 4y + 5z - 104 = 0$ ◄▪▪▪► point Q on the plane (26, 0, 0)

$20x - 20y + 25z - 100 = 0$ ◄▪▪▪► $\langle a, b, c \rangle = \langle 20, -20, 25 \rangle$

a. $\dfrac{-84}{\sqrt{57}}$

$D = \dfrac{|20(26) + (-20)(0) + (25)(0) - 100|}{\sqrt{(20)^2 + (-20)^2 + (25)^2}} = \dfrac{84}{\sqrt{57}}$

b. $\dfrac{84}{\sqrt{57}}$ ⟵(circled)

or just extend the concept from #64
by finding a point from the first plane

c. $\dfrac{89}{\sqrt{57}}$

The distance between a plane and a point Q (not in the plane) is

$$D = \|\text{proj}_{\mathbf{n}}\overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

$= \dfrac{420}{\sqrt{1425}} = \dfrac{84}{\sqrt{57}}$

d. $\dfrac{-168}{\sqrt{57}}$

where P is a point in the plane and $\mathbf{n}$ is normal to the plane.

e. $\dfrac{168}{\sqrt{57}}$

$Q = (26, 0, 0)$ $P = (5, 0, 0)$ $n = \langle 20, -20, 25 \rangle$

____ 66. Find the distance between the point $(1, 1, 1)$ and the line given by the set of parametric equations

$x = 1 - 3t,\ y = 3 + t,\ z = -t.$

$D = \dfrac{\|\overrightarrow{PQ} \times u\|}{\|u\|}$ $u = \langle -3, 1, -1 \rangle$ $Q = (1,1,1)$

a. $5\sqrt{22}/11$

to find a point on the line, let $t = 0$

b. 2

$P = (1, 3, 0)$

c. $3\sqrt{110}/11$ ⟵

so $\overrightarrow{PQ} = \langle 1-1, 1-3, 1-0 \rangle = \langle 0, -2, 1 \rangle$

d. 4

$\overrightarrow{PQ} \times u = \begin{vmatrix} i & j & k \\ 0 & -2 & 1 \\ -3 & 1 & -1 \end{vmatrix} = \langle 1, -3, -6 \rangle = \dfrac{\sqrt{46}}{\sqrt{11}}$

e. 1

____ 67. Identify the following quadric surface.

$$z = \frac{x^2}{6} - \frac{y^2}{16}$$

Hyperbolic Paraboloid

a. hyperboloid of one sheet
b. elliptic cone
c. elliptic paraboloid
d. hyperboloid of two sheets
e. hyperbolic paraboloid ⟵(circled)

$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

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_____ 68. Identify the following quadric surface.

$$\frac{x^2}{3} + \frac{y^2}{2} - \frac{z^2}{4} = 0$$

Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

    (a.) Elliptic cone
    b. Hyperboloid of two sheets
    c. Elliptic paraboloid
    d. ellipsoid
    e. Hyperboloid of one sheet

_____ 69. Find an equation for the surface of revolution generated by revolving the curve $xz = 3$ in the xz-plane about the z-axis.

a. $y^2 + z^2 = \dfrac{9}{x^2}$

1. Revolved about the x-axis: $y^2 + z^2 = [r(x)]^2$
2. Revolved about the y-axis: $x^2 + z^2 = [r(y)]^2$
3. Revolved about the z-axis: $x^2 + y^2 = [r(z)]^2$

b. $z^2 + x^2 = \dfrac{9}{y^2}$

$$x = \frac{3}{z}$$

c. $y^2 + z^2 = \dfrac{3}{x^2}$

$$x^2 + y^2 = \left(\frac{3}{z}\right)^2$$

d. $y^2 + x^2 = \dfrac{3}{z}$

(e.) $y^2 + x^2 = \dfrac{9}{z^2}$

_____ 70. Given the equation of a surface of revolution $x^2 + z^2 - 3y = 0$, find the equation of its generating curve in the yz-plane rotating about the y-axis.

    (a.) $x = \sqrt{3y}$ or $z = \sqrt{3y}$

1. Revolved about the x-axis: $y^2 + z^2 = [r(x)]^2$
2. Revolved about the y-axis: $x^2 + z^2 = [r(y)]^2$
3. Revolved about the z-axis: $x^2 + y^2 = [r(z)]^2$

    b. $x^2 + z^2 = 9y^2$

    c. $x^2 + z^2 = \sqrt{3y}$

$$x^2 + z^2 = \sqrt{3y}^2$$

    d. $x = 9y^2$ or $z = 9z^2$

$$r(y) = \sqrt{3y}$$

    e. $x = 3y$ or $z = 3y$

$$so \quad x = \sqrt{3y} \quad or \quad z = \sqrt{3y}$$

11-PT solutions

_____ 71. Convert the following point from cylindrical coordinates to rectangular coordinates.

$$\left(4, \frac{\pi}{3}, 2\right) = (r, \theta, z)$$

$x = r\cos\theta = 4\cos\frac{\pi}{3} = 2$

$y = r\sin\theta = 4\sin\frac{\pi}{3} = 2\sqrt{3}$

$z = z = 2$

- a. $\left(4, 4\sqrt{3}, 2\right)$
- b. $\left(8, 8\sqrt{3}, 2\right)$
- c. $\left(2\sqrt{3}, 2, 2\right)$
- d. $\left(2, 2\sqrt{3}, 2\right)$
- e. $\left(4\sqrt{3}, 4, 2\right)$

_____ 72. Convert the following point from rectangular coordinates to cylindrical coordinates. Give any angles in radians.

$$(-4, -4, 5) = (x, y, z)$$

$\tan\theta = \frac{y}{x}$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

$(-4, -4)$ is in quadrant 3

so choose $\frac{5\pi}{4}$

$r^2 = x^2 + y^2$

$r^2 = (-4)^2 + (-4)^2$

$4\sqrt{2} = r$

- a. $\left(4\sqrt{2}, \frac{5\pi}{2}, 5\right)$
- b. $\left(8\sqrt{2}, \frac{5\pi}{4}, 5\right)$
- c. $\left(4\sqrt{2}, \frac{5\pi}{4}, 5\right)$
- d. $\left(4, \frac{5\pi}{4}, 5\right)$
- e. $\left(4, \frac{5\pi}{2}, 5\right)$

11-PT solutions

_____ 73. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

$z = 9x^2 + 9y^2 - 2$

     a. $z = 9r + 2$

     b. $z = 9r^2 - 2$

     c. $z^2 = 9r - 2$

     d. $z = \dfrac{9}{2}r^2 + 2$

     e. $z = \dfrac{9}{2}r^2 - 2$

Handwritten:
$$x^2 + y^2 = \frac{z+2}{9}$$
$$r^2 = \frac{z+2}{9}$$
$$z = 9r^2 - 2$$

_____ 74. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

$9x^2 + 9y^2 = 8x$

     a. $r^2 = \dfrac{8}{9}\sin\theta$

     b. $r = \dfrac{8}{9}\sin\theta$

     c. $r = \dfrac{8}{9}\cos^{-1}\theta$

     d. $r = \dfrac{8}{9}\cos\theta$

     e. $r = \dfrac{9}{8}\cos\theta$

Handwritten:
$$x^2 + y^2 = \frac{8}{9}x$$
$$r^2 = \frac{8}{9}r\cos\theta$$
$$r = \frac{8}{9}\cos\theta$$

11-PT solutions

____ 75. Find an equation in rectangular coordinates for the equation given in cylindrical coordinates.

$r = 6\sin\theta$

$r^2 = 6r\sin\theta$

$x^2 + y^2 = 6y$

a. $x^2 + y^2 = 6y$

b. $x^2 + y^2 = 6x$

c. $x^2 + y^2 = \dfrac{x}{6}$

d. $x^2 + y^2 = \dfrac{y}{6}$

e. $x^2 + z^2 = 6x$

____ 76. Find an equation in rectangular coordinates for the equation given in cylindrical coordinates.

$r = 4z$

$r^2 = 16z^2$

a. $x^2 + y^2 = \dfrac{z}{4}$

b. $x^2 + y^2 = \dfrac{y}{4}$

$x^2 + y^2 = 16z^2$

c. $x^2 + z^2 = 4y$

d. $x^2 + y^2 = 16z^2$

e. $x^2 + y^2 = 4z$

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11-PT solutions

_____ 77. Find an equation in rectangular coordinates for the equation given in cylindrical coordinates.

$r^2 + z^2 = 4$ $x^2 + y^2 + z^2 = 4$

   a. $x + y + z = 2$

   (b.) $x^2 + y^2 + z^2 = 4$

   c. $x^2 + y^2 + \left(\dfrac{z}{2}\right)^2 = 0$

   d. $x^2 + y^2 = -4z^2$

   e. $x^2 + y^2 + z^2 = 2$

_____ 78. Convert the point $\left(-7, 7\sqrt{3}, 14\right)$ from rectangular coordinates to spherical coordinates.

   a. $\left(14\sqrt{2}, \dfrac{\pi}{4}, \dfrac{2\pi}{3}\right)$ $\rho^2 = x^2 + y^2 + z^2, \quad \tan\theta = \dfrac{y}{x}, \quad \phi = \arccos\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

   b. $\left(28\sqrt{2}, \dfrac{2\pi}{3}, \dfrac{\pi}{4}\right)$ $\rho^2 = (-7)^2 + (7\sqrt{3})^2 + (14)^2 = 392$

   (c.) $\left(14\sqrt{2}, \dfrac{2\pi}{3}, \dfrac{\pi}{4}\right)$ $\rho = 14\sqrt{2}$

$\tan\theta = \dfrac{7\sqrt{3}}{-7}$

   d. $\left(28\sqrt{2}, \dfrac{\pi}{4}, \dfrac{2\pi}{3}\right)$ $\theta = \dfrac{2\pi}{3}$

   e. $\left(14\sqrt{2}, \dfrac{\pi}{3}, \dfrac{\pi}{4}\right)$ $\phi = \arccos\left(\dfrac{14}{\sqrt{(-7)^2 + (7\sqrt{3})^2 + (14)^2}}\right) = \dfrac{14}{14\sqrt{2}} = \dfrac{\pi}{4}$

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11-PT solutions

____ 79. Convert the point from spherical coordinates to rectangular coordinates.

$$\left(9, \frac{2\pi}{3}, \frac{5\pi}{6}\right)$$

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$$

a. $\left(\frac{9}{4}, -\frac{9\sqrt{3}}{4}, \frac{9\sqrt{3}}{2}\right)$

b. $\left(\frac{9}{4}, \frac{9\sqrt{3}}{4}, \frac{9\sqrt{3}}{2}\right)$

c. $\left(-\frac{9}{4}, \frac{9\sqrt{3}}{4}, -\frac{9\sqrt{3}}{2}\right)$ (circled)

d. $\left(\frac{9}{4}, -\frac{9\sqrt{3}}{4}, -\frac{9\sqrt{3}}{2}\right)$

e. $\left(-\frac{9}{4}, -\frac{9\sqrt{3}}{4}, \frac{9\sqrt{3}}{2}\right)$

Handwritten work:

$x = 9 \sin \frac{5\pi}{6} \cos \frac{2\pi}{3} = 9\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{9}{4}$

$y = 9 \sin \frac{5\pi}{6} \sin \frac{2\pi}{3} = 9\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{9\sqrt{3}}{4}$

$z = 9 \cos \frac{5\pi}{6} = 9\left(\frac{-\sqrt{3}}{2}\right) = \frac{-9\sqrt{3}}{2}$

$\left(-\frac{9}{4}, \frac{9\sqrt{3}}{4}, \frac{-9\sqrt{3}}{2}\right)$

____ 80. Find an equation in spherical coordinates for the equation given in rectangular coordinates.

$y = 4$

a. $\rho = 4\cos \phi \cos \theta$
b. $\rho = 4\sec \phi \sec \theta$
c. $\rho = 4\sin \phi \sin \theta$
d. $\rho = 4\sec \phi \csc \theta$
e. $\rho = 4\csc \phi \csc \theta$ (circled)

Handwritten work:

$y = \rho \sin \phi \sin \theta$

$4 = \rho \sin \phi \sin \theta$

$\rho = 4 \csc \phi \csc \theta$

_____ 81. Find an equation in spherical coordinates for the equation given in rectangular coordinates.

$$x^2 + y^2 - 4z^2 = 3$$

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$$

a. $\rho^2 = \dfrac{3}{\left(\sin^2 \phi - 4\cos^2 \phi\right)}$

b. $\rho^2 = \dfrac{3}{\left(\sin^2 \theta - 4\cos^2 \theta\right)}$

c. $\rho^2 = \dfrac{3}{\left(\sin^2 \phi - 4\cos^2 \theta\right)}$

d. $\rho^2 = \dfrac{3}{\left(\cos^2 \phi - 4\sin^2 \phi\right)}$

e. $\rho = -\sqrt{\dfrac{3}{4}}$

Handwritten work:

$\rho^2 \sin\phi^2 \cos\theta^2 + r^2 \sin\phi s \sin\theta^2 - 4\rho^2 \cos\phi^2 = 3$

$\rho^2 \left(\sin^2\phi \cos^2\theta + \sin^2\phi \sin^2\theta - 4\cos^2\phi \right) = 3$

$\rho^2 \left(\sin^2\phi \left(\cos^2\theta + \sin^2\theta \right) - 4\cos^2\phi \right) = 3$

$\rho^2 = \dfrac{3}{\sin^2\phi - 4\cos^2\phi}$

_____ 82. Find an equation in rectangular coordinates for the equation given in spherical coordinates. Express the coefficients to two decimal places.

$$\theta = \frac{\pi}{16}$$

a. $y = 0.20x$
b. $x = 0.20y$
c. $y = 5.03x$
d. $z = 0.20x$
e. $y = 0.20z$

Handwritten work:

$\tan \theta = \dfrac{y}{x}$

$\tan \dfrac{\pi}{16} = \dfrac{y}{x}$

$0.20 = \dfrac{y}{x}$

11-PT solutions

_____ 83. Find an equation in rectangular coordinates for the equation given in spherical coordinates.

$\rho = -7 \csc \phi \sec \theta$

$-7 = \rho \sin \phi \cos \theta$

    a. $y = -7$

    b. $x = -\dfrac{1}{7}$

$-7 = x$

    c. $x = -7$

    d. $z = -7$

    e. $y = -\dfrac{1}{7}$

_____ 84. Convert the following point from spherical coordinates to cylindrical coordinates.

$\left(7, \dfrac{\pi}{4}, \dfrac{5\pi}{6}\right)$

$\rho \quad \theta \quad \phi$

$r^2 = \rho^2 \sin^2 \phi, \qquad \theta = \theta, \qquad z = \rho \cos \phi$

$r = \sqrt{7^2 \sin^2\left(\dfrac{5\pi}{6}\right)} = \sqrt{49\left(\dfrac{1}{4}\right)} = \dfrac{7}{2}$

    a. $\left(\dfrac{7}{2}, \dfrac{\pi}{4}, \dfrac{-7\sqrt{3}}{2}\right)$

$\theta = \dfrac{\pi}{4}$

    b. $\left(\dfrac{7}{2}, \dfrac{\pi}{4}, \dfrac{7\sqrt{3}}{2}\right)$

$z = 7\cos\dfrac{5\pi}{6} = 7\left(\dfrac{-\sqrt{3}}{2}\right) = \dfrac{-7\sqrt{3}}{2}$

    c. $\left(7, \dfrac{\pi}{4}, \dfrac{-7\sqrt{3}}{2}\right)$

    d. $\left(\dfrac{7\sqrt{3}}{2}, \dfrac{\pi}{4}, \dfrac{7}{2}\right)$

    e. $\left(\dfrac{-7\sqrt{3}}{2}, \dfrac{\pi}{4}, \dfrac{7}{2}\right)$

11-PT solutions

Multivariable Calculus Chapter 11 Practice Test
Answer Section

MULTIPLE CHOICE

1. C
2. B
3. E
4. E
5. C
6. E
7. C
8. D
9. D
10. E
11. C
12. A
13. E
14. E
15. A
16. E
17. E
18. A
19. A
20. D
21. B
22. C
23. A
24. A
25. A
26. A
27. D
28. C
29. E
30. A
31. A
32. B
33. C
34. B
35. D
36. B
37. D
38. A
39. A

11-PT solutions

40. B
41. C
42. C
43. E
44. D
45. C
46. C
47. D
48. B
49. C
50. B
51. A
52. E
53. B
54. D
55. E
56. A
57. A
58. D
59. D
60. C
61. E
62. C
63. D
64. C
65. B
66. C
67. E
68. A
69. E
70. A
71. D
72. C
73. B
74. D
75. A
76. D
77. B
78. C
79. C
80. E
81. A
82. A
83. C
84. A