

AP Calculus AB Chapter 6 Test (Practice)

1. Find the particular solution of the differential equation $3x + 20yy' = 0$ that satisfies the initial condition $y = 5$ when $x = 2$, where $3x^2 + 20y^2 = C$ is the general solution.

a. $3x^2 + 20y^2 = 504$ b. $3x^2 + 20y^2 = 155$ c. $3x^2 + 20y^2 = 112$ d. $3x^2 + 20y^2 = 37$ e. $3x^2 + 20y^2 = 512$

$$3(2)^2 + 20(5)^2 = C \quad C = 512$$

$$12 + 500 = C$$

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2. Use integration to find a general solution of the differential equation.

$$\int \frac{dy}{dx} = \int x\sqrt{x+2} \, dx$$

a. $y = \frac{2}{5}(x+2)^{3/2} - \frac{4}{3}(x+2)^{5/2} + C$ b. $y = \frac{2}{5}(x+2)^{3/2} - (x+2)^{5/2} + C$
 c. $y = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$ d. $y = \frac{2}{5}(x+2)^{3/2} + \frac{4}{3}(x+2)^{5/2} + C$
 e. $y = \frac{2}{5}(x+2)^2 - \frac{4}{3}(x+2) + C$

$$u = x+2$$

$$du = dx$$

$$x = u-2$$

$$\int dy = \int(u-2)u^{1/2} du$$

$$\int dy = \int(u^{3/2} - 2u^{1/2}) du$$

$$y = \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$

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3. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x\sqrt{4-x^2}$$

a. $y = -\frac{1}{3}x(4-x^2)^{\frac{3}{2}} + C$

b. $y = -\frac{1}{5}x(4-x^2)^{\frac{5}{2}} + C$

c. $y = -\frac{1}{3}(4-x^2)^{\frac{3}{2}} + C$

d. $y = \frac{1}{5}x(4-x^2)^{\frac{5}{2}} + C$

e. $y = \frac{1}{3}(4-x^2)^{\frac{3}{2}} + C$

$$dy = x\sqrt{4-x^2} dx$$

$$= -\frac{1}{2} \int \sqrt{u} du$$

$$U = 4 - x^2$$

$$dv = -2x dx$$

$$-\frac{1}{2} dv = x dx$$

$$y = -\frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} \right) + C$$

$$y = -\frac{1}{3} (4-x^2)^{\frac{3}{2}} + C$$

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4. The initial investment in a savings account in which interest is compounded continuously is \$604. If the time required to double the amount is $9\frac{1}{2}$ years, what is the amount after 15 years? Round your answer to the nearest cent.

- a. \$1,917.58 b. \$1,804.46 c. \$1,907.37
d. \$1,404.46 e. \$8,278.18

$$y = Ce^{kt}$$

$$604 = Ce^{k0}$$

$$C = 604$$

$$1208 = Ce^{k9.5}$$

$$\frac{1208}{604} = e^{k9.5}$$

$$\ln 2 = \frac{k9.5}{9.5}$$

$$k = 0.0729$$

$$y = 604e^{0.0729(15)}$$

$$y = 1804.46$$

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5. Find the orthogonal trajectories of the family

$$y = Ce^{8x}$$

- a. $8y^2 = -2x + C$ b. $\ln y = 8x + C$ c. $y = 8Ce^{8x}$
d. $y = ke^{-8x}$ e. $y = C \ln(8x)$

$$C = \frac{y}{e^{8x}}$$

$$\int 8y \, dy = \int -1 \, dx$$

$$\frac{dy}{dx} = -\frac{1}{8y}$$

$$\frac{8y^2}{2} = -x + C$$

$$8y^2 = -2x + C$$

$$y' = 8Ce^{8x}$$

$$\frac{dy}{dx} = -\frac{1}{8Ce^{8x}} = -\frac{1}{8(\frac{y}{e^{8x}})e^{8x}} = -\frac{1}{8y}$$

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6. The rate of change of N is proportional to N . When $t = 0$, $N = 200$ and when $t = 1$, $N = 360$. What is the value of N when $t = 4$? Round your answer to three decimal places.

- a. 2,129.520 b. 2,099.520 c. 2,049.520
d. 491.383 e. 262,440.000

$$\frac{dN}{dt} = kN$$

$$\int \frac{dN}{N} = \int k \, dt$$

$$\ln|N| = kt + C$$

$$\begin{aligned} N &= Ce^{kt} \\ 360 &= 200e^{k \cdot 1} \\ 1.8 &= e^k \\ k &= \ln(1.8) \\ &\approx 0.58779 \end{aligned}$$

$$\begin{aligned} N &= 200e^{0.58779t} \\ N &= 200e^{0.58779 \cdot 4} \\ &\approx 200e^{2.35116} \\ &\approx 200 \cdot 10.000 \\ &\approx 2000 \end{aligned}$$

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7. Solve the differential equation.

$$y' = x(1+y)$$

- a. $\ln|1+y| = x^2 + C$ b. $3\ln|1+y| = x^3 + C$
 c. $4\ln|1+y| = x^4 + C$ d. $\ln|1+y| = x + C$
 e. $2\ln|1+y| = x^2 + C$

$$\frac{dy}{dx} = x(1+y)$$

$$\frac{dy}{1+y} = x dx$$

$$\ln|1+y| = \frac{x^2}{2} + C$$

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8. The isotope ^{14}C has a half-life of 5,715 years.
 Given an initial amount of 11 grams of the isotope,
 how many grams will remain after 500 years? After
 5,000 years? Round your answers to four decimal
 places.
- a. 7.2469 gm, 4.1988 gm b. 6.2117 gm, 3.5989 gm
 c. 10.3528 gm, 5.9982 gm d. 4.1411 gm,
 2.3993 gm e. 12.4233 gm, 7.1979 gm

half-life

$$y_{\text{final}} = 11 \cdot \left(\frac{1}{2}\right)^{\frac{500}{5715}}$$

$$b. y_{\text{final}} = 11 \cdot \left(\frac{1}{2}\right)^{\frac{5000}{5715}}$$

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9. The isotope ^{14}C has a half-life of 5,715 years. After 2,000 years, a sample of the isotope is reduced to 1.2 grams. What was the initial size of the sample (in grams)? How much will remain after 20,000 years (i.e., after another 18,000 years)? Round your answers to four decimal places.

- a. 1.0706, 0.0947 b. 2.4471, 0.2164
 c. 1.5294, 0.1352 d. 2.1412, 0.1893
 e. 1.9883, 0.1758

$$\text{a) } C_0 \left(\frac{1}{2}\right)^{\frac{2000}{5715}} = 1.2$$

$$C_0 = \frac{1.2}{\left(\frac{1}{2}\right)^{\frac{2000}{5715}}} \approx 1.5294$$

$$\text{b) } 1.5294 \left(\frac{1}{2}\right)^{\frac{20000}{5715}} \approx 0.1352$$

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10. A container of hot liquid is placed in a freezer that is kept at a constant temperature of 30° F. The initial temperature of the liquid is 190° F. After 6 minutes, the liquid's temperature is 64° F. How much longer will it take for its temperature to decrease to 32° F? Round your answer to two decimal places.

- ~~a. 16.98 minutes b. 6.59 minutes c. 9.88 minutes d. 4.39 minutes e. 12.07 minutes~~

$$\frac{dT}{dt} = k(T_0 - T_A)$$

$$\frac{dT}{T_0 - 30} = k dt$$

$$\ln(T_0 - 30) = kt + \ln 160$$

$$\ln(T_0 - 30) = kt + C$$

$$\ln 160 = C$$

$$\ln(32 - 30) = \frac{1}{6} \ln \frac{34}{160} t + \ln 160$$

$$\ln 34 = 6k + \ln 160$$

$$\frac{1}{6} \ln \frac{34}{160} = k$$

$$t = 16.98 \text{ min}$$

So 10.98 more minutes

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11. Find the particular solution of the differential equation $\frac{dr}{ds} = e^{r-7s}$ that satisfies the initial condition $r(0) = 0$.

a. $r = \ln(7 + e^{-7s}) + C$

b. $r = \ln(7) - \ln(6 + e^{-7s})$ c. $r = e^{r-7s}$

d. $r = \ln\left(\frac{8 + e^{-7s}}{7}\right)$ e. $r = (1 + e^{-7s})^7$

$$\begin{aligned} r &= \ln\left(\frac{e^{-7s} + 6}{7}\right) \\ r &= -\ln\left(\frac{e^{-7s} + 6}{7}\right) \\ r &= \ln\left(\frac{7}{e^{-7s} + 6}\right) \end{aligned}$$

$$\frac{dr}{ds} = \frac{e^r}{e^{-7s}} \quad \int \frac{dr}{e^r} = \int \frac{ds}{e^{-7s}}$$

$$-e^{-r} = -\frac{1}{7}e^{-7s} + C$$

$$\ln e^{-r} = \ln\left(\frac{1}{7}e^{-7s} + C\right)$$

$$-r = \ln\left(\frac{1}{7}e^{-7s} + C\right)$$

$$0 = -\ln\left(\frac{1}{7}e^{-7s} + C\right)$$

$$0 = \ln\left(\frac{1}{7}e^{-7s} + C\right)$$

$$1 = \frac{1}{7} + C$$

$$C = \frac{6}{7}$$

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12. Find the orthogonal trajectories of the family $3x^2 + 6y^2 = C$.

a. $6y^2 - 3x^2 = k$ b. $|y|^3 = k|x|^6$

c. $|y|^6 = k|x|^{-6}$ d. $6y^2 + 3x^2 = k$

e. $|y|^3 = k|x|^{-6}$

$$6x + 12xy' = 0$$

$$y' = \frac{-x}{2y}$$

so our $y' = \frac{2y}{x}$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\int \frac{dy}{2y} = \int \frac{dx}{x}$$

$$\frac{1}{2} \ln|y| = \ln|x| + C$$

$$e^{\ln|y|} = e^{\ln x^2 + C}$$

$$y = Ce^{\ln x^2}$$

$$y = Cx^2$$

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13. The logistic function $P(t) = \frac{24}{1+3e^{-2t}}$ models the growth of a population. Identify the initial population.

- a. 6 b. 8 c. 3 d. 24 e. 2

$$3 = A$$

$$3 = \frac{24 - P_0}{P_0}$$

$$\cancel{P_0} = \cancel{24} - \cancel{3}$$

$$6$$

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14. The logistic function $P(t) = \frac{10}{1+3e^{-2t}}$ models the

growth of a population. Determine when the population reaches one-half of the maximum carrying capacity. Round your answer to three decimal places.

- a. 0.549 b. 3.333 c. 1.151 d. 5.000
e. 1.000

$$5 = \frac{1}{2} \text{ max capacity}$$

$$5 = \frac{10}{1+3e^{-2t}}$$

$$1+3e^{-2t} = 2$$

$$3e^{-2t} = 1$$

$$e^{-2t} = \frac{1}{3}$$

$$t = \frac{\ln(1/3)}{-2}$$

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15. A conservation organization releases 40 coyotes into a preserve. After 4 years, there are 70 coyotes in the preserve. The preserve has a carrying capacity of 175. Write a logistic function that models the population of coyotes in the preserve.

a. $y = \frac{175}{1 + 3.375e^{-0.202733t}}$

b. $y = \frac{175}{1 + 3.375e^{-0.702733t}}$

c. $y = \frac{175}{1 + 3.375e^{-0.402733t}}$

d. $y = \frac{175}{1 + 4.875e^{-0.352733t}}$

e. $y = \frac{175}{1 + 3.375e^{-1.002733t}}$

$$y = \frac{175}{1 + b e^{-kt}}$$

$$y = \frac{175}{1 + b e^{-kt}} (4, 70)$$

$$40 = \frac{175}{1 + b}$$

$$b = 3.375$$

Plug in
solve for
 k

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16. A conservation organization releases 30 panthers into a preserve. After 3 years, there are 50 panthers in the preserve. The preserve has a carrying capacity of 150. Determine the time it takes for the population to reach 110.

- a. 13.139 years b. 8.994 years c. 10.378 years
d. 7.811 years e. 12.003 years

$$P_0 = 30 \quad (3, 50)$$

$$K = 150$$

$$\frac{150 - 30}{30} = 4 = A$$

$$110 - \frac{150}{1 + 4e^{-2.3105t}} = 150$$

$$t = 10.378$$

$$50 = \frac{150}{1 + 4e^{-3k}}$$

$$k = .23105$$

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17. Solve the first order linear differential equation.

$$\frac{dy}{dx} + \frac{8}{x}y = 7x + 2$$

integrating factor

$$e^{\int \frac{8}{x} dx} = e^{8 \ln x} = e^{\ln x^8} = x^8$$

a. $y = \frac{7}{10}x^2 - \frac{2}{9}x + Cx^{-8}$

b. $y = \frac{10}{7}x^2 + \frac{9}{2}x + Cx^{-8}$

c. $y = -\frac{7}{10}x^2 + \frac{2}{9}x + Cx^{-8}$

d. $y = -\frac{10}{7}x^2 + \frac{9}{2}x + Cx^{-8}$

e. $y = \frac{7}{10}x^2 + \frac{2}{9}x + Cx^{-8}$

$$\int x^8 \left(\frac{dx}{dx} + f' \right) + x^{-8} y = \int 7x^9 + 2x^8$$

$$y x^8 = \frac{7}{10} x^{10} + \frac{2}{9} x^9 + C$$

$$y = \frac{7}{10} x^2 + \frac{2}{9} x + C x^{-8}$$

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18. Solve the first-order linear differential equation

$$\frac{dy}{dx} = (y - 1) \sin(2x).$$

- a. $y = 1 + Ce^{\frac{\sin(2x)}{2}}$
 b. $y = 1 + Ce^{-\frac{\cos(2x)}{2}}$
 c. $y = 1 + Ce^{\frac{\sin(2x)}{2}}$
 d. $y = 1 + Ce^{-\frac{\sin(2x)}{2}}$
 e. $y = 1 + Ce^{\cos(2x)}$

$$\frac{dy}{y-1} = \sin 2x dx$$

$$e^{-\int \sin 2x dx}$$

$$\frac{dy}{dx} = y \sin 2x - \sin(2x)$$

$$\frac{dy}{dx} - y \sin 2x = -\sin(2x)$$

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19. Find the particular solution of the differential equation $x dy = (x + y + 6)dx$ that satisfies the boundary condition $y(1) = 5$.

- a. $y = 6x \ln|x| - 10x - 5$ b. $y = x \ln|x| + 11x - 6$
 c. $y = x \ln|x| + 11x + 5$ d. $y = 6x \ln|x| + 10x - 5$
 e. $y = x \ln|x| - 11x + 6$

$$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{6}{x}$$

$$\int \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = \int \frac{6}{x^2} + \frac{1}{x}$$

$$\frac{y}{x} = 6x^{-1} + \ln x + C$$

$$y = -6 + x \ln x + xC$$

$$5 = -6 + \ln 1 + 1C \quad C = 11$$

$$y = -6 + x \ln x + 11x$$

integrating factor

$$e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

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20. Find the particular solution of the differential equation $4xy' - y = x^3 - 2x$ that satisfies the boundary condition $y\left(\sqrt{\frac{22}{3}}\right) = 0$.

- a. $y = \frac{x^3}{3} - \frac{2}{3}x$ b. $y = \frac{x^3}{11} - \frac{2}{3}x$
 c. $y = \frac{x^3}{3} + \frac{2}{11}x$ d. $y = \frac{x^3}{3} - \frac{2}{11}x$
 e. $y = \frac{x^3}{11} + \frac{2}{3}x$

$$\frac{dy}{dx} - \frac{1}{4x}y = \frac{x^2}{4} - \frac{1}{2}$$

$$\int \frac{1}{4x} \frac{dy}{dx} - \frac{1}{4x} \frac{y}{x} = \int \frac{x^2}{4} - \frac{1}{2}$$

$$x^{\frac{1}{4}} \cdot x^{-\frac{1}{4}} y = \frac{x^{\frac{11}{4}}}{11} - \frac{2x^{\frac{3}{4}}}{3} + C$$

$$y = \frac{x^{\frac{3}{4}}}{11} - \frac{2x^{\frac{1}{4}}}{3} + x^{\frac{1}{4}}C$$

$$0 = \frac{\sqrt{\frac{22}{3}}^3}{11} - \frac{2\sqrt{\frac{22}{3}}}{3} - \sqrt{\frac{22}{3}}^{\frac{1}{4}}C$$

$$0 = \frac{22}{3}^{\frac{3}{4}} - \frac{2}{3} - \sqrt{\frac{22}{3}}^{\frac{1}{4}}C$$

$$C = 0$$

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21. Find the particular solution of the differential equation $\frac{dy}{dx} + 7x^3y = x^3$ passing through the point $\left(0, \frac{3}{2}\right)$.

- a. $y = \frac{1}{14} - \frac{23}{14}e^{-1.75x^4}$ b. $y = \frac{1}{7} + \frac{5}{14}e^{-1.75x^4}$
c. $y = \frac{1}{7} + \frac{23}{14}e^{-1.75x^4}$ d. $y = \frac{1}{7} + \frac{19}{14}e^{-1.75x^4}$
e. $y = \frac{1}{14} - \frac{19}{14}e^{-1.75x^4}$

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