Calculus Name_____ ID: 1

Assignment Date_____ Period____

For each problem, find the average rate of change of the function over the given interval.

1) $y = x^2 + 2x + 1$; $[-1, 0]$ $[a, b]$ $\frac{f(b) - f(a)}{b - a} = \frac{1 - 0}{0 - -1} = 1$

Use the definition of the derivative to find the derivative of each function with respect to x.

2) $y = -2x^2 + 2$

$$\lim_{h \to 0} \frac{-2(x+h)^2 + 2 - (-2x^2 + 2)}{h}$$

$$\lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) + 2 + 2x^2 - 2}{h}$$

$$\lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 2x^2}{h}$$

$$\lim_{h \to 0} \frac{h(-4x - 2h)}{h} = -4x$$

Differentiate each function with respect to x.

3) $f(x) = 2\sqrt[5]{x} + 4x^{-3}$ $= 2x^{\frac{1}{5}} + 4x^{-3}$

$f'(x) = \frac{2}{5}x^{-\frac{4}{5}} - 12x^{-4}$

For each problem, find the indicated derivative with respect to x.

4) $y = 3x^5 + 5x^4$ Find $\frac{d^2y}{dx^2}$ means second derivative

$\frac{dy}{dx} = 15x^4 + 20x^3$

$\frac{d^2y}{dx^2} = 60x^3 + 60x^2$

Differentiate each function with respect to x.

5) $y = (-2x^2 + 5) \cdot -x^2$

f g

$f'g + g'f$

$(-4x)(-x^2) + (-2x)(-2x^2+5)$

6) $y = \frac{5x^3 + 5}{2x^4 + 2}$ $\frac{f'g - g'f}{g^2}$

$\frac{(15x^2)(2x^4+2) - (8x^3)(5x^3+5)}{(2x^4+2)^2}$

7) $y = (4x^5 - 1)^4$ chain rule

$y' = 4(4x^5-1)^3(20x^4)$

8) $y = \sqrt[3]{\csc 4x^5}$ $= [\csc(4x^5)]^{1/3}$

$y' = \frac{1}{3}[\csc(4x^5)]^{-2/3}(-\csc(4x^5)\cot(4x^5))(20x^4)$

9) $y = \left((-3x^3 + 5)^2 + 4\right)^3$

$y' = 3\left((-3x^3+5)^2+4\right)^2\left(2(-3x^3+5)\right)(-9x^2)$

10) $y = \sqrt[5]{-x^5 + 5}(4x^3 + 1)$ $= \underset{f}{(-x^5+5)^{\frac{1}{5}}}\ \underset{g}{(4x^3+1)}$

$y' = \left(\frac{1}{5}(-x^5+5)^{-\frac{4}{5}}(-5x^4)\right)(4x^3+1) + (12x^2)\left((-x^5+5)^{\frac{1}{5}}\right)$

f' g g' f

11) $y = \csc(\underbrace{\cos 5x^4}_{u})$

$y' = -\csc(\underset{u}{\cos 5x^4})\cot(\underset{u}{\cos 5x^4})\underbrace{(-\sin(5x^4)(20x^3))}_{u'}$

12) $f(x) = (4x^2 + 5)\csc 4x^4$

$\underset{f}{\ }\ \underset{g}{\ }$

$f'(x) = (8x)(\csc 4x^2) + (-\csc 4x^4 \cot 4x^4 (16x^3))(4x^2+5)$

f' g g' f

f g

13) $y = \tan x^4 \csc 3x^5$

$y' = (\sec^2 x^4 (4x^3))(\csc 3x^5) + (-\csc 3x^5 \cot 3x^5 (15x^4))(\tan x^4)$

f′ g g′ f

f g

14) $y = \sin x^5 \cdot (x^2 + 2)$

$y' = (\cos x^5 (5x^4))(x^2+2) + (2x)(\sin x^5)$

f′ g g′ f

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

15)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−1	2	1
2	2	−1	3	1
3	1	0	4	$-\frac{1}{2}$
4	2	1	2	−2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(1)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(4)$

Part 3) Given $h_3(x) = f(x) \cdot g(x)$, find $h_3'(4)$

Part 4) Given $h_4(x) = \frac{f(x)}{g(x)}$, find $h_4'(2)$

Part 5) Given $h_5(x) = (f(x))^2$, find $h_5'(1)$

Part 6) Given $h_6(x) = f(g(x))$, find $h_6'(3)$

1) $h'(1) = f'(1) + g'(1)$
$= -1 + 1 = 0$

2) $h'(4) = f'(4) - g'(4)$
$= 1 - -2 = 3$

3) $h'(4) = f'(4)g(4) + g'(4)f(4)$
$= (1)(2) + (-2)(2) = -2$

4) $h'(2) = \frac{f'(2)g(2) - g'(2)f(2)}{(g(2))^2} = \frac{(-1)(3) - (1)(2)}{3^2} = \frac{-5}{9}$

5) $h'(1) = 2f(1)f'(1) = 2(3)(-1) = -6$

6) $h'(3) = f'(g(3))g'(3) = f'(4)(-\frac{1}{2})$
$= (1)(-\frac{1}{2}) = -\frac{1}{2}$

Answers to Assignment (ID: 1)

1) 1

2) $\frac{dy}{dx} = -4x$

3) $f'(x) = \frac{2}{5}x^{-\frac{4}{5}} - 12x^{-4}$

4) $\frac{d^2y}{dx^2} = 60x^3 + 60x^2$

5) $\frac{dy}{dx} = (-2x^2 + 5) \cdot -2x - x^2 \cdot -4x$
$= 8x^3 - 10x$

6) $\frac{dy}{dx} = \frac{(2x^4 + 2) \cdot 15x^2 - (5x^3 + 5) \cdot 8x^3}{(2x^4 + 2)^2}$
$= \frac{-5x^6 - 20x^3 + 15x^2}{2x^8 + 4x^4 + 2}$

7) $\frac{dy}{dx} = 4(4x^5 - 1)^3 \cdot 20x^4$
$= 80x^4(4x^5 - 1)^3$

8) $\frac{dy}{dx} = \frac{1}{3} \cdot (\csc 4x^5)^{-\frac{2}{3}} \cdot -\csc 4x^5 \cot 4x^5 \cdot 20x^4$
$= -\frac{20x^4(\csc 4x^5)^{\frac{1}{3}} \cdot \cot 4x^5}{3}$

9) $\frac{dy}{dx} = 3\left(\left(-3x^3 + 5\right)^2 + 4\right)^2 \cdot 2(-3x^3 + 5) \cdot -9x^2$
$= -54x^2\left(\left(-3x^3 + 5\right)^2 + 4\right)^2(-3x^3 + 5)$

10) $\frac{dy}{dx} = (-x^5 + 5)^{\frac{1}{5}} \cdot 12x^2 + (4x^3 + 1) \cdot \frac{1}{5}(-x^5 + 5)^{-\frac{4}{5}} \cdot -5x^4$

11) $\frac{dy}{dx} = -\csc(\cos 5x^4)\cot(\cos 5x^4) \cdot -\sin 5x^4 \cdot 20x^3$
$= 20x^3 \csc(\cos 5x^4)\cot(\cos 5x^4)\sin 5x^4$

12) $f'(x) = (4x^2 + 5) \cdot -\csc 4x^4 \cot 4x^4 \cdot 16x^3 + \csc 4x^4 \cdot 8x$
$= 8x\csc 4x^4 \cdot (-8x^4 \cot 4x^4 - 10x^2 \cot 4x^4 + 1)$

13) $\frac{dy}{dx} = \tan x^4 \cdot -\csc 3x^5 \cot 3x^5 \cdot 15x^4 + \csc 3x^5 \cdot \sec^2 x^4 \cdot 4x^3$
$= x^3 \csc 3x^5 \cdot (-15x \tan x^4 \cot 3x^5 + 4\sec^2 x^4)$

14) $\frac{dy}{dx} = \sin x^5 \cdot 2x + (x^2 + 2) \cdot \cos x^5 \cdot 5x^4$
$= x(2\sin x^5 + 5x^5 \cos x^5 + 10x^3 \cos x^5)$

15) $h_1'(1) = f'(1) + g'(1) = 0$
$h_2'(4) = f'(4) - g'(4) = 3$
$h_3'(4) = f(4) \cdot g'(4) + g(4) \cdot f'(4) = -2$
$h_4'(2) = \frac{g(2) \cdot f'(2) - f(2) \cdot g'(2)}{(g(2))^2} = -\frac{5}{9}$
$h_5'(1) = 2 \cdot f(1) \cdot f'(1) = -6$
$h_6'(3) = f'(g(3)) \cdot g'(3) = -\frac{1}{2}$