AP CALCULUS Worksheet 5.3
Derivatives of Inverse Functions

1)

If $f(x) = x^3 + x$ and $h(x)$ is the inverse of $f(x)$, then $h'(2)$ is

A) $\frac{1}{13}$ B) $\frac{1}{4}$ C) 1 D) 4 E) 13

2)

Let f be a differentiable function such that $f(3) = 15, f'(3) = -8$, and $f'(6) = -2$, $f(6) = 3$
The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

A) $-\frac{1}{2}$ B) $-\frac{1}{8}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) The value of $g'(3)$ cannot be determined from the information given.

3)

Use your calculator to determine the derivative of the inverse function of $f(x)$ at $x = 4$ where $f(x) = x^5 + x^3 + 2x - 2$.

4)

Suppose f is a one-to-one function, which is differentiable for all real numbers x. The table below gives some of the values of $f(x)$ and $f'(x)$:

x	$f(x)$	$f'(x)$
1	2	$\frac{7}{6}$
2	3	$\frac{7}{6}$
3	5	$\frac{19}{6}$
4	10	$\frac{43}{6}$

(a) Write an equation of the tangent line, T_1, to the function $f(x)$ at $x = 3$.

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(b) Write an equation of the normal line, N_1, to the function $f(x)$ at $x = 3$.

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(c) Write an equation of the tangent line, T_2, to the function $f^{-1}(x)$ at $x = 3$.

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5)

The function used in Problems 4 and 5 is $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{19}{6}x$.

(a) $f'(0) =$ __________.

(b) If $g(x) = f^{-1}(x)$, then $g'(0) =$ __________.

(c) Write an equation for the normal line to $g(x)$ at $x = 0$.

6)

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.