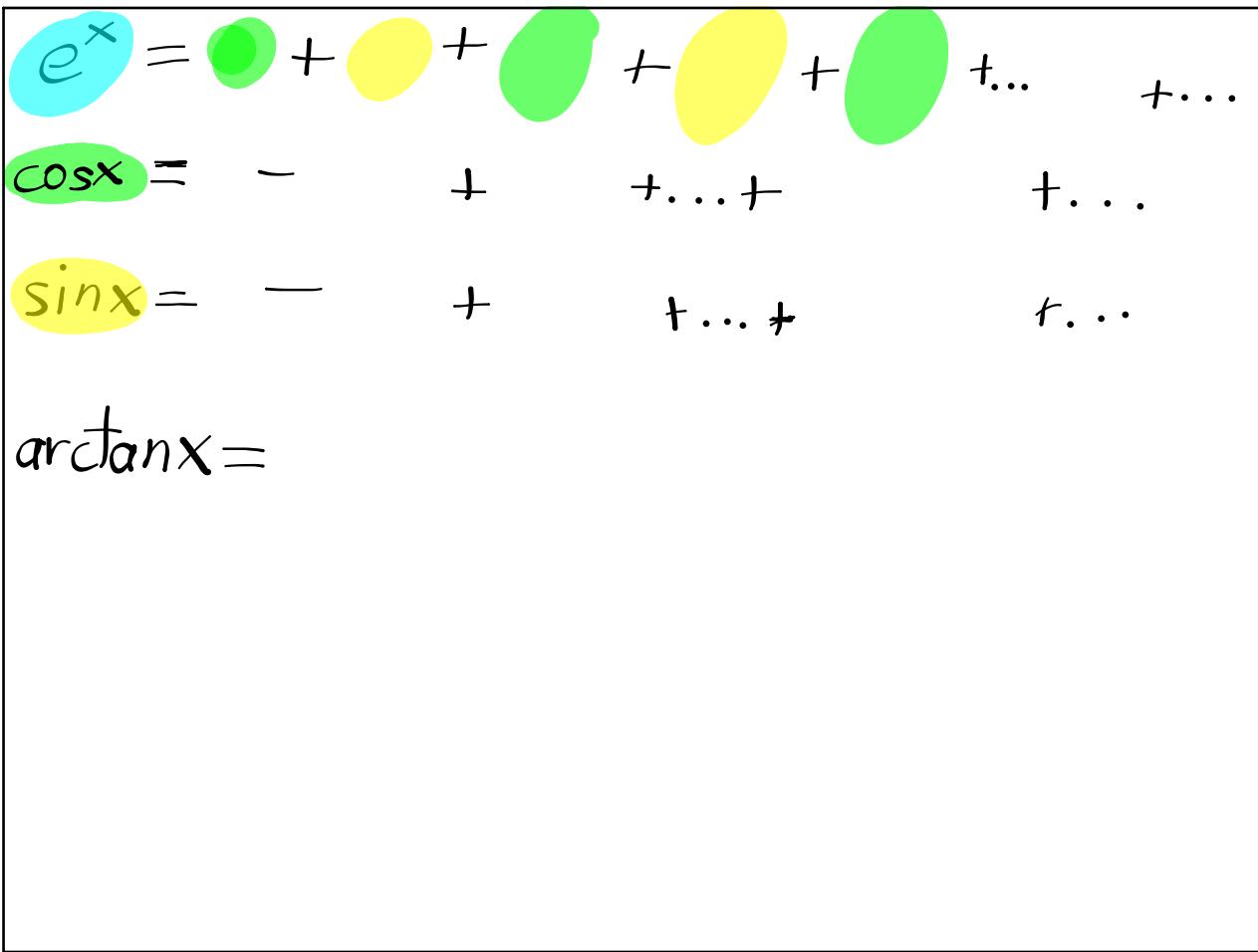


9-recap



POWER SERIES FOR ELEMENTARY FUNCTIONS

Function	Interval of Convergence
$\frac{1}{x} = 1 - (x - 1) + (x - 1)^2 - (x - 1)^3 + (x - 1)^4 - \dots + (-1)^n(x - 1)^n + \dots$	$0 < x < 2$
$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots + (-1)^n x^n + \dots$	$-1 < x < 1$
$\ln x = (x - 1) - \frac{(x - 1)^2}{2} + \frac{(x - 1)^3}{3} - \frac{(x - 1)^4}{4} + \dots + \frac{(-1)^{n-1}(x - 1)^n}{n} + \dots$	$0 < x \leq 2$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots + \frac{x^n}{n!} + \dots$	$-\infty < x < \infty$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$	$-\infty < x < \infty$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$	$-\infty < x < \infty$
$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots + \frac{(-1)^n x^{2n+1}}{2n+1} + \dots$	$-1 \leq x \leq 1$
$\arcsin x = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots + \frac{(2n)! x^{2n+1}}{(2^n n!)^2 (2n+1)} + \dots$	$-1 \leq x \leq 1$
$(1+x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \dots + \frac{k(k-1)\dots(k-n+1)x^n}{n!} + \dots$	$-1 < x < 1^*$

* The convergence at $x = \pm 1$ depends on the value of k .

$$\frac{d}{dx} \arctan x = \boxed{}$$

$$\frac{1}{1+x} = \frac{1}{1+(-x)} =$$

$$\frac{1}{1+x^2} =$$

$$\arctan x =$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

9-recap

write a power series for $f(x)$ centered at $c=0$

$$f(x) = \frac{x^3}{x+2} =$$

write a power series for $f(x)$ centered at $c=0$

$$f(x) = \frac{x}{1-2x} =$$

$$\frac{1}{1-x} =$$

$$\frac{3}{2+x} = \frac{\frac{3}{2} - \frac{3}{4}x}{2+x}$$

$$= \frac{(3 + \frac{3}{2}x)}{-\frac{3}{2}x}$$

$$= \frac{(-\frac{3}{2}x - \frac{3}{4}x^2)}{\frac{3}{4}x^2}$$

9-recap

$$\boxed{f(x) = \frac{1}{4-x} \quad c=1}$$

which method would you use
to find a power series for $f(x)$?

$$f(1) =$$

$$f'(1) =$$

$$f''(1) =$$

$$\left(\frac{1}{4-x}\right)^2 \quad \dots + \frac{(x-1)^n}{3^{n+1}}$$

$$(4-x)^{-2} \quad \frac{f''(1)}{2!} (x-1)^2 \\ + 2(4-x)^{-3}$$

$$f(t) = \frac{4}{1+t^2} \quad G(x) = \int_0^x f(t) dt$$

find the first 4 non zero terms and general term
of a) $f(t)$ b) $G(x)$ and its interval of convergence

$$\text{a) } f(t) =$$

$$\text{b) } G(x) =$$

$f(x) = e^{-2x^2}$

- find the first 4 non-zero terms and general term for $f(x)$ about $x=0$
- and its interval of convergence
- is the error of this partial sum < 0.02 for $-0.6 < x < 0.6$ explain. (use calculator)

$$e^{-2x^2} =$$

a)

b)

c)

$$f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots + \frac{x^n}{(n+1)!} + \dots$$

find $f'(0)$ and $f^{17}(0)$

$$f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots + \frac{x^n}{(n+1)!} + \dots$$

$$g(x) = x f(x)$$

what is $g(x)$

as a series and
as a familiar term?

what, then, is $f(x)$?

What is the sum?

$$1 + \frac{2}{1!} + \frac{4}{2!} + \frac{8}{3!} + \dots + \frac{2^n}{n!} + \dots =$$

$$1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots + \frac{(-1)^n}{(2n+1)!} + \dots =$$

$$1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots + \left(\frac{1}{4}\right)^n + \dots =$$

$$1 - \frac{100}{2!} + \frac{10000}{4!} + \dots + \frac{(-1)^n 10^{2n}}{(2n)!} + \dots =$$

approximate $\int_0^1 e^{-x^2} dx$ with an error less than .01

Use your calculator to
help with decimals.

Another way this question could be worded is
"How many terms are required to approximate with an error < .01?"

$$\frac{x e^x}{x e^x} \quad vs \quad \frac{\sin x e^x}{\sin x e^x}$$