1. Find the particular solution of the differential equation $3x + 20yy' = 0$ that satisfies the initial condition $y = 5$ when $x = 2$, where $3x^2 + 20y^2 = C$ is the general solution.

   a. $3x^2 + 20y^2 = 504$
   b. $3x^2 + 20y^2 = 155$
   c. $3x^2 + 20y^2 = 112$
   d. $3x^2 + 20y^2 = 37$
   e. $3x^2 + 20y^2 = 512$

$3(2)^2 + 20(5)^2 = C$

$12 + 500 = C$

$C = 512$

2. Use integration to find a general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{3x}{3+x^2}$.

   a. $y = \frac{3}{2}\ln\left(\left|3+x^2\right|\right) + C$
   b. $y = \frac{3}{2x}\ln\left(\left|6+x^2\right|\right) + C$
   c. $y = \frac{6}{x^2}\ln\left(\left|3+x^2\right|\right) + C$
   d. $y = \dfrac{3}{x\ln\left(\left|3+x^2\right|\right)} + C$
   e. $y = \dfrac{3x^2}{\ln\left(\left|3+x^2\right|\right)} + C$

$\dfrac{dy}{dx} = \dfrac{3x}{3+x^2}$

$\int dy = \int \dfrac{3x}{3+x^2}\,dx$

$y = 3\int \dfrac{x}{3+x^2}\,dx$

$u = 3 + x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\frac{3}{2}\int \frac{1}{u}\,du$

$\frac{3}{2}\ln|u| + C$

$\frac{3}{2}\ln|3+x^2| + C$

3. Use integration to find a general solution of the differential equation $\frac{dy}{dx} = 13x\cos\left(8x^2\right)$.

a. $y = \frac{13x\sin\left(16x^2\right)}{2x} + C$

b. $y = \frac{13x\cos\left(8x^2\right)}{16} + C$

c. $y = \frac{13\sin\left(8x^2\right)}{16} + C$

d. $y = \frac{13\cos\left(8x^2\right)}{16x} + C$

e. $y = \frac{13\sin\left(16x^2\right)}{4} + C$

$u = 8x^2$

$\int dy = \int 13x\cos(8x^2)dx$

$du = 16x\,dx$

$y = \frac{13}{16}\int \cos(u)\,du$

$y = \frac{13}{16}\sin(8x^2) + C$

4. Use integration to find a general solution of the differential equation.

$\frac{dy}{dx} = x\sqrt{x+2}$

$\int dy = \int x\sqrt{x+2}\,dx$

$u = x+2$

$du = dx$

$x = u-2$

a. $y = \frac{2}{5}(x+2)^{3/2} - \frac{4}{3}(x+2)^{5/2} + C$

b. $y = \frac{2}{5}(x+2)^{3/2} - (x+2)^{5/2} + C$

c. $y = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

d. $y = \frac{2}{5}(x+2)^{3/2} + \frac{4}{3}(x+2)^{5/2} + C$

e. $y = \frac{2}{5}(x+2)^2 - \frac{4}{3}(x+2) + C$

$y = \int (u-2)(u)^{0.5}\,du$

$y = \int u^{1.5} - 2u^{0.5}\,du$

$y = \frac{2}{5}(x+2)^{2.5} - \frac{4}{3}(x+2)^{1.5} + C$

5. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x\sqrt{4-x^2}$$

a. $y = -\frac{1}{3}x(4-x^2)^{\frac{3}{2}} + C$

b. $y = -\frac{1}{5}x\left(4-x^2\right)^{\frac{5}{2}} + C$

c. $y = -\frac{1}{3}\left(4-x^2\right)^{\frac{3}{2}} + C$

d. $y = \frac{1}{5}x\left(4-x^2\right)^{\frac{5}{2}} + C$

e. $y = \frac{1}{3}\left(4-x^2\right)^{\frac{3}{2}} + C$

$dy = x\sqrt{-x^2+4}\,dx$ $\quad u = -x^2+4$

$du = -2x\,dx$

$\int dy = \int x\sqrt{u}\,dx$ $\quad -\frac{1}{2}du = x\,dx$

$y = -\frac{1}{2}\left(\int u^{.5}\,du\right) + C$

$y = -\frac{1}{2}\left(\frac{2}{3}u^{1.5}\right) + C_1$

$y = -\frac{1}{3}(4-x^2)^{1.5} + C_1$

6. Solve the differential equation.

$$\frac{dy}{dx} = -y - 8$$

a. $y = Ce^{-x} + 8$

b. $y = Ce^{x} - 8$

c. $y = Ce^{x} + 8$

d. $y = Ce^{-x} - 8$

e. $y = Ce^{x} + 16$

$\int \frac{dy}{y+8} = -\int dx$

$e^{\ln|y+8|} = e^{-x+c}$

$y+8 = Ce^{-x}$

$y = Ce^{-x} - 8$

7. Solve the differential equation $y' = \frac{\sqrt{x}}{-5y}$.

a. $4y^2 = -15x^{\frac{3}{2}} + C$

b. $4y^2 = -5x^{\frac{3}{2}} + C$

c. $-5y^2 = 4x^{\frac{3}{2}} + C$

d. $-15y^2 = 4x^{\frac{3}{2}} + C$

e. $-15y^2 = 2x^{\frac{3}{2}} + C$

$$\frac{dy}{dx} = \frac{\sqrt{x}}{-5y}$$

$$\int y\,dy = -\frac{1}{5}\int x^{\frac{1}{2}}\,dx$$

$$30\left[\frac{1}{2}y^2 = -\frac{1}{5}\left(\frac{2}{3}x^{\frac{3}{2}}\right) + C\right]$$

$$15y^2 = -4x^{\frac{3}{2}} + C$$

8. Solve the differential equation.

$y' = x(1+y)$

a. $\ln|1+y| = x^2 + C$

b. $3\ln|1+y| = x^3 + C$

c. $4\ln|1+y| = x^4 + C$

d. $\ln|1+y| = x + C$

e. $2\ln|1+y| = x^2 + C$

$$\frac{dy}{dx} = x(1+y)$$

$$\int \frac{dy}{1+y} = \int x\,dx$$

$$2\left[\ln|1+y| = \frac{1}{2}x^2 + C\right]$$

$$2\ln|1+y| = x^2 + C$$

9. Find the function $y = f(t)$ passing through the point $(0, 15)$ with the first derivative $\frac{dy}{dt} = \frac{1}{4}t$.

a. $y(t) = \frac{t^2}{8} + 15$

b. $y(t) = 8t^2 + 15$

c. $y(t) = 4t + 15$

d. $y(t) = \frac{t^2}{4} + 15$

e. $y(t) = \frac{t}{4} + 15$

$\int dy = \frac{1}{4}\int t\,dt$

$y = \frac{1}{4}\left(\frac{1}{2}t^2\right) + c$

$15 = \frac{1}{8}(0)^2 + c$

$c = 15$

$y(t) = \frac{1}{8}t^2 + 15$

10. Find the function $y = f(t)$ passing through the point $(0, 12)$ with the first derivative $\frac{dy}{dt} = \frac{6}{7}y$.

a. $y(t) = e^{\frac{6}{7}t^2} + 12$

b. $y(t) = \frac{6}{7}t^2 + 12$

c. $y(t) = 12e^{\frac{6}{7}t^2}$

d. $y(t) = 12e^{\frac{6}{7}t}$

e. $y(t) = e^{\frac{6}{7}t} + 12$

$\int \frac{dy}{y} = \int \frac{6}{7}\,dt$

$\ln y = \frac{6}{7}t + c$

$\ln 12 = \frac{6}{7}(0) + c$

$c = \ln 12$

$e^{\ln y} = e^{\frac{6}{7}t + \ln 12}$

$y = e^{\frac{6}{7}t} \cdot e^{\ln 12}$

$y = 12e^{\frac{6}{7}t}$

11. Write and solve the differential equation that models the following verbal statement. Evaluate the solution at the specified value of the independent variable, rounding your answer to four decimal places:

The rate of change of V is proportional to V. When $s = 0$, $V = 60$ and when $s = 4$, $V = 116$. What is the value of V when $s = 12$?

a. $V(12) = 433.5822$
b. $V(12) = 437.3322$
c. $V(12) = 424.6622$
d. $V(12) = 439.7922$
e. $V(12) = 429.8322$

$\frac{dV}{ds} = kV$

$V = Ce^{kt}$

$V = 60e^{kt}$

$116 = 60e^{4t}$

$e^{4t} = \frac{116}{60}$

$4t = \ln\frac{116}{60}$ $t = \frac{1}{4}\ln\frac{116}{60}$

$V = 60e^{\frac{1}{4}\ln\frac{116}{60}t}$

$(12, 433.582)$

12. The half-life of the radium isotope Ra-226 is approximately 1,599 years. If the initial quantity of the isotope is 38 g, what is the amount left after 1,000 years? Round your answer to two decimal places.

a. 24.63 g
b. 30.60 g
c. 25.13 g
d. 11.88 g
e. 12.32 g

$Y = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$Y = 38\left(\frac{1}{2}\right)^{\frac{1000}{1599}}$

$Y \approx 24.63$

13. The half-life of the radium isotope Ra-226 is approximately 1,599 years. What percent of a given amount remains after 400 years? Round your answer to two decimal places.

a. 84.08 %
b. 12.51 %
c. 86.08 %
d. 0.84 %
e. 5.84 %

$y = 100\left(\frac{1}{2}\right)^{\frac{400}{1599}}$

$y \approx 84.08$

14. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{5x^2}{8y^2}$.

a. $y = \frac{5}{8}x^3 + C$
b. $y = \sqrt[3]{\frac{5}{8}x^3 + C}$
c. $y = \sqrt{\frac{8}{5}x^3 + C}$
d. $y = \sqrt[3]{\frac{x^2}{8} + C}$
e. $y = \sqrt[3]{5x^3 + 8C}$

$\int 8y^2 dy = \int 5x^2 dx$

$\frac{3}{8}\left[\frac{8}{3}y^3 = \frac{5}{3}x^3 + C\right]$

$y^3 = \frac{5}{8}x^3 + C$

$y = \sqrt[3]{\frac{5}{8}x^3 + C}$

15. Find the particular solution of the differential equation $\frac{dr}{ds} = e^{r-7s}$ that satisfies the initial condition $r(0) = 0$.

a. $r = \ln\left(7+e^{-7s}\right)+C$

b. $r = \ln(7) - \ln\left(6+e^{-7s}\right)$

c. $r = e^{r-7s}$

d. $r = \ln\left(\frac{8+e^{-7s}}{7}\right)$

e. $r = \left(1+e^{-7s}\right)^7$

$\frac{dr}{ds} = \frac{e^r}{e^{7s}}$

$\frac{dr}{e^r} = \frac{ds}{e^{7s}}$

$\int e^{-r}dr = \int e^{-7s}ds$

$-1\left[-e^{-r} = -\frac{1}{7}e^{-7s} + C\right]$

$e^{-r} = \frac{1}{7}e^{-7s} + C$

$\ln e^{-r} = \ln\left(\frac{1}{7}e^{-7s}+C\right)$

$-r = \ln\left(\frac{1}{7}e^{-7s}+C\right)$

$r = -\ln\left(\frac{1}{7}e^{-7s}+C\right)$

$0 = -\ln\left(\frac{1}{7}e^{0}+C\right)$

$e^{0} = e^{+\ln\left(\frac{1}{7}+C\right)}$

$1 = \frac{1}{7}+C$

$C = \frac{6}{7}$

finally $r = -\ln\left(\frac{e^{-7s}+6}{7}\right)$

$r = -\left(\ln\left(e^{-7s}+6\right) - \ln 7\right)$

$\boxed{r = \ln 7 - \ln\left(e^{-7s}+6\right)}$

16. Find an equation of the graph that passes through the point (7, 3) and has the slope $y' = \frac{5y}{2x}$.

a. $y = 3(7x)^{\frac{2}{5}}$

b. $y = xe^{\frac{5}{2}+\frac{7}{3}}$

c. $y = 3\left(\frac{x}{7}\right)^{\frac{5}{2}}$

d. $y = 7\left(\frac{5x}{2}\right)^3$

e. $y = \frac{2}{5x} - \ln(7x) + 3$

$\frac{dy}{dx} = \frac{5y}{2x}$

$\int \frac{1}{5y}dy = \int \frac{1}{2x}dx$

$\frac{1}{5}\ln y = \frac{1}{2}\ln x + C$

$\ln y = \frac{5}{2}\ln x + C$

$e^{\ln y} = e^{\frac{5}{2}\ln x + C}$ $(7,3)$

$y = e^{\ln x^{\frac{5}{2}}} + C_1$

$y = C_1 x^{\frac{5}{2}}$

$3 = C_2(7)^{5/2}$

$C_2 = 3/7^{5/2}$

$y = \frac{3}{7^{5/2}}x^{5/2}$

$y = 3\left(\frac{x^{\frac{5}{2}}}{7^{\frac{5}{2}}}\right)$

1. The number of bacteria in a culture is increasing according to the law of exponential growth. After 5 hours there are 175 bacteria in the culture and after 10 hours there are 425 bacteria in the culture. Answer the following questions, rounding numerical answers to four decimal places.

$y = Ce^{kt}$ $(5, 175)$ $(10, 425)$

(i) Find the initial population.

(ii) Write an exponential growth model for the bacteria population. Let t represent time in hours.

(iii) Use the model to determine the number of bacteria after 20 hours.

(iv) After how many hours will the bacteria count be 15,000?

i) $\frac{175}{e^{5k}} = \frac{Ce^{5k}}{e^{5k}}$ 1

$C = \frac{175}{e^{5k}}$ 2

$425 = Ce^{10k}$

$425 = \frac{175}{e^{5k}}(e^{10k})$

$\frac{425}{175} = \frac{e^{10k}}{e^{5k}}$

$\frac{17}{7} = e^{5k}$

3

$175 = C\left(\frac{17}{7}\right)$

$C = 175\left(\frac{7}{17}\right)$

$C = 72.0588$

$y = 72.0588e^{kt}$

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ii) $\frac{17}{7} = e^{5k}$

$\ln\frac{17}{7} = \ln e^{5k}$

$5k = .8873$

$k \approx .1775$

$y = 72.0588e^{0.1775t}$

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iii) $y = 72.0588 e^{0.1775t}$ $t = 20$

$y = 72.0588 e^{0.1775(20)} \approx 2508.6059$

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iv) $y = 72.0588 e^{0.1775t}$

$15000 = 72.0588 e^{0.1775t}$

$t \approx 30.075$