You have a small 10 question Quiz today on anti differentiation (indefinite integrals).

For example:

① $\int 5x^{\frac{2}{3}}\,dx$

③ $\int (6+6\tan^2\theta)\,d\theta$

② $\int 7\sec^2\theta\,d\theta$

④ $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

Section 4.4 **The Fundamental Theorem of Calculus**

- Evaluate a definite integral using the Fundamental Theorem of Calculus.
- Understand and use the Mean Value Theorem for Integrals.
- Find the average value of a function over a closed interval.
- Understand and use the Second Fundamental Theorem of Calculus.

THEOREM 4.9 The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

EXAMPLE 1 Evaluating a Definite Integral

Evaluate each definite integral.

a. $\int_1^2 (x^2 - 3)\,dx$ **b.** $\int_1^4 3\sqrt{x}\,dx$ **c.** $\int_0^{\pi/4} \sec^2 x\,dx$

EXAMPLE 2 A Definite Integral Involving Absolute Value

Evaluate $\int_0^2 |2x - 1|\,dx$.

EXAMPLE 3 Using the Fundamental Theorem to Find Area

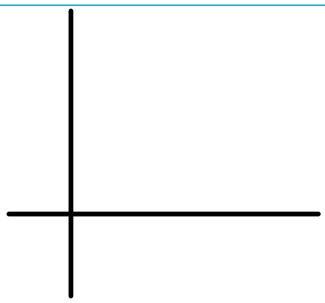
Find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$, as shown in Figure 4.29.

THEOREM 4.10 Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b - a).$$

let's construct this from scratch



Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

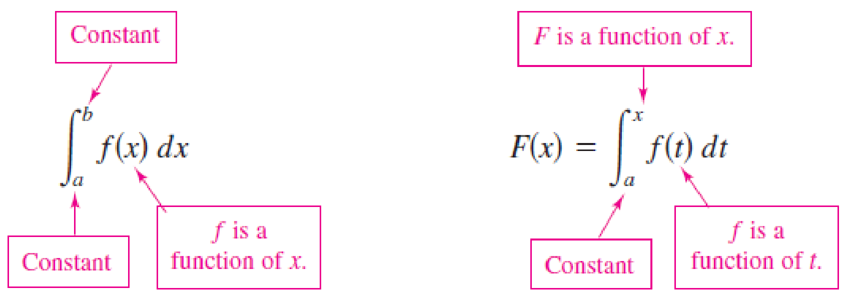
$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

EXAMPLE 4 Finding the Average Value of a Function

Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$.

The Second Fundamental Theorem of Calculus

The Definite Integral as a Number | *The Definite Integral as a Function of x*



EXAMPLE 6 The Definite Integral as a Function

Evaluate the function

$$F(x) = \int_0^x \cos t \, dt$$

at $x = 0, \pi/6, \pi/4, \pi/3$, and $\pi/2$.

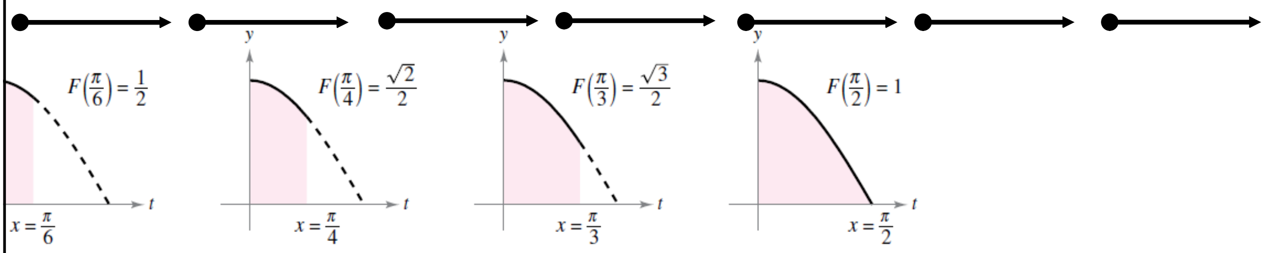
$F(0) = \sin 0 =$

$F(\frac{\pi}{6}) = \sin \frac{\pi}{6} =$

$F(\frac{\pi}{4}) = \sin \frac{\pi}{4} =$

$F(\frac{\pi}{3}) = \sin \frac{\pi}{3} =$

$F(\frac{\pi}{2}) = \sin \frac{\pi}{2} =$



THEOREM 4.11 The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t) \, dt\right] = f(x).$$

EXAMPLE 7 Using the Second Fundamental Theorem of Calculus

Evaluate $\frac{d}{dx}\left[\int_0^x \sqrt{t^2 + 1} \, dt\right]$.

EXAMPLE 8 Using the Second Fundamental Theorem of Calculus

Find the derivative of $F(x) = \int_{\pi/2}^{x^3} \cos t\, dt$.

Solution Using $u = x^3$, you can apply the Second Fundamental Theorem of Calculus with the Chain Rule as shown.

$$
\begin{aligned}
F'(x) &= \frac{dF}{du}\frac{du}{dx} && \text{Chain Rule} \\
&= \frac{d}{du}[F(x)]\frac{du}{dx} && \text{Definition of } \frac{dF}{du} \\
&= \frac{d}{du}\left[\int_{\pi/2}^{x^3} \cos t\, dt\right]\frac{du}{dx} && \text{Substitute } \int_{\pi/2}^{x^3} \cos t\, dt \text{ for } F(x). \\
&= \frac{d}{du}\left[\int_{\pi/2}^{u} \cos t\, dt\right]\frac{du}{dx} && \text{Substitute } u \text{ for } x^3. \\
&= (\cos u)(3x^2) && \text{Apply Second Fundamental Theorem of Calculus.} \\
&= (\cos x^3)(3x^2) && \text{Rewrite as function of } x.
\end{aligned}
$$

Net Change Theorem

The Fundamental Theorem of Calculus (Theorem 4.9) states that if f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

But because $F'(x) = f(x)$, this statement can be rewritten as

$$\int_a^b F'(x)\, dx = F(b) - F(a)$$

where the quantity $F(b) - F(a)$ represents the *net change of* $F(x)$ on the interval $[a, b]$.

THEOREM 4.12 The Net Change Theorem

If $F'(x)$ is the rate of change of a quantity $F(x)$, then the definite integral of $F'(x)$ from a to b gives the total change, or **net change**, of $F(x)$ on the interval $[a, b]$.

$$\int_a^b F'(x)\, dx = F(b) - F(a) \qquad \text{Net change of } F(x)$$

A chemical flows into a storage tank at a rate of $(180 + 3t)$ liters per minute, where t is the time in minutes and $0 \le t \le 60$. Find the amount of the chemical that flows into the tank during the first 20 minutes.

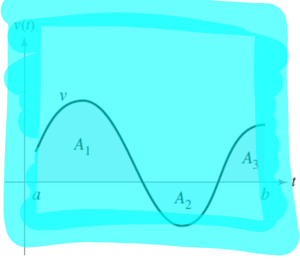
Solution Let $c(t)$ be the amount of the chemical in the tank at time t. Then $c'(t)$ represents the rate at which the chemical flows into the tank at time t. During the first 20 minutes, the amount that flows into the tank is

$$
\begin{aligned}
\int_0^{20} c'(t)\, dt &= \int_0^{20} (180 + 3t)\, dt \\
&= \left[180t + \frac{3}{2}t^2\right]_0^{20} \\
&= 3600 + 600 \\
&= 4200.
\end{aligned}
$$

Another way to illustrate the Net Change Theorem is to examine the velocity of a particle moving along a straight line, where $s(t)$ is the position at time t. Then its velocity is $v(t) = s'(t)$ and

$$\int_a^b v(t)\,dt = s(b) - s(a).$$

This definite integral represents the net change in position, or **displacement,** of the particle.



A_1, A_2, and A_3 are the areas of the shaded regions.
Figure 4.36

When calculating the *total* distance traveled by the particle, you must consider the intervals where $v(t) \le 0$ and the intervals where $v(t) \ge 0$. When $v(t) \le 0$, the particle moves to the left, and when $v(t) \ge 0$, the particle moves to the right. To calculate the total distance traveled, integrate the absolute value of velocity $|v(t)|$. So, the **displacement** of the particle on the interval $[a, b]$ is

$$\text{Displacement on } [a, b] = \int_a^b v(t)\,dt = A_1 - A_2 + A_3$$

and the **total distance traveled** by the particle on $[a, b]$ is

$$\text{Total distance traveled on } [a, b] = \int_a^b |v(t)|\,dt = A_1 + A_2 + A_3.$$

(See Figure 4.36.)

HW 4.4 part 1 - Page 292 #1-61 eoo

4.4 part 2 - Page 293 #65-109 eoo

67. Analyzing a Function Let

$$g(x) = \int_0^x f(t)\,dt$$

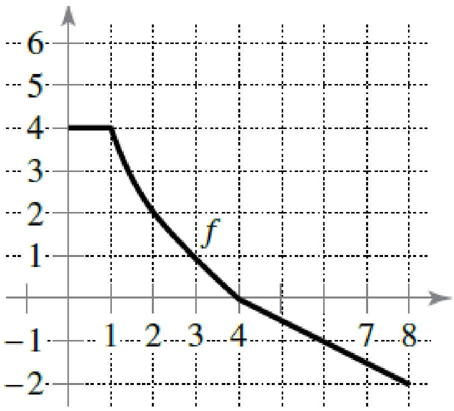
where f is the function whose graph is shown in the figure.

(a) Estimate $g(0)$, $g(2)$, $g(4)$, $g(6)$, and $g(8)$.

(b) Find the largest open interval on which g is increasing. Find the largest open interval on which g is decreasing.

(c) Identify any extrema of g.

(d) Sketch a rough graph of g.



67. $g(x) = \int_0^x f(t)\,dt$

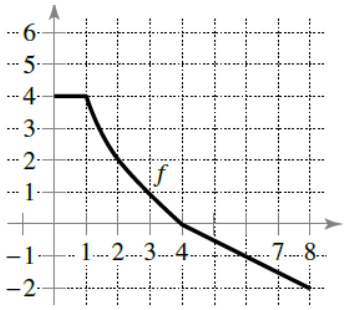
(a) $g(0) = \int_0^0 f(t)\,dt = 0$

$g(2) = \int_0^2 f(t)\,dt \approx 4 + 2 + 1 = 7$

$g(4) = \int_0^4 f(t)\,dt \approx 7 + 2 = 9$

$g(6) = \int_0^6 f(t)\,dt \approx 9 + (-1) = 8$

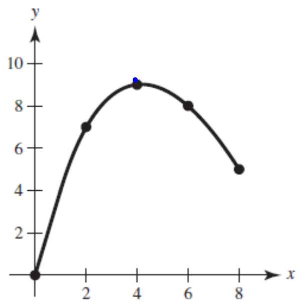
$g(8) = \int_0^8 f(t)\,dt \approx 8 - 3 = 5$



(b) g increasing on $(0, 4)$ and decreasing on $(4, 8)$

(c) g is a maximum of 9 at $x = 4$.

(d)



Finding and Checking an Integral In Exercises 69–74, (a) integrate to find F as a function of x, and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result in part (a).

71. $F(x) = \int_8^x \sqrt[3]{t}\,dt$

find $F'(x)$.

79. $F(x) = \int_1^x \sqrt{t} \csc t \, dt$

83. $F(x) = \int_0^{\sin x} \sqrt{t} \, dt$

Particle Motion In Exercises 93–98, the velocity function, in feet per second, is given for a particle moving along a straight line, where t is the time in seconds. Find (a) the displacement and (b) the total distance that the particle travels over the given interval.

95. $v(t) = t^3 - 10t^2 + 27t - 18, \quad 1 \leq t \leq 7$

just set up

a)

b)

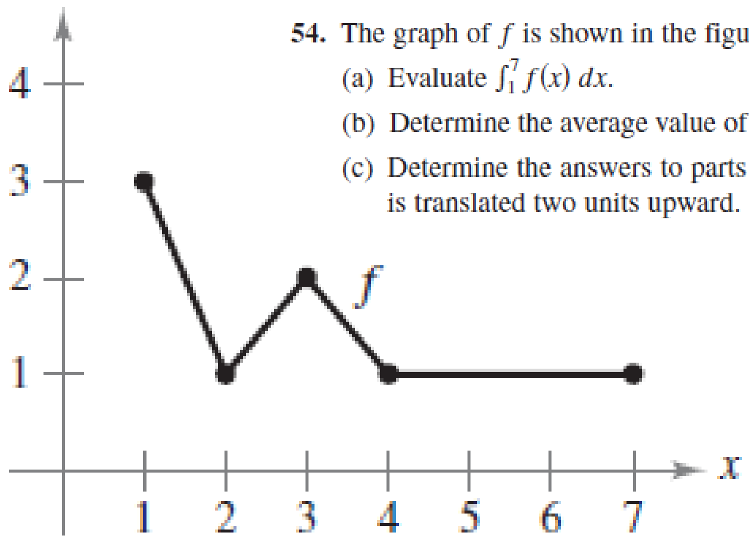
99. **Particle Motion** Describe a situation where the displacement and the total distance traveled for a particle are equal.

107. $\int_{\pi/4}^{3\pi/4} \sec^2 x \, dx = \left[\tan x\right]_{\pi/4}^{3\pi/4} = -2$ X

111. Analyzing a Function Show that the function

$$f(x) = \int_0^{1/x} \frac{1}{t^2 + 1}\,dt + \int_0^{x} \frac{1}{t^2 + 1}\,dt$$

is constant for $x > 0$.



54. The graph of f is shown in the figure.

(a) Evaluate $\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.

55. $\int_0^2 f(x)\,dx =$ ▢

56. $\int_2^6 f(x)\,dx =$ ▢

57. $\int_0^6 |f(x)|\,dx =$ ▢

58. $\int_0^2 -2f(x)\,dx =$ ▢

59. $\int_0^6 [2 + f(x)]\,dx =$ ▢

60. The average value of f over the interval $[0, 6]$ is ▢.