Semester 1 Practice Final

1. Evaluate the function at the specified value of the independent variable and simplify.

$$q(x) = \frac{-2x}{3x+5}$$

$$q(y-3)$$

ⓐ $\frac{-2y-6}{3y-4}$ ⓑ $\frac{-2x+6}{3x-4}$ ⓒ $\frac{-2y+6}{3y-4}$ ⓓ $-\frac{3}{2}$ ⓔ $\frac{3}{7}$

$$q(y-3) = \frac{-2(y-3)}{3(y-3)+5} = \frac{-2y+6}{3y-4}$$

2. Find all real values of x such that $f(x) = 0$.

$$f(x) = \frac{-6x-9}{2}$$

ⓐ $-\frac{3}{4}$ ⓑ $\pm\frac{3}{4}$ ⓒ $\pm\frac{3}{2}$ ⓓ $-\frac{3}{2}$ ⓔ $\frac{3}{2}$

$$0 = \frac{-6x-9}{2}$$

$$0 = -6x-9$$

$$0 = 6x+9$$

$$-9 = 6x$$

$$\frac{-9}{6} = x$$

$$x = -\frac{3}{2}$$

3. Find the value(s) of x for which $f(x) = g(x)$.

$f(x) = x^2 + 7x + 33 \qquad g(x) = -6x - 9$

ⓐ $6,\ 7$ ⓑ $33,\ 7,\ -\frac{3}{2}$ ⓒ $-40,\ -\frac{3}{2}$ ⓓ $-6,\ -7$

ⓔ $33,\ 26,\ -\frac{3}{2}$

$$\overbrace{x^2+7x+33}^{f(x)} = \overbrace{-6x-9}^{g(x)}$$

$$x^2+13x+42=0$$

$$(x+6)(x+7)=0$$

$$x=-6 \quad x=-7$$

4. Find the domain of the function.

$q(s) = \dfrac{8s}{s-6}$ $\qquad s \neq 6$

ⓐ $s = 6,\ s = 0$ ⓑ all real numbers $s \neq 6$
ⓒ all real numbers $s \neq 6,\ s \neq 0$ ⓓ all real numbers ⓔ $s = 6$

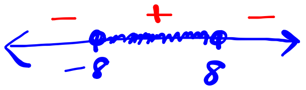
5. Find the domain of the function.

$q(s) = \sqrt{64 - s^2}$

ⓐ $s \leq 8$ ⓑ $s \leq -8$ or $s \geq 8$ ⓒ $s \geq 0$ ⓓ all real numbers ⓔ $-8 \leq s \leq 8$

$64 - s^2 = 0$

$s = \pm 8$



test point $q(0) = \sqrt{64}$ positive in the $\sqrt{}$

$q(-9) = \sqrt{64-81} =$ not real

$q(9) = \sqrt{64-81} =$ not real

6. Let the function f be defined by the equation $y = f(x)$, where x and $f(x)$ are real numbers. Find the domain and range of the function

$f(x) = \sqrt{16x^2 - 2}$.

ⓐ domain: $(-\infty, -\frac{\sqrt{2}}{4}] \cup [\frac{\sqrt{2}}{4}\}, +\infty)$

ⓑ domain: $(-\infty, -\frac{2}{16}) \cup (\frac{2}{16}, +\infty)$

ⓒ domain: $(-\infty, -\frac{2}{4}] \cup [\frac{2}{4}, +\infty)$

ⓓ domain: $(-\infty, -\frac{2}{16}] \cup [\frac{2}{16}, +\infty)$

ⓔ domain: $(-\infty, -\frac{\sqrt{2}}{4}) \cup (\frac{\sqrt{2}}{4}\}, +\infty)$

$16x^2 - 2 = 0$

$x = \pm \frac{\sqrt{2}}{4}$

$f(0) = \sqrt{-2}$

no negatives allowed in square roots

7. Use the functions given by $f(x)=\frac{x}{8}+1$ and $g(x)=x^3$ to find the indicated value.

$(f \circ g)^{-1}(5)$

ⓐ undefined ⓑ $2\sqrt[3]{6}$ ⓒ $3\sqrt[3]{2}$ ⓓ $2\sqrt[3]{5}-1$ ⓔ $\frac{637}{512}$

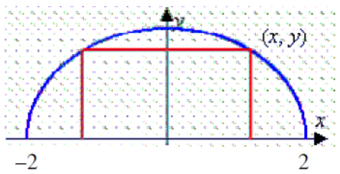
$f(g(x)) = \frac{x^3}{8}+1$

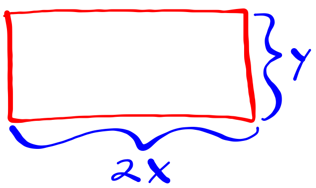
$f(g(x))^{-1} = \sqrt[3]{8(x-1)}$

$f(g(5))^{-1} = \sqrt[3]{8(5-1)}$

8. A rectangle is bounded by the x-axis and the semicircle $y=\sqrt{4-x^2}$ (see figure). Write the area A of the rectangle as a function of x and determine the domain of the function.

$y=\sqrt{4-x^2}$





ⓐ $A(x)=x\sqrt{4-x^2},\quad x\ge 0$
ⓑ $A(x)=2|x|\sqrt{4-x^2},\quad -2\le x\le 2$
ⓒ $A(x)=|x|\sqrt{4-x^2},\quad$ all real numbers
ⓓ $A(x)=2x\sqrt{4-x^2},\quad x\ge 0$
ⓔ $A(x)=2x\sqrt{4-x^2},\quad -2\le x\le 2$

$A = 2xy$

$A = 2x(\sqrt{4-x^2})$ $\quad 0 \le x \le 2$

or $A = 2|x|(\sqrt{4-x^2})$ $\quad -2 \le x \le 2$

9. Evaluate the difference quotient for the function.

$$\frac{f(x+h)-f(x)}{h}$$

$f(x) = 2x - 9$

ⓐ 2 ⓑ $\frac{9}{2}$ ⓒ $2x+9$ ⓓ 9 ⓔ -9

$$\frac{\overbrace{2(x+h)-9}^{f(x+h)} - \overbrace{(2x-9)}^{f(x)}}{h} = \frac{\cancel{2x}+2h\cancel{-9}\cancel{-2x}\cancel{+9}}{h} = \frac{2h}{h} = 2$$

10. Find the difference quotient and simplify your answer.

$f(y) = -4y^2 + 6y$, $\frac{f(1+h)-f(1)}{h}$, $h \neq 0$

ⓐ $-2-4h$ ⓑ $6-4h$ ⓒ $6-4y+\frac{12}{y}$ ⓓ $8+h$

ⓔ $-2-4y+\frac{12}{y}$

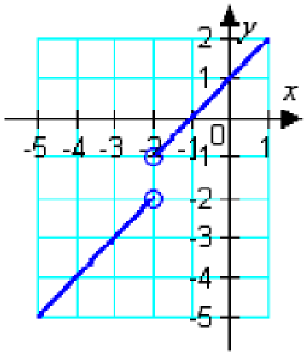
$$\frac{\overbrace{-4(1+h)^2+6(1+h)}^{f(1+h)} - \overbrace{(-4(1)+6(1))}^{f(1)}}{h}$$

$$= \frac{-4(1+2h+h^2)+6+6h-2}{h}$$

$$= \frac{-4-8h-4h^2+6+6h-2}{h}$$

$$= \frac{-4h^2-2h}{h} = \boxed{-4h-2}$$

11. Use the graph of the function to find the domain and range of f.



ⓐ domain: all real numbers
range: $(-\infty, -2) \cup (-1, \infty)$

ⓑ domain: all real numbers
range: all real numbers

ⓒ domain: $(-\infty, -2) \cup (-2, \infty)$
range: $(-\infty, -2) \cup (-1, \infty)$

ⓓ domain: all real numbers
range: $(-\infty, -2] \cup [-1, \infty)$

ⓔ domain: $(-\infty, -2) \cup (-1, \infty)$
range: $(-\infty, -2) \cup (-2, \infty)$

12. Determine whether the function is even, odd, or neither.

$$f(x) = 7x^{\frac{3}{4}}$$

when $f(-x) = f(x) \rightarrow$ even

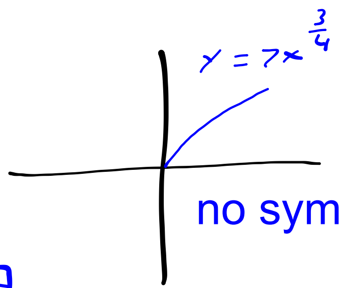
when $f(-x) = -f(x) \rightarrow$ odd

ⓐ odd ⓑ even ⓒ neither

$f(-x) = 7\sqrt[4]{(-x)^3}$

$f(-x) = 7\sqrt[4]{-x^3}$ $x < 0$

$f(-x) = 7(-x)^{\frac{3}{4}}$ $x < 0$ $\boxed{\text{neither}}$



no symmetry

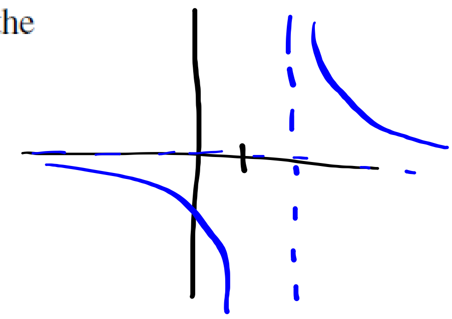
13. Describe the sequence of transformations from the related common function $f(x) = \sqrt{x}$ to g.

$g(x) = -\sqrt{x} + 6$

ⓐ reflection in the y-axis; then horizontal shift 6 units right ⓑ reflection in the y-axis; then vertical shift 6 units up ⓒ reflection in the y-axis; then horizontal shift 6 units left ⓓ reflection in the x-axis; then vertical shift 6 units up ⓔ reflection in the x-axis; then vertical shift 6 units down

14. Find all vertical and horizontal asymptotes of the function.

$f(x) = \dfrac{1}{x-2}$



ⓐ Horizontal : $y = -1$, Vertical : $x = 2$
ⓑ Horizontal : $y = 0$, Vertical : $x = -2$
ⓒ Horizontal : $y = 1$, Vertical : $x = -2$
ⓓ Horizontal : $y = 0$, Vertical : $x = 2$
ⓔ Horizontal : $y = 1$, Vertical : $x = 2$

15. Find $g \circ f$.

$f(x) = x - 3 \qquad g(x) = x^2$

$g(f(x)) = (x-3)^2$

$= x^2 - 6x + 9$

ⓐ $(g \circ f)(x) = x^2 - 6x + 9$

ⓑ $(g \circ f)(x) = x^2 + 9$

ⓒ $(g \circ f)(x) = x^2 - 3$

ⓓ $(g \circ f)(x) = x^2 - 3x + 9$

ⓔ $(g \circ f)(x) = x^2 - 9$

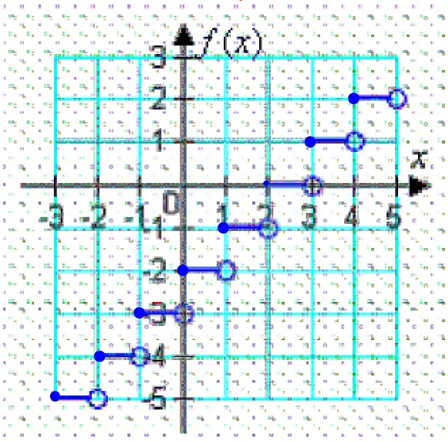
16. Let $f(x) = \frac{1}{x}$, $g(x) = x + 5$. Find the composite function which expresses the given correspondence correctly.

$f(g(x))$ because we want to plug g into f.

$$\frac{1}{x+5}$$

ⓐ $(g \circ g)(x)$ ⓑ $(f \circ g)(x)$ ⓒ $(f \circ f)(x)$
ⓓ $(g \circ f)(x)$ ⓔ none of the above

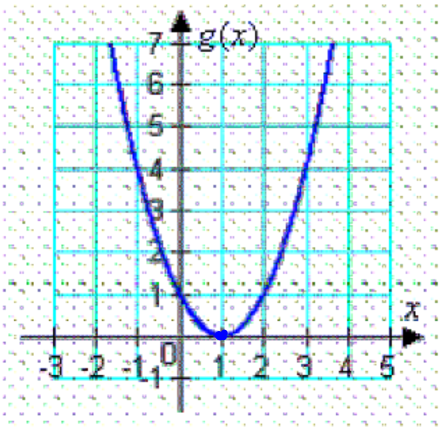
17. Use the graphs of f and g to evaluate the function. step- function



$f(g(1)) = ?$

$g(1) = 0$

$f(0) = -2$



$(f \circ g)(1) \to f(g(1))$

ⓐ -1 ⓑ 9 ⓒ 0 ⓓ -4 ⓔ -2

18. Find the inverse function of f.

$f(x) = x^5 + 2$ switch x and y

$x = y^5 + 2$

$y = \sqrt[5]{x-2}$

ⓐ $f^{-1}(x) = -\sqrt[5]{x} + 2$ ⓑ $f^{-1}(x) = \sqrt[5]{x} - 2$

ⓒ $f^{-1}(x) = -\sqrt[5]{x+2}$ ⓓ $f^{-1}(x) = \sqrt[5]{x-2}$

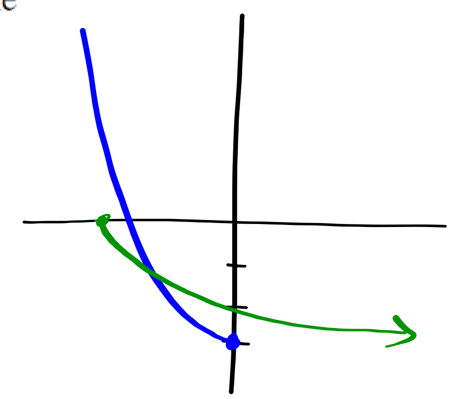
ⓔ $f^{-1}(x) = \sqrt[5]{x} + 2$

19. The function $f(x) = x^2 - 3$ is one-to-one on the domain $(x \le 0)$. Find $f^{-1}(x)$.

ⓐ $f^{-1}(x) = x^2 + 3$ ⓑ $f^{-1}(x) = \sqrt{x+3}$

ⓒ $f^{-1}(x) = \sqrt{x-3}$ ⓓ $f^{-1}(x) = \dfrac{1}{x^2 - 3}$

ⓔ $f^{-1}(x) = -\sqrt{x+3}$



$x = y^2 - 3$

$y^{-1} = \pm\sqrt{x+3}$ range must be $y \le 0$

so $y^{-1} = -\sqrt{x+3}$

20. Find all real zeros of the polynomial $f(x) = x^3 + 5x^2 - 16x - 80$ and determine the multiplicity of each.

ⓐ $x = -5$, multiplicity 2; $x = -4$, multiplicity 1 ⓑ $x = -5$, multiplicity 3

ⓒ $x = -4$, multiplicity 1; $x = 5$, multiplicity 1; $x = -5$, multiplicity 1

ⓓ $x = 4$, multiplicity 2; $x = -5$, multiplicity 1

ⓔ $x = 4$, multiplicity 1; $x = -4$, multiplicity 1; $x = -5$, multiplicity 1

$0 = x^2(x+5) - 16(x+5)$

$0 = (x^2 - 16)(x+5)$

$0 = (x+4)(x-4)(x+5)$

$x = -4$ $x = 4$ $x = -5$

21. Find all real zeros of the polynomial $f(x) = x^4 + 8x^3 + 7x^2$ and determine the multiplicity of each.

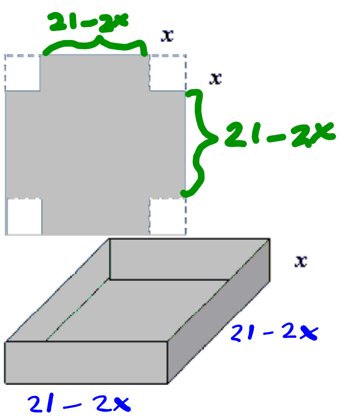
ⓐ $x = 0$, multiplicity 1; $x = 7$, multiplicity 1; $x = -7$, multiplicity 1; $x = 1$, multiplicity 1
ⓑ $x = 0$, multiplicity 2; $x = -7$, multiplicity 1; $x = -1$, multiplicity 1
ⓒ $x = 7$, multiplicity 2; $x = 1$, multiplicity 2
ⓓ $x = 0$, multiplicity 2; $x = 7$, multiplicity 1; $x = 1$, multiplicity 1
ⓔ $x = -7$, multiplicity 2; $x = -1$, multiplicity 2

$0 = x^2(x^2+8x+7)$

$0 = x^2(x+1)(x+7)$

$x=0$ mult 2 $x=-1$ mult 1 $x=-7$ mult 1

22. An open box is to be made from a square piece of cardboard, 21 inches on a side, by cutting equal squares with sides of length x from the corners and turning up the sides (see figure below). Determine the function, V, in terms of x, that represents the volume of the box.



ⓐ $V(x) = -4x^3 + 42x^2 - 21x$
ⓑ $V(x) = -2x^3 + 21x^2$
ⓒ $V(x) = 4x^3 - 84x^2 + 441x$
ⓓ $V(x) = 4x^3 - 42x^2 + 21x$
ⓔ $V(x) = -4x^3 + 42x^2$

$V(x) = x(21-2x)^2$

23. Do the operation and express the answer in $a + bi$ form.

$$(2 - \sqrt{-64}) + (6 + \sqrt{-36})$$

ⓐ $8 + 2i$ ⓑ $8 - 2i$ ⓒ $-8 + 2i$ ⓓ $-16 - 4i$ ⓔ $-16 + 4i$

$2 - 8i + 6 + 6i$

$8 - 2i$

24. Determine the domain of the function

$$f(x) = \frac{x^2 - 25}{x^2 + 4x - 5}.$$

denominator factors: $(x+5)(x-1)$

$x \neq -5 \quad x \neq 1$

ⓐ Domain: all real numbers ⓑ Domain: all real numbers except $x = 5$ and -1
ⓒ Domain: all real numbers except $x = -1$
ⓓ Domain: all real numbers except $x = -5$ and -1 ⓔ Domain: all real numbers except $x = -5$ and 1

25. Determine the zeros (if any) of the rational function $f(x) = \dfrac{x^2 - 49}{x - 4}$.

set numerator = 0

ⓐ $x = -7, x = 7$ ⓑ $x = -49, x = 49$
ⓒ $x = \dfrac{7}{4}, x = -\dfrac{7}{4}$ ⓓ no zeros ⓔ $x = 4$

$x^2 - 49 = 0$

$x^2 = 49$

$x = \pm 7$

26. Let Q represent a mass of radioactive radium (^{226}Ra) (in grams), whose half-life is 1599 years. The quantity of radium present after t years is

$$Q = 5\left(\frac{1}{2}\right)^{t/1599}$$

plug in 300 into t

Determine the quantity present after 300 years. Round to the nearest hundredth of a gram.

ⓐ 0.88 g ⓑ 0.47 g ⓒ 5.00 g ⓓ 0.00 g
ⓔ 4.39 g

27. Rewrite the logarithmic equation $\log_4 \frac{1}{16} = -2$ in exponential form.

ⓐ $4^{16} = -2$ ⓑ $4^{-2} = -\frac{1}{16}$ ⓒ $4^{-2} = \frac{1}{16}$

ⓓ $4^{1/16} = -2$ ⓔ $\left(\frac{1}{16}\right)^{-2} = 4$

$4^{-2} = \frac{1}{16}$

28. Write the logarithmic equation $\ln 5 = 1.609\ldots$ in exponential form.

ⓐ $2.303e^{1.609\ldots} = 5$ ⓑ $e^5 = 1.609\ldots$

ⓒ $2.303 \times 10^5 = 1.609\ldots$ ⓓ $10^5 = 1.609\ldots$

ⓔ $e^{1.609\ldots} = 5$

$e^{1.609} = 5$

29. Rewrite the exponential equation $5^{-3} = \frac{1}{125}$ in logarithmic form.

ⓐ $\log_{125} 5 = -3$ ⓑ $\log_5 125 = -3$

ⓒ $\log_3 125 = -3$ ⓓ $\log_5 \frac{1}{125} = 3$

ⓔ $\log_5 \frac{1}{125} = -3$

$\log_5 \frac{1}{125} = -3$

30. Write the exponential equation $e^{1/2} = 1.6487\ldots$ in logarithmic form.

ⓐ $\ln(1) = \frac{1.6487\ldots}{2}$ ⓑ $\log_{10}(1.6487\ldots) = \frac{1}{2}$

ⓒ $\ln(1.6487\ldots) = \frac{1}{2}$ ⓓ $\ln\left(\frac{1}{2}\right) = 1.6487\ldots$

ⓔ $2.303\log\left(\frac{1}{2}\right) = 1.6487\ldots$

$\ln 1.6487 = \frac{1}{2}$

31. Solve the equation $\log(1-x) = \log(10)$ for x using the One-to-One Property.
ⓐ -11 ⓑ 0 ⓒ 11 ⓓ The equation has no solution. ⓔ -9

$1 - x = 10$

$x = -9$

32. Identify the x-intercept of the function $f(x) = 2\ \ln(x-3)$.
ⓐ $x = 3$ ⓑ $x = 0$ ⓒ $x = 2$ ⓓ $x = 4$ ⓔ The function has no x-intercept.

$0 = 2\ln(x-3)$

$0 = \ln(x-3)$

$e^0 = e^{\ln(x-3)}$

$1 = x - 3$

$x = 4$

33. A population growing at an annual rate r will triple in a time t given by the formula $t = \frac{\ln 3}{r}$. If the growth rate remains constant and equals 9% per year, how long will it take the population of the town to triple?

$t = \frac{\ln 3}{.09}$

$t \approx 12.2$

ⓐ 6.6 years ⓑ 1 years ⓒ 5.3 years ⓓ 2.2 years
ⓔ 12.2 years

34. Simplify the expression $\log_5 150$.

ⓐ The expression cannot be simplified. ⓑ 6
ⓒ $2 \log_5 6$ ⓓ $2 + \log_5 6$ ⓔ $30 \log_5 2$

$\log_5(25 \cdot 6)$

$\log_5 25 + \log_5 6$

$2 + \log_5 6$

35. Simplify the expression $\log_3\left(\frac{1}{27}\right)^4$.

ⓐ -108 ⓑ 4 ⓒ -12 ⓓ 1 ⓔ The expression cannot be simplified.

$= \log_3(3^{-3})^4$

$= \log_3 3^{-12}$

$= -12\log_3 3$

$= -12$

36. Assume that x, y, z and b are positive numbers. Use the properties of logarithms to write the expression $\log_b \sqrt[4]{\frac{x^7 y^2}{z^4}}$ in terms of the logarithms of x, y, and z.

ⓐ $28\log_b x + 8\log_b y - 16\log_b z$ ⓑ $\frac{7}{4}\log_b x + \frac{1}{2}\log_b y - \log_b z$ ⓒ $\frac{7}{4}\log_b (x+y-z)$

ⓓ $\log_b x + \frac{1}{2}\log_b y - \log_b z$ ⓔ $\frac{7}{4}\log_b x + \log_b y - \log_b z$

$= \log_b\left(\frac{x^7y^2}{z^4}\right)^{\frac{1}{4}}$

$= \log_b x^{\frac{7}{4}} + \log_b y^{\frac{1}{2}} - \log_b z$

$= \frac{7}{4}\log_b x + \frac{1}{2}\log_b y - \log_b z$

37. Condense the expression $\frac{1}{5}\left[\log_4 x + \log_4 7\right] - \left[\log_4 y\right]$ to the logarithm of a single term.

$\log_4 (7x)^{\frac{1}{5}} - \log_4 y$

$\log_4 \frac{\sqrt[5]{7x}}{y}$

ⓐ $\log_4 \sqrt[5]{\frac{7x}{y}}$ ⓑ $\log_4 \frac{(7x)^5}{y}$ ⓒ $\log_4 \frac{\sqrt[5]{7x}}{y}$

ⓓ $\log_4 \frac{7x}{5y}$ ⓔ $\log_4 \sqrt[5]{7x} - \log_4 y$

38. Find the exact value of $\log_8 \sqrt[3]{64}$ without using a calculator.

ⓐ $\frac{64}{3}$ ⓑ $\frac{2}{3}$ ⓒ -1 ⓓ $\frac{3}{64}$ ⓔ $\frac{16}{3}$

$8^x = \sqrt[3]{64}$

$8^x = \sqrt[3]{8^2}$

$8^x = 8^{\frac{2}{3}}$

$x = \frac{2}{3}$

39. Find the exact value of $\log_4 36 - \log_4 9$ without using a calculator.

ⓐ 9 ⓑ $\frac{1}{2}$ ⓒ 4 ⓓ $\frac{9}{2}$ ⓔ 1

$\log_4 \frac{36}{9}$

$\log_4 4$

1

40. Find the exact value of $\ln e^{2.50} - \ln\sqrt{e}$ without using a calculator.

ⓐ 5 ⓑ 1.25 ⓒ 2.5 ⓓ 3 ● 2

$\ln\left|e^{2.5-.5}\right|$

$\ln e^2$

2

41. Solve the equation.

$2^x = 5$

ⓐ $x = 0.5541$ ⓑ $x = 5.8628$ ⓒ $x = 0.4307$ ⓓ $x = 1.4307$ ⓔ $x = 1.6652$ ⓕ $x = 2.3$

$\log_2 5 = x$

$x \approx 2.3$

42. Solve $\left(\frac{1}{5}\right)^x = 125$ for x.

ⓐ 1 ⓑ -5 ⓒ -3 ⓓ -1 ⓔ no solution

$(5^{-1})^x = 5^3$

$5^{-x} = 5^3$

$-x = 3$

$x = -3$

43. Solve for x: $9\left(10^{x-2}\right) = 23$. Round to 3 decimal places.

ⓐ 2.407 ⓑ −1.362 ⓒ 1.362 ⓓ 0.407 ⓔ no solution

$$10^{x-2} = \frac{23}{9}$$

$$(x-2)\cancel{\log 10} = \log \frac{23}{9}$$

$$x-2 = \log \frac{23}{9}$$

$$x = \log \frac{23}{9} + 2$$

44. Simplify the expression.

$\log_3 3^6$

ⓐ 6 ⓑ 36 ⓒ 3 ⓓ 18 ⓔ none of these

$$6 \log_3 3$$

$$6(1)$$

$$6$$

45. Simplify the expression.

$9^{\log_9 2}$ $=2$

ⓐ 2 ⓑ 18 ⓒ 4 ⓓ 9 ⓔ none of these

46. Use the One-to-One Property to solve the following equation for x.

$2^{3x} = 128$

ⓐ $\frac{3}{7}$ ⓑ $\frac{7}{3}$ ⓒ $-\frac{64}{3}$ ⓓ 2 ⓔ $\frac{128}{3}$

$2^{3x} = 2^7$

$3x = 7$

$x = \frac{7}{3}$

47. Solve for x: $4^{-x/2} = 0.0052$. Round to 3 decimal places.

ⓐ 10.518 ⓑ −3.794 ⓒ −13.291 ⓓ 13.291
ⓔ 7.587

$\ln 4^{-\frac{x}{2}} = \ln 0.0052$

$-\frac{x}{2} = \ln 0.0052$

$x = -2\ln 0.0052$

$x \approx 10.518$

48. An initial investment of $9000 grows at an annual interest rate of 5% compounded continuously. How long will it take to double the investment?

ⓐ 13.86 years ⓑ 14.40 years ⓒ 13.40 years
ⓓ 1 year ⓔ 14.86 years

$A = Pe^{rt}$

$18000 = 9000e^{.05t}$

$2 = e^{.05t}$

$\ln 2 = \ln e^{.05t}$

$\ln 2 = .05t$

$\frac{\ln 2}{.05} = t$

$t \approx 13.86$

49. Carbon dating presumes that, as long as a plant or animal is alive, the proportion of its carbon that is ^{14}C is constant. The amount of ^{14}C in an object made from harvested plants, like paper, will decline exponentially according to the equation $A = A_o e^{-0.0001213t}$, where A represents the amount of ^{14}C in the object, A_o represents the amount of ^{14}C in living organisms, and t is the time in years since the plant was harvested. If an archeological artifact has 40% as much ^{14}C as a living organism, how old would you predict it to be? Round to the nearest year.

ⓐ 5715 years ⓑ 87 years ⓒ 7554 years
ⓓ 16,006 years ⓔ 30,411 years

$40 = 100 e^{-0.0001213t}$

$\ln \frac{2}{5} = \ln e^{-0.0001213t}$

$\ln \frac{2}{5} = -0.0001213t$

$t = \dfrac{\ln \frac{2}{5}}{-0.0001213}$

$t \approx 7553.9$

50. Tritium, a radioactive isotope of hydrogen, has a half-life of 12.4 years. Of an initial sample of 69 grams, how much will remain after 75 years?

ⓐ 1.5848 grams ⓑ 61.5289 grams ⓒ 0.0000 grams ⓓ 1.0426 grams ⓔ 17.2500 grams

$A = 69\left(\frac{1}{2}\right)^{\frac{75}{12.4}}$

$A \approx 1.0426$

51. The chemical acidity of a solution is measured in units of pH: $\text{pH} = -\log\left[H^+\right]$, where $\left[H^+\right]$ is the hydrogen ion concentration in the solution. What is $\left[H^+\right]$ if the $\text{pH} = 8.8$?

ⓐ 6.31×10^{-8} ⓑ 1.58×10^{-9} ⓒ 8.800
ⓓ 1.58×10^{-8} ⓔ 6.31×10^{-9}

$8.8 = -\log H$
$-8.8 = \log H$
$10^{-8.8} = H$

52. The chemical acidity of a solution is measured in units of pH: $\text{pH} = -\log\left[H^+\right]$, where $\left[H^+\right]$ is the hydrogen ion concentration in the solution. If a sample of rain has a pH of 3.2, how many times higher is its $\left[H^+\right]$ than pure water's, which has a pH of 7?

ⓐ 7 ⓑ 1.6×10^{4} ⓒ 1.6×10^{3} ⓓ 6.3×10^{3}
ⓔ 6.3×10^{4}

$3.2 = -\log H$
$-3.2 = \log H$
$10^{-3.2} = H$
$H_s = 10^{-3.2}$ solution
$H_w = 10^{-7}$ water
$H_w(x) = H_s$
$x = \frac{H_s}{H_w} \approx 6309.6$

53. Determine two coterminal angles (one positive and one negative) for $\theta = \frac{2\pi}{3}$.

(a) $\frac{10\pi}{3}, -\frac{8\pi}{3}$ (b) $\frac{4\pi}{3}, -\frac{4\pi}{9}$ (c) $\frac{8\pi}{3}, -\frac{4\pi}{3}$

(d) $\frac{5\pi}{3}, -\frac{7\pi}{3}$ (e) $\frac{7\pi}{3}, -\frac{2\pi}{3}$

$\frac{2\pi}{3} + 2\pi$ $\frac{2\pi}{3} - 2\pi$

$\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$ $\frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$

54. Find (if possible) the supplement of $\frac{11\pi}{13}$.

(a) $\frac{12\pi}{13}$ (b) $\frac{11\pi}{26}$ (c) not possible (d) $\frac{2\pi}{13}$

(e) $\frac{5\pi}{13}$

$\frac{11\pi}{13} + x = \pi$

$\pi - \frac{11\pi}{13}$

$\frac{13\pi}{13} - \frac{11\pi}{13} = \frac{2\pi}{13}$

55. Rewrite $\frac{4\pi}{9}$ in degree measure.

ⓐ 53° ⓑ 120° ⓒ 160° ⓓ 40° ⓔ 80°

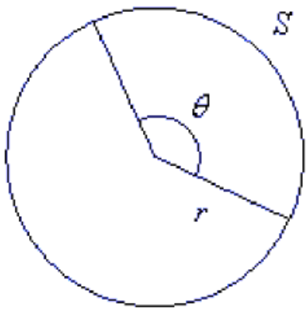
$\frac{4\pi}{9} \cdot \frac{180}{\pi} = 80$

56. Rewrite 47.10° in radian measure. Round to three decimal places.

ⓐ 0.164 ⓑ 0.099 ⓒ 0.822 ⓓ 0.123
ⓔ 0.148

$47.10\left(\frac{\pi}{180}\right) =$

57. Find the angle, in radians, in the figure below if $S = 12$ and $r = 8$.



$S = r\theta$

$\theta = \frac{S}{r} = \frac{12}{8} = \frac{3}{2}$

ⓐ $\frac{2\pi}{3}$ ⓑ $\frac{3}{2}$ ⓒ $\frac{3\pi}{2}$ ⓓ $\frac{2}{3}$ ⓔ $\frac{5\pi}{2}$

58. Find the length of the arc, S, on a circle of radius 7 feet intercepted by a central angle of 330°. Round to two decimal places.

ⓐ $S = 53.76$ feet ⓑ $S = 80.63$ feet
ⓒ $S = 40.32$ feet ⓓ $S = 26.88$ feet
ⓔ $S = 32.25$ feet

$S = r\theta$

$S = 7\left(\frac{11\pi}{6}\right)$

$S \approx 40.317$

$330°\left(\frac{\pi}{180}\right) = \frac{11\pi}{6}$

59. The displacement from equilibrium of an oscillating weight suspended by a spring is given by $y(t) = 2\cos 6t$, where y is the displacement in centimeters and t is the time in seconds. Find the displacement when $t = 1.45$, rounding answer to four decimal places.

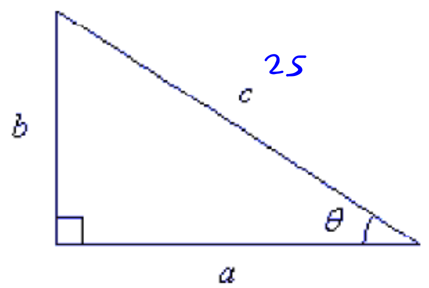
ⓐ 2.7845 cm ⓑ –1.4973 cm ⓒ –5.8257 cm
ⓓ –3.6205 cm ⓔ 1.4460 cm

calculator in radian mode

$y(1.45) = 2\cos[6(1.45)]$

$y(1.45) \approx -1.4973$

60. Find the exact value of $\csc\theta$, using the triangle shown in the figure below, if $a = 24$ and $b = 7$.



ⓐ $\frac{25}{24}$ ⓑ $\frac{24}{7}$ ⓒ $\frac{24}{25}$ ⓓ $\frac{25}{7}$ ⓔ $\frac{7}{25}$

$\csc\theta = \frac{c}{b}$

$\csc\theta = \frac{25}{7}$

61. Given $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$, determine the following:

$\csc 30°$

ⓐ undefined ⓑ $\csc 30° = 2$ ⓒ $\csc 30° = \frac{\sqrt{3}}{3}$

ⓓ $\csc 30° = \frac{\sqrt{2}}{2}$ ⓔ $\csc 30° = \sqrt{3}$

$\csc 30° = \frac{1}{\sin 30°}$

$\csc 30° = \frac{1}{\frac{1}{2}} = 2$

62. Given $\sec\theta = \sqrt{10}$ and $\tan\theta = 3$, determine the following.

$\csc(90° - \theta)$

ⓐ $\csc(90° - \theta) = \sqrt{10}$ ⓑ undefined

ⓒ $\csc(90° - \theta) = \frac{1}{3}$ ⓓ $\csc(90° - \theta) = \frac{\sqrt{10}}{3}$

ⓔ $\csc(90° - \theta) = \frac{3\sqrt{10}}{10}$

$\csc(90° - \theta) = \sec\theta$

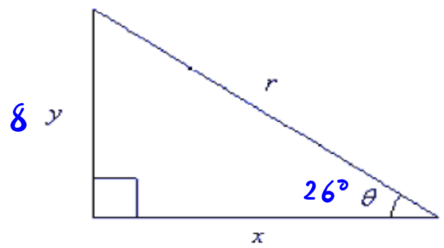
$\csc(90° - \theta = \sqrt{10}$

63. If $\sin\theta = \dfrac{\sqrt{3}}{2}$, find the value of θ in degrees ($0 < \theta < 90°$) without the aid of a calculator.

ⓐ $\theta = 15°$ ⓑ $\theta = 90°$ ⓒ $\theta = 60°$ ⓓ $\theta = 75°$
ⓔ $\theta = 45°$

$\theta = \arcsin \frac{\sqrt{3}}{2} = 60°$

64. Using the figure below, if $\theta = 26°$ and $y = 8$, determine the exact value of x.



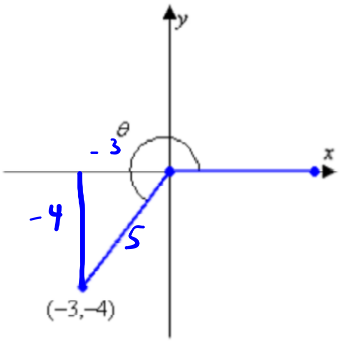
$\tan 26° = \frac{8}{x}$

$x \tan 26° = 8$

$x = \frac{8}{\tan 26°}$

ⓐ $x = \dfrac{8}{\tan 26°}$ ⓑ $x = \dfrac{4}{\sin 13°}$ ⓒ $x = \dfrac{26}{\csc 8°}$
ⓓ $x = \dfrac{8}{\cot 26°}$ ⓔ $x = \dfrac{13}{\tan 4°}$

65. Given the figure below, determine the value of $\sin\theta$.

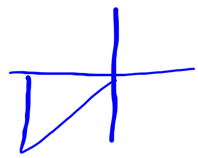


$\sin\theta = -\frac{4}{5}$

ⓐ $\sin\theta = \frac{3}{4}$ ⓑ $\sin\theta = -\frac{3}{5}$ ⓒ $\sin\theta = -\frac{3}{4}$

ⓓ $\sin\theta = -\frac{4}{5}$ ⓔ $\sin\theta = \frac{4}{3}$

66. The point $(-5, -12)$ is on the terminal side of an angle in standard position. Determine the exact value of $\tan\theta$.



$(-5, -12)$

ⓐ $\tan\theta = \frac{12}{5}$ ⓑ $\tan\theta = \frac{17}{12}$ ⓒ $\tan\theta = -\frac{13}{12}$

ⓓ $\tan\theta = -\frac{12}{13}$ ⓔ $\tan\theta = -\frac{1}{12}$

$\tan\theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$

67. Given the equation below, determine two solutions such that $0° \le \theta < 360°$.

$\csc\theta = 2$

ⓐ $\theta = 225°, 315°$ ⓑ $\theta = 30°, 210°$
ⓒ $\theta = 30°, 150°$ ⓓ $\theta = 60°, 240°$
ⓔ $\theta = 45°, 225°$

where is $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$30°$ $150°$

68. Given the equation below, determine two solutions such that $0 \le \theta < 2\pi$.

$\sec\theta = \frac{2\sqrt{3}}{3}$

ⓐ $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$ ⓑ $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$
ⓒ $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$ ⓓ $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$
ⓔ $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

$\cos\theta = \frac{1}{\sec\theta} = \frac{3 \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}}$

$= \frac{\cancel{3}\sqrt{3}}{\cancel{3} \cdot 2}$

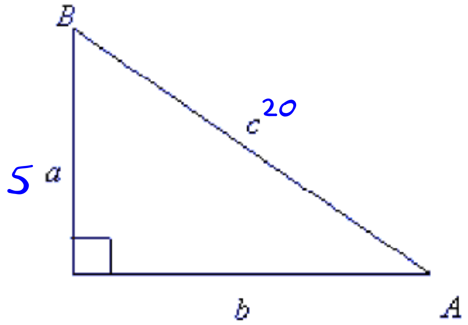
$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

69. If $a = 5$ and $c = 20$, determine the value of A. Round to two decimal places.

calculator in degree mode

$A = \arcsin \frac{5}{20} \approx 14.48^\circ$



ⓐ 75.96° ⓑ 80.52° ⓒ 14.04° ⓓ 14.48°
ⓔ 75.52°

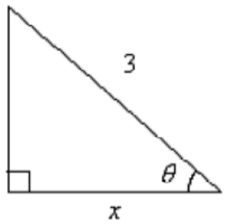
70. Evaluate $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ without using a calculator.

ⓐ $-\frac{5\pi}{6}$ ⓑ $-\frac{2\pi}{3}$ ⓒ $\frac{5\pi}{6}$ ⓓ $-\frac{\pi}{3}$ ⓔ $\frac{3\pi}{4}$

71. Evaluate $\arctan\frac{\sqrt{3}}{3}$ without using a calculator.

ⓐ $-\frac{3\pi}{4}$ ⓑ $\frac{\pi}{3}$ ⓒ $-\frac{\pi}{6}$ ⓓ $\frac{\pi}{6}$ ⓔ $\frac{\pi}{4}$

72. Use an inverse function to write θ as a function of x.

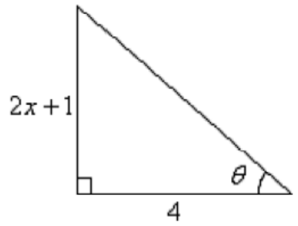


ⓐ $\theta = \arccos\frac{3}{\sqrt{x^2+9}}$ ⓑ $\theta = \arccos\frac{x}{3}$

ⓒ $\theta = \arccos\frac{\sqrt{x^2+9}}{3}$ ⓓ $\theta = \arccos\frac{3\pi}{x}$

ⓔ $\theta = \arccos\frac{3}{x}$

73. Use an inverse function to write θ as a function of x.



ⓐ $\theta = \tan^{-1}\left(\frac{2x+1}{4}\right)$ ⓑ $\theta = \tan^{-1}\left(\frac{4}{2x+1}\right)$

ⓒ $\theta = \tan^{-1}\left(\frac{x+1}{2}\right)$ ⓓ $\theta = \tan^{-1}\left(\frac{1}{x+1}\right)$

ⓔ $\theta = \sin^{-1}(2x+1)$

74. Use the properties of inverse trigonometric functions to evaluate $\sin[\arcsin(-0.63)]$.

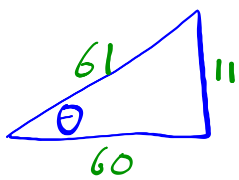
ⓐ -0.63 ⓑ -0.40 ⓒ -0.93 ⓓ -1.07
ⓔ -0.59

75. Use the properties of inverse trigonometric functions to evaluate $\arctan\left[\tan\left(\frac{2\pi}{9}\right)\right]$.

ⓐ $\frac{2\pi}{7}$ ⓑ $\frac{2\pi}{9}$ ⓒ $-\frac{7\pi}{9}$ ⓓ $\frac{\pi}{9}$ ⓔ $\frac{9\pi}{2}$

76. Find the exact value of $\sin\left(\arctan\frac{11}{60}\right)$.

ⓐ $\frac{61}{72}$ ⓑ $\frac{72}{11}$ ⓒ $\frac{60}{11}$ ⓓ $\frac{61}{11}$ ⓔ $\frac{11}{60}$ ⓕ $\frac{11}{61}$



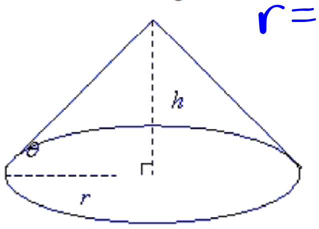
$\sin(\underbrace{\arctan\frac{11}{60}}_{\theta})$

$\tan\theta = \frac{11}{60}$

$\sin\theta = \frac{11}{61}$

77. A granular substance such as sand naturally settles into a cone-shaped pile when poured from a small aperture. Its height depends on the humidity and adhesion between granules. The angle of elevation of a pile, θ, is called the angle of repose. If the height of a pile of sand is 15 feet and its diameter is approximately 52 feet, determine the angle of repose. Round answer to nearest degree.

$r = 26$

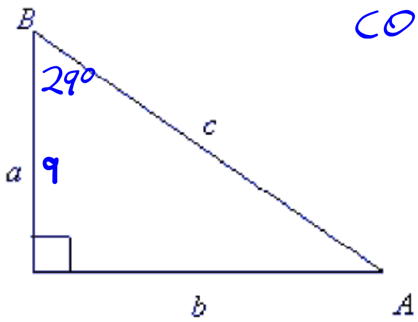


ⓐ 27° ⓑ 28° ⓒ 26° ⓓ 30° ⓔ 29°

$\theta = \arctan \frac{15}{26}$

$\theta \approx 29.98°$

78. If $B = 29°$ and $a = 9$, determine the value of c. Round to two decimal places.

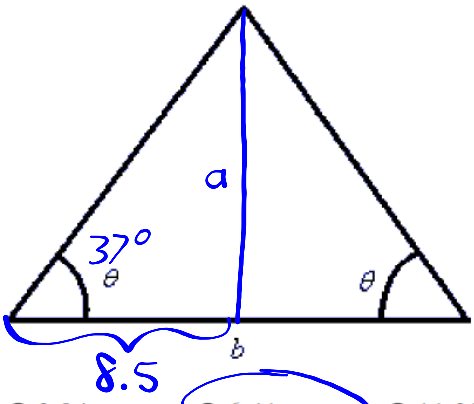


$\cos 29° = \frac{9}{c}$

$c = \frac{9}{\cos 29°} \approx 10.29$

ⓐ 10.29 ⓑ 4.99 ⓒ 18.56 ⓓ 4.36 ⓔ 16.24

79. Find the altitude of the isosceles triangle shown below if $\theta = 37°$ and $b = 17$ meters. Round answer to two decimal places.

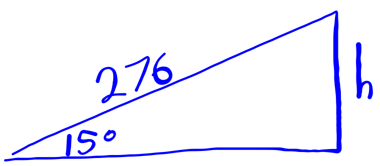


ⓐ 2.84 meters ⓑ 6.41 meters ⓒ 11.28 meters
ⓓ 12.81 meters ⓔ 5.12 meters

$\tan 37° = \frac{a}{8.5}$

$a = 8.5 \tan 37° \approx 6.4$

80. After leaving the runway, a plane's angle of ascent is 15° and its speed is 276 feet per second. How many minutes will it take for the airplane to climb to a height of 15,000 feet? Round answer to two decimal places.
ⓐ 1.93 minutes ⓑ 1.39 minutes ⓒ 3.50 minutes ⓓ 2.72 minutes ⓔ 0.91 minutes



$\sin 15° = \frac{h}{276}$

$h = 276 \sin 15°$

$h \approx 71.43$ ft/s

15000 ft
in ≈ 210 seconds

3.5 min