1. $$\frac{\sin^2 x}{1-\cos x} = \frac{1-\cos^2 x}{1-\cos x}$$

$$= \frac{(1+\cos x)\cancel{(1-\cos x)}}{\cancel{1-\cos x}}$$

$$= 1+\cos x$$

2 $x = 6\sin\theta$

$$\sqrt{36 - x^2}$$

$$= \sqrt{36 - (6\sin\theta)^2}$$

$$= \sqrt{36 - 36\sin^2\theta}$$

$$= \sqrt{36(1 - \sin^2\theta)}$$

$$= \sqrt{36\cos^2\theta}$$

$$= 6\cos\theta$$

4

$$\ln|\sin\theta| - \ln|\cos\theta|$$

$$\ln\left|\frac{\sin\theta}{\cos\theta}\right|$$

$$\ln|\tan\theta|$$

5

$$\sin^2 21^\circ + \sin^2 61^\circ + \sin^2 69^\circ + \sin^2 29^\circ$$

$$1 - \cancel{\cos^2 21^\circ} + 1 - \cancel{\cos^2 61^\circ} + \cancel{\sin^2 69^\circ} + \cancel{\sin^2 29^\circ}$$

$$1 + 1 = 2$$

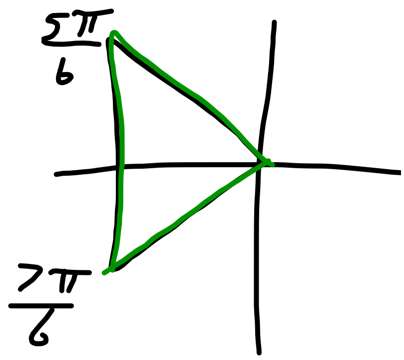
6 $\sec x = 2$

$\cos x = \frac{1}{2}$

$\frac{\pi}{3}$

$\frac{5\pi}{3}$

9) $\cos\frac{x}{2} = -\frac{\sqrt{3}}{2}$



$\frac{x}{2} = \frac{5\pi}{6} \pm 2\pi n$

$\frac{x}{2} = \frac{7\pi}{6} \pm 2\pi n$

$$\boxed{\begin{aligned} x &= \frac{5\pi}{3} \pm 4\pi n \\ x &= \frac{7\pi}{3} \pm 4\pi n \end{aligned}}$$

$\frac{x}{2} = \pm\frac{5\pi}{6} \pm 2\pi n$

$x = \pm\frac{5\pi}{3} \pm 4\pi n$

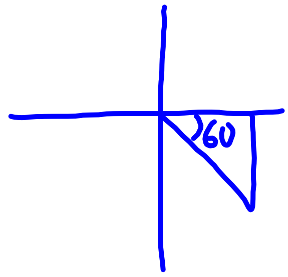
extra credit

(10) $\sin\frac{5\pi}{3}\cos\frac{\pi}{4} - \cos\frac{5\pi}{3}\sin\frac{\pi}{4}$

$$-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$\frac{-\sqrt{6}-\sqrt{2}}{4}$$

(11) $\sin 345^\circ = \sin(300^\circ + 45^\circ)$



$\sin 300^\circ \cos 45^\circ + \cos 300^\circ \sin 45^\circ$

$$-\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{-\sqrt{6} + \sqrt{2}}{4}$$

(12) $\cos \frac{19\pi}{12} = \cos(285^\circ) = \cos(240^\circ + 45^\circ)$

$\cos 240^\circ \cos 45^\circ - \sin 240^\circ \sin 45^\circ$

$-\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{-\sqrt{2} + \sqrt{6}}{4}$

(13) $\sin(\underbrace{\arcsin x}_{u} + \underbrace{\arccos x}_{v})$

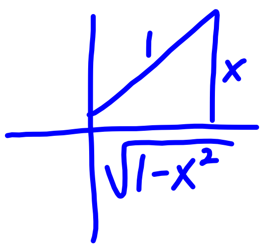
$$\sin(\arcsin x)\cos(\arccos x) + \cos(\arcsin x)\sin(\arccos x)$$

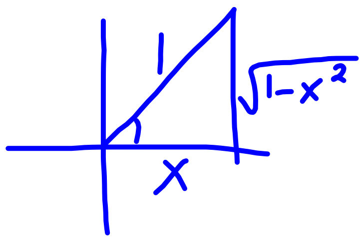
$$x \cdot x + \sqrt{1-x^2}\,\sqrt{1-x^2}$$

$$x^2 + \sqrt{1-x^2}^{\,2}$$

$$x^2 + 1 - x^2$$

$$\boxed{1}$$





(14) $\sin 2x = \sin x \quad [0, 2\pi)$

$2\sin x\cos x = \sin x$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = \frac{1}{2}$

$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

15) $\sin 4x = -2\sin 2x \quad [0, 2\pi)$

$\sin(2x+2x) = -2\sin 2x$

$\sin 2x\cos 2x + \cos 2x\sin 2x = -2\sin 2x$

$2\sin 2x\cos 2x = -2\sin 2x$

$2\sin 2x\cos 2x + 2\sin 2x = 0$

$2\sin 2x(\cos 2x + 1) = 0$

$2\sin 2x = 0 \quad \cos 2x = -1$

$\sin 2x = 0$

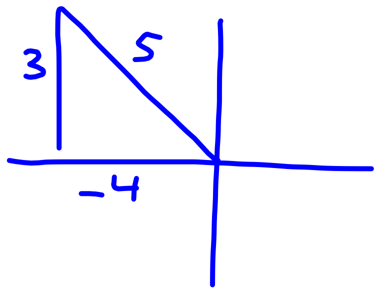
$2\sin x\cos x = 0$

$\sin x = 0 \quad \cos x = 0$

$2x = 0$ and π and $2\pi \quad 3\pi \quad 4\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

16 $\cos 2u \quad \sin u \ \frac{3}{5} \quad \frac{\pi}{2} < u < \pi$



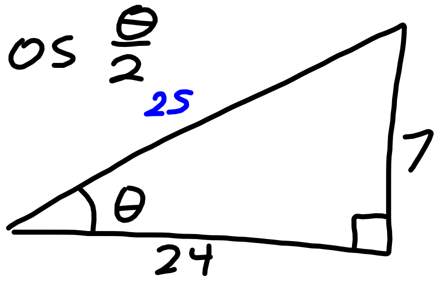
$\cos^2 u - \sin^2 u$

$\left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\frac{16}{25} - \frac{9}{25}$

$\boxed{\frac{7}{25}}$

17 $\cos\frac{\theta}{2}$



$$\pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\pm\sqrt{\frac{1+\left(\frac{24}{25}\right)}{2}}$$

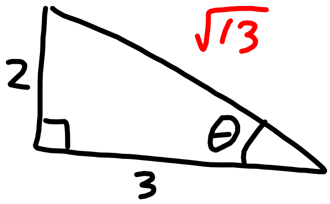
$$\pm\sqrt{\frac{\frac{49}{25}}{2}}$$

$$\pm\sqrt{\frac{49}{50}}$$

$$\pm\frac{7}{5\sqrt{2}}$$

$$\boxed{\frac{7\sqrt{2}}{10}}$$

(18) $\cos 2\theta$



$\cos^2\theta - \sin^2\theta$

$= \left(\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2$

$= \frac{9}{13} - \frac{4}{13}$

$\boxed{\frac{5}{13}}$

(19) $\tan\frac{3\pi}{8}$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\tan\left(\frac{\frac{3\pi}{4}}{2}\right) = \pm\sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}} = \pm\sqrt{\frac{1-\frac{-\sqrt{2}}{2}}{1+\frac{-\sqrt{2}}{2}}}$$

$$\pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}$$

$$\pm\sqrt{\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}}$$

$$\pm\sqrt{\frac{(2+\sqrt{2})^2}{4-2}}$$

$$\pm\frac{2+\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$$

$$\frac{2\sqrt{2}+2}{2}$$

$$\boxed{\sqrt{2}+1}$$

20 $\sqrt{\frac{1+\cos 12x}{2}}$ $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

if $\theta = 12x$,

then $\frac{\theta}{2} = \frac{12x}{2} = 6x$

(21) $\cos\frac{x}{2} - \sin x = 0$

$$\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = \sin x^2$$

$$\frac{1+\cos x}{2} = \sin^2 x$$

$$1+\cos x = 2(\sin^2 x)$$

$$1+\cos x = 2(1-\cos^2 x)$$

$$1+\cos x = 2-2\cos^2 x$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

(22) $6 \sin \underset{a}{\frac{\pi}{10}} \cos \underset{b}{\frac{\pi}{10}}$

$\frac{1}{2}\left[\sin(a+b)+\sin(a-b)\right]$

$6 \cdot \frac{1}{2}\left[\sin\frac{\pi}{5}+\sin 0\right]$

$3\left[\sin\frac{\pi}{5}+0\right]$

$\boxed{3\sin\frac{\pi}{5}}$

23 $\sin 4\theta - \sin 2\theta$

$2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$

$2\cos\left(\frac{4\theta+2\theta}{2}\right)\sin\left(\frac{4\theta-2\theta}{2}\right)$

$2\cos(3\theta)\sin\theta$