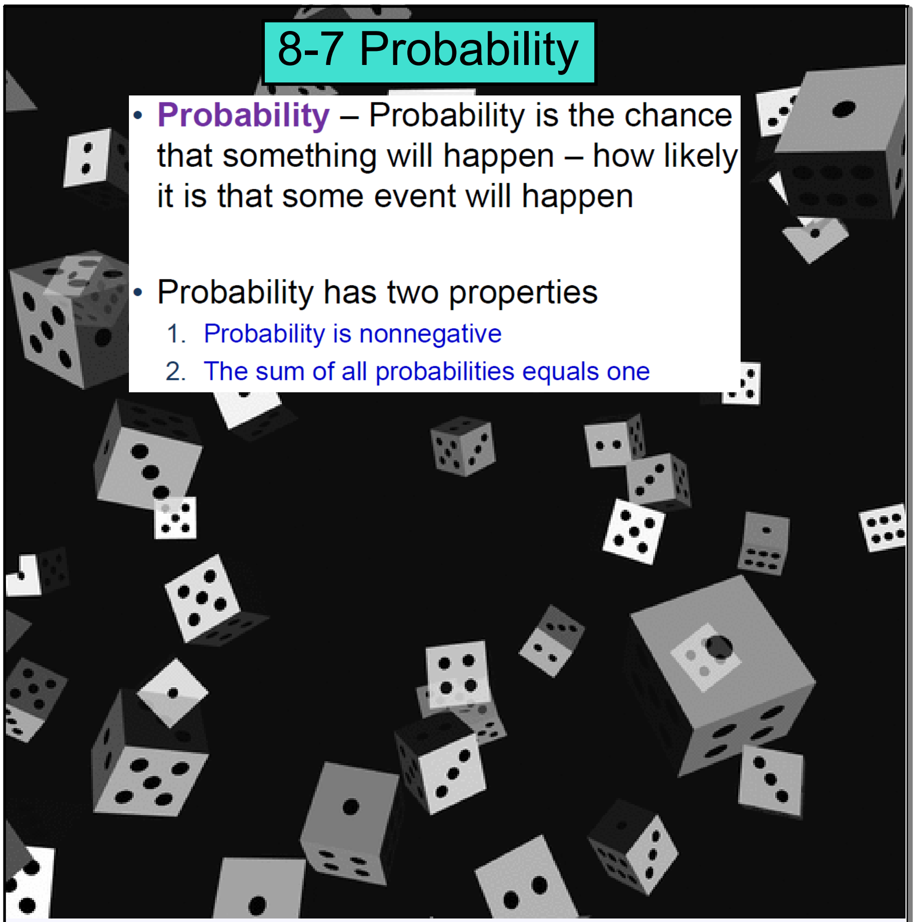


8-7 Probability

- **Probability** – Probability is the chance that something will happen – how likely it is that some event will happen

- Probability has two properties
  1. Probability is nonnegative
  2. The sum of all probabilities equals one

In a bag of M&Ms,™ the candies are colored red, green, blue, brown, yellow, and orange. A candy is drawn from the bag and the color is recorded. The sample space of this experiment is {red, green, blue, brown, yellow, orange}. Determine which of the following are probability models.

(a)

Outcome	Probability
red	0.3
green	0.15
blue	0
brown	0.15
yellow	0.2
orange	0.2

(b)

Outcome	Probability
red	0.1
green	0.1
blue	0.1
brown	0.4
yellow	0.2
orange	0.3

(c)

Outcome	Probability
red	0.3
green	−0.3
blue	0.2
brown	0.4
yellow	0.2
orange	0.2

(d)

Outcome	Probability
red	0
green	0
blue	0
brown	0
yellow	1
orange	0

In a bag of M&Ms,[TM] the candies are colored red, green, blue, brown, yellow, and orange. A candy is drawn from the bag and the color is recorded. The sample space of this experiment is {red, green, blue, brown, yellow, orange}. Determine which of the following are probability models.

(a)

Outcome	Probability
red	0.3
green	0.15
blue	0
brown	0.15
yellow	0.2
orange	0.2

(b)

Outcome	Probability
red	0.1
green	0.1
blue	0.1
brown	0.4
yellow	0.2
orange	0.3

Sums to more than one

(c)

Outcome	Probability
red	0.3
green	−0.3
blue	0.2
brown	0.4
yellow	0.2
orange	0.2

Negative probability

(d)

Outcome	Probability
red	0
green	0
blue	0
brown	0
yellow	1
orange	0

Probability models

Constructing a Probability Model

An experiment consists of rolling a fair die once. A die is a cube with each face having either 1, 2, 3, 4, 5, or 6 dots on it. Construct a probability model for this experiment.

A sample space S consists of all the possibilities that can occur. Because rolling the die will result in one of six faces showing, the sample space S consists of

$$S = \{1, 2, 3, 4, 5, 6\}$$

Constructing a Probability Model

An experiment consists of tossing a coin. The coin is weighted so that heads (H) is three times as likely to occur as tails (T). Construct a probability model for this experiment.

The sample space S is $S = \{H, T\}$. If x denotes the probability that a tail occurs,

$$P(T) = x \quad \text{and} \quad P(H) = 3x$$

Probability for Equally Likely Outcomes

If an experiment has n equally likely outcomes and if the number of ways that an event E can occur is m, then the probability of E is

$$P(E) = \frac{\text{Number of ways that } E \text{ can occur}}{\text{Number of possible outcomes}} = \frac{m}{n}$$

If S is the sample space of this experiment,

$$P(E) = \frac{n(E)}{n(S)}$$

Calculating Probabilities of Events Involving Equally Likely Outcomes

Calculate the probability that in a 3-child family there are 2 boys and 1 girl. Assume equally likely outcomes.

EXAMPLE

Rolling Two Dice

Consider an experiment of rolling two dice. A convenient sample space that will enable us to answer many questions about events of interest is shown in Figure 1. Let S be the set of all ordered pairs listed in the figure. Note that the simple event (3, 2) is to be distinguished from the simple event (2, 3). The former indicates a 3 turned up on the first die and a 2 on the second, whereas the latter indicates a 2 turned up on the first die and a 3 on the second. What is the event that corresponds to each of the following outcomes?

(A) A sum of 7 turns up.

(B) A sum of 11 turns up.

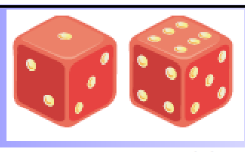
(C) A sum less than 4 turns up.

(D) A sum of 12 turns up.

SECOND DIE

FIRST DIE	⚀	⚁	⚂	⚃	⚄	⚅
⚀	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
⚁	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
⚂	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
⚃	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
⚄	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
⚅	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

A sample space for rolling two dice.



(A) A sum of 7 turns up. (B) A sum of 11 turns up.

(C) A sum less than 4 turns up. (D) A sum of 12 turns up.

a=

b=

c=

d=

Probability of the Union of Two Events

If A and B are events in the same sample space, the probability of A *or* B occurring is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B).$$

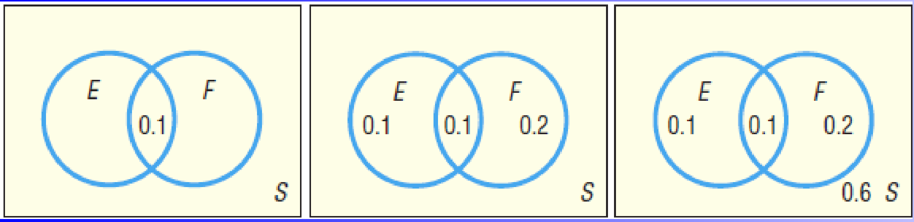
EXAMPLE

Computing Probabilities of the Union of Two Events

If $P(E) = 0.2$, $P(F) = 0.3$, and $P(E \cap F) = 0.1$, find the probability of E or F. That is, find $P(E \cup F)$.

Probability of E or F $= P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.2 + 0.3 - 0.1 = 0.4$



EXAMPLE

Computing Probabilities of the Union of Two Mutually Exclusive Events

If $P(E) = 0.4$ and $P(F) = 0.25$, and E and F are mutually exclusive, find $P(E \cup F)$.

Since E and F are mutually exclusive,

$P(E \cup F) = P(E) + P(F) = 0.4 + 0.25 = 0.65$

Probability of Independent Events

If A and B are **independent events,** the probability that both A and B will occur is given by

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

A random number generator on a computer selects three integers from 1 to 20. What is the probability that all three numbers are less than or equal to 5?

Solution

The probability of selecting a number from 1 to 5 is

$$P(A) = \frac{5}{20} = \frac{1}{4}.$$

So, the probability that all three numbers are less than or equal to 5 is

$$P(A) \cdot P(A) \cdot P(A) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{64}.$$

In 2004, approximately 65% of the population of the United States was 25 years old or older. In a survey, 10 people were chosen at random from the population. What is the probability that all 10 were 25 years old or older? (Source: U.S. Census Bureau)

Solution

Let A represent choosing a person who was 25 years old or older. The probability of choosing a person who was 25 years old or older is 0.65, the probability of choosing a second person who was 25 years old or older is 0.65, and so on. Because these events are independent, you can conclude that the probability that all 10 people were 25 years old or older is

$$[P(A)]^{10} = (0.65)^{10} \approx 0.01.$$

Probability of a Complement

Let A be an event and let A' be its complement. If the probability of A is $P(A)$, then the probability of the complement is given by

$$P(A') = 1 - P(A).$$

EXAMPLE

Computing Probabilities Using Complements

On the local news the weather reporter stated that the probability of rain tomorrow is 40%. What is the probability that it will not rain?

The complement of the event "rain" is "no rain."

$$P(\text{no rain}) = 1 - P(\text{rain}) = 1 - 0.4 = 0.6$$

There is a 60% chance of no rain tomorrow.

A manufacturer has determined that a machine averages one faulty unit for every 1000 it produces. What is the probability that an order of 200 units will have one or more faulty units?

Solution

To solve this problem as stated, you would need to find the probabilities of having exactly one faulty unit, exactly two faulty units, exactly three faulty units, and so on. However, using complements, you can simply find the probability that all units are perfect and then subtract this value from 1. Because the probability that any given unit is perfect is 999/1000, the probability that all 200 units are perfect is

$$P(A) = \left(\frac{999}{1000}\right)^{200}$$

$$\approx 0.82.$$

So, the probability that at least one unit is faulty is

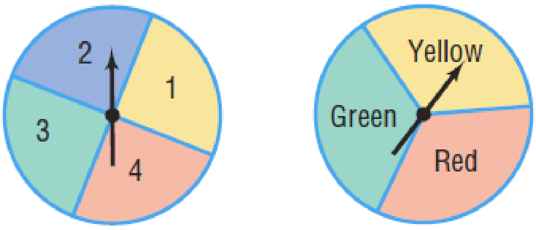
$$P(A') = 1 - P(A)$$

$$\approx 0.18.$$

Exploration

You are in a class with 22 other people. What is the probability that at least two out of the 23 people will have a birthday on the same day of the year? What if you know the probability of everyone having the same birthday? Do you think this information would help you to find the answer?

use the following spinners to construct a probability model for each experiment.

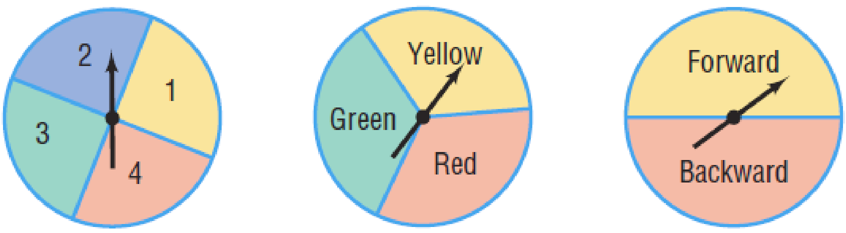


Spinner I
(4 equal areas)

Spinner II
(3 equal areas)

Spin the first spinner and then the second. What is the probability of getting a 2 or a 4, followed by red?

Spin spinner I, then II, then III. What is the probability of getting a 1, followed by Red or Green, followed by Backward?



Spinner I
(4 equal areas)

Spinner II
(3 equal areas)

Spinner III
(2 equal areas)

an urn contains 5 white marbles, 10 green marbles, 8 yellow marbles, and 7 black marbles.

If one marble is selected, determine the probability that it is white.

assume equally likely outcomes

Determine the probability of having 3 boys in a 3-child family.

Determine the probability of having 1 girl and 3 boys in a 4-child family.

find the probability of the indicated event if $P(A) = 0.25$ *and* $P(B) = 0.45$.

$P(A \cup B)$ if $P(A \cap B) = 0.15$

$P(A \cup B)$ if A, B are mutually exclusive

If $P(A) = 0.60$, $P(A \cup B) = 0.85$, and $P(A \cap B) = 0.05$, find $P(B)$.

Hw: p 645, # 1 – 77 eoo